CHAPTER - 1

RELATIONS AND FUNCTIONS

Exercise - 1.1

1. Find $A \times B$, $A \times A$ and $B \times A$

(i) $A = \{2, -2, 3\}$ and $B = \{1, -4\}$

★ $A \times B = \{2, -2, 3\} \times \{1, -4\}$

$$= \{(2,1), (2,-4), (-2,1), (-2,-4), (3,1), (3,-4)\}$$

★ $A \times A = \{2, -2, 3\} \times \{2, -2, 3\}$

$$= \{(2,2), (2,-2), (2,3), (-2,2), (-2,-2), (-2,3), (3,2), (3,-2), (3,3)\}$$

★ $B \times A = \{1, -4\} \times \{2, -2, 3\}$

$$= \{(1,2), (1,-2), (1,3), (-4,2), (-4,-2), (-4,3)\}$$

(ii) $A = B = \{p, q\}$

Sol:-

* $A \times B = \{p, q\} \times \{p, q\}$.

  $A \times B = \{(p,p), (p,q), (q,p), (q,q)\}$

Since $A = \{p, q\}$, $B = \{p, q\}$

* $A \times A = B \times A = \{(p,p), (p,q) (q,p), (q,q)\}$

(iii) $A = \{m, n\}$, $B = \phi = \{\ \}$

Sol:-

* $A \times B = \{m, n\} \times \{\ \}$

  $A \times B = \{\ \}$.

* $A \times A = \{m, n\} \times \{m, n\}$

  $A \times A = \{(m,m), (m,n), (n,m), (n,n)\}$

* $B \times A = \{\ \} \times \{m, n\}$

  $B \times A = \{\ \}$

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2. Let $A = \{1, 2, 3\}$ and $B = \{x / x$ is a prime number less than $10\}$. Find $A \times B$ and $B \times A$.

Sol:-

$A = \{1, 2, 3\}$

$B = \{2, 3, 5, 7\}$

★ $A \times B = \{1, 2, 3\} \times \{2, 3, 5, 7\}$

$$\boxed{\begin{aligned} A \times B = \{&(1,2), (1,3), (1,5), (1,7) \\ &(2,2), (2,3), (2,5), (2,7) \\ &(3,2), (3,3), (3,5), (3,7)\} \end{aligned}}$$

★ $B \times A = \{2, 3, 5, 7\} \times \{1, 2, 3\}$

$$\boxed{\begin{aligned} B \times A = \{&(2,1) (2,2), (2,3), (3,1), (3,2), (3,3) \\ &(5,1), (5,2), (5,3), (7,1), (7,2), (7,3)\} \end{aligned}}$$

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3. If $B \times A = \{(-2,3),(-2,4),(0,3),(0,4),(3,3),(3,4)\}$ find A and B.

Sol:-

$B \times A = \{(-2,3),(-2,4),(0,3),(0,4),(3,3),(3,4)\}$

$B = \{-2, 0, 3\}$

$A = \{3, 4\}$

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4. If $A = \{5,6\}$, $B = \{4,5,6\}$, $C = \{5,6,7\}$ Show that $A \times A = (B \times B) \cap (C \times C)$

Sol:

$A = \{5, 6\}$

$B = \{4, 5, 6\}$

$C = \{5, 6, 7\}$

To Prove: $A \times A = (B \times B) \cap (C \times C)$

LHS: $A \times A$

$A \times A = \{5,6\} \times \{5,6\}$

$\boxed{A \times A = \{(5,5),(5,6),(6,5),(6,6)\}} \rightarrow ①$

RHS: $(B \times B) \cap (C \times C)$

$B \times B = \{4, 5, 6\} \times \{4, 5, 6\}$

$B \times B = \{(4,4), (4,5), (4,6),$
$(5,4), (5,5), (5,6),$
$(6,4), (6,5), (6,6)\}$

$C \times C = \{5, 6, 7\} \times \{5, 6, 7\}$

$C \times C = \{(5,5), (5,6), (5,7),$
$(6,5), (6,6), (6,7),$
$(7,5), (7,6), (7,7)\}$

$$\boxed{(B \times B) \cap (C \times C) = \{(5,5), (5,6), (6,5), (6,6)\}} \to ②$$

① = ②

∴ LHS = RHS

$$\therefore \boxed{A \times A = (B \times B) \cap (C \times C)}$$

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5. Given $A = \{1, 2, 3\}$, $B = \{2, 3, 5\}$, $C = \{3, 4\}$ and $D = \{1, 3, 5\}$, Check if $(A \cap C) \times (B \cap D) = (A \times B) \cap (C \times D)$ is true?

Sol:

$$\boxed{\begin{aligned} A &= \{1, 2, 3\} \\ B &= \{2, 3, 5\} \\ C &= \{3, 4\} \\ D &= \{1, 3, 5\}. \end{aligned}}$$

$$(A \cap C) \times (B \cap D) = (A \times B) \cap (C \times D)$$

LHS:- $(A \cap C) \times (B \cap D)$

$$A \cap C = \{1, 2, 3\} \cap \{3, 4\}$$

$$A \cap C = \{3\}$$

$$B \cap D = \{2, 3, 5\} \cap \{1, 3, 5\}$$

$$B \cap D = \{3, 5\}$$

$$(A \cap C) \times (B \cap D) = \{3\} \times \{3, 5\}$$

$$\boxed{(A \cap C) \times (B \cap D) = \{(3, 3)\ (3, 5)\}} \longrightarrow ①$$

R.H.S : $(A\times B)\cap(C\times D)$

$A\times B = \{1,2,3\}\times\{2,3,5\}$

$A\times B = \{(1,2),(1,3),(1,5),(2,2),(2,3)$
$((2,5),(3,2),(3,3),(3,5)\}$

$C\times D = \{3,4\}\times\{1,3,5\}$

$= \{(3,1),(3,3),(3,5),(4,1),(4,3),(4,5)\}$

$$\boxed{(A\times B)\cap(C\times D) = \{(3,3),(3,5)\}} \longrightarrow ②$$

$$① = ②$$

$$\therefore \boxed{(A\cap C)\times(B\cap D) = (A\times B)\cap(C\times D)}$$

∴ It is true.

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6. Let $A = \{x\in W / x<2\}$
$B = \{x\in N / 1<x\le 4\}$ and $C=\{3,5\}$
Verify that
(i) $A\times(B\cup C) = (A\times B)\cup(A\times C)$

Sol:

$A = \{0, 1\}$

$B = \{2, 3, 4\}$

$C = \{3, 5\}$

(i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

LHS: $A \times (B \cup C)$

$B \cup C = \{2, 3, 4\} \cup \{3, 5\}$

$B \cup C = \{2, 3, 4, 5\}$

$A \times (B \cup C) = \{0, 1\} \times \{2, 3, 4, 5\}$

$$A \times (B \cup C) = \{(0,2), (0,3), (0,4), (0,5) \\ (1,2), (1,3), (1,4), (1,5)\} \rightarrow ①$$

R.H.S: $(A \times B) \cup (A \times C)$

$A \times B = \{0, 1\} \times \{2, 3, 4\}$

$A \times B = \{(0, 2), (0, 3), (0, 4), (1, 2), (1, 3), (1, 4)\}$

$A \times C = \{0, 1\} \times \{3, 5\}$

$A \times C = \{(0, 3), (0, 5), (1, 3), (1, 5)\}$

$$(A \times B) \cup (A \times C) = \{(0,2), (0,3), (0,4), (0,5), (1,2), \\ (1,3), (1,4), (1,5)\} \rightarrow ②$$

$$① = ②$$

$$\therefore \boxed{A\times(B\cup C) = (A\times B)\cup(A\times C)}$$

(ii) $A\times(B\cap C) = (A\times B)\cap(A\times C)$

$A = \{0,1\}$

$B = \{2,3,4\}$

$C = \{3,5\}$

LHS:- $A\times(B\cap C)$

$B\cap C = \{2,3,4\}\cap\{3,5\}$

$B\cap C = \{3\}$

$A\times(B\cap C) = \{0,1\}\times\{3\}$

$\boxed{A\times(B\cap C) = \{(0,3),\ (1,3)\}} \rightarrow ①$

RHS:- $(A\times B)\cap(A\times C)$

$A\times B = \{0,1\}\times\{2,3,4\}$

$A\times B = \{(0,2),(0,3),(0,4),(1,2),(1,3),(1,4)\}$

$A \times C = \{0,1\} \times \{3,5\}$

$A \times C = \{(0,3),(0,5),(1,3),(1,5)\}$

$\boxed{(A \times B) \cap (A \times C) = \{(0,3),(1,3)\}} \rightarrow$ ②

① = ②

$\therefore \boxed{A \times (B \cap C) = (A \times B) \cap (A \times C)}$

(iii) $(A \cup B) \times C = (A \times C) \cup (B \times C)$

$A = \{0,1\}$

$B = \{2,3,4\}$

$C = \{3,5\}$

LHS :- $(A \cup B) \times C$

$A \cup B = \{0,1\} \cup \{2,3,4\}$

$A \cup B = \{0,1,2,3,4\}$

$(A \cup B) \times C = \{0,1,2,3,4\} \times \{3,5\}$

$$\boxed{\begin{aligned}(A \cup B) \times C = \{&(0,3),(0,5),(1,3),(1,5)\\ &(2,3),(2,5),(3,3),(3,5)\\ &(4,3),(4,5)\}\end{aligned}} \rightarrow \text{①}$$

R.H.S :- $(A\times C) \cup (B\times C)$

$A\times C = \{0,1\} \times \{3,5\}$

$A\times C = \{(0,3),(0,5),(1,3),(1,5)\}$

$B\times C = \{2,3,4\} \times \{3,5\}$

$B\times C = \{(2,3),(2,5),(3,3),(3,5),$
$(4,3),(4,5)\}$

$$\boxed{(A\times C)\cup(B\times C) = \{(0,3),(0,5),(1,3),(1,5),(2,3),(2,5),(3,3),(3,5),(4,3),(4,5)\}} \longrightarrow ②$$

① = ②

$$\boxed{(A\cup B)\times C = (A\times C)\cup(B\times C)}$$

7. Let A = The Set of all natural numbers less than 8.

B = The Set of all Prime numbers less than 8.

C = The Set of even Prime number.

Verify that :-

Sol :-

$A = \{1, 2, 3, 4, 5, 6, 7\}$

$B = \{2, 3, 5, 7\}$

$C = \{2\}$

(i) $(A \cap B) \times C = (A \times C) \cap (B \times C)$

LHS :- $(A \cap B) \times C$

$A \cap B = \{1, 2, 3, 4, 5, 6, 7\} \cap \{2, 3, 5, 7\}$

$A \cap B = \{2, 3, 5, 7\}$

$$(A \cap B) \times C = \{2, 3, 5, 7\} \times \{2\}$$

$$\boxed{(A \cap B) \times C = \{(2,2), (3,2), (5,2), (7,2)\}} \hookrightarrow ①$$

RHS: - $(A \times C) \cap (B \times C)$

$$A \times C = \{1, 2, 3, 4, 5, 6, 7\} \times \{2\}$$

$$A \times C = \{(1,2), (2,2), (3,2), (4,2), (5,2), (6,2), (7,2)\}$$

$$B \times C = \{2, 3, 5, 7\} \times \{2\}$$

$$B \times C = \{(2,2), (3,2), (5,2), (7,2)\}$$

$$\boxed{(A \times C) \cap (B \times C) = \{(2,2), (3,2), (5,2), (7,2)\}} \hookrightarrow ②$$

① = ②

$$\boxed{(A \cap B) \times C = (A \times C) \cap (B \times C)}$$

(ii) $A \times (B - C) = (A \times B) - (A \times C)$

$A = \{1, 2, 3, 4, 5, 6, 7\}$

$B = \{2, 3, 5, 7\}$

$C = \{2\}.$

$\underline{\underline{LHS}} = A \times (B - C)$

$B - C = \{\cancel{2}, 3, 5, 7\} - \{\cancel{2}\}$

$B - C = \{3, 5, 7\}$

$A \times (B - C) = \{1, 2, 3, 4, 5, 6, 7\} \times \{3, 5, 7\}$

$$\boxed{\begin{aligned} A \times (B - C) = \{&(1,3), (1,5), (1,7) \\ &(2,3), (2,5), (2,7) \\ &(3,3), (3,5), (3,7) \\ &(4,3), (4,5), (4,7) \\ &(5,3), (5,5), (5,7) \\ &(6,3), (6,5), (6,7) \\ &(7,3), (7,5), (7,7)\} \rightarrow ① \end{aligned}}$$

$\underline{\underline{RHs}}:- (A \times B) - (A \times C)$

$A \times B = \{1, 2, 3, 4, 5, 6, 7\} \times \{2, 3, 5, 7\}$

$$
\begin{aligned}
= \{ & (1,2), (1,3), (1,5), (1,7) \\
& (2,2)\ (2,3)\ (2,5)\ (2,7) \\
& (3,2)\ (3,3)\ (3,5)\ (3,7) \\
& (4,2)\ (4,3)\ (4,5)\ (4,7) \\
& (5,2)\ (5,3)\ (5,5)\ (5,7) \\
& (6,2)\ (6,3)\ (6,5)\ (6,7) \\
& (7,2)\ (7,3)\ (7,5)\ (7,7) \}
\end{aligned}
$$

$$
\begin{aligned}
A \times C &= \{1, 2, 3, 4, 5, 6, 7\} \times \{2\} \\
&= \{(1,2), (2,2), (3,2), (4,2), (5,2), (6,2), (7,2)\}
\end{aligned}
$$

$$
\boxed{
\begin{aligned}
(A \times B) - (A \times C) = \{ & (1,3)\ (1,5)\ (1,7) \\
& (2,3)\ (2,5)\ (2,7) \\
& (3,3)\ (3,5)\ (3,7) \\
& (4,3)\ (4,5)\ (4,7) \\
& (5,3)\ (5,5)\ (5,7) \\
& (6,3)\ (6,5)\ (6,7) \\
& (7,3)\ (6,5)\ (6,7) \}
\end{aligned}
} \longrightarrow ②
$$

$$① = ②$$

$$\therefore A \times (B - C) = (A \times B) - (A \times C)$$

Exercise- 1.2

① Let $A = \{1, 2, 3, 7\}$ and $B = \{3, 0, -1, 7\}$, which of the following are relation from A to B ?

(i) $R_1 = \{(2,1), (7,1)\}$

Sol:
$A = \{1, 2, 3, 7\}$
$B = \{3, 0, -1, 7\}$

Since '1' is not an element of Co-domain B
R_1 is not a relation from A to B.

(ii) $R_2 = \{(-1,1)\}$

Sol:

Since '-1' is not an element of Domain 'A' and '1' is not an element of Co-domain 'B'
R_2 is not a relation from A to B.

(iii) $R_3 = \{(2,-1), (7,7), (1,3)\}$

Sol:

R_3 is a relation from A to B.

(iv) $R_4 = \{(7,-1), (0,3), (3,3), (0,7)\}$

Sol: Since '0' is not an element of Domain A.

R_4 is not a relation from A to B.

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② Let $A = \{1,2,3,4,\ldots,45\}$ and R be the relation defined as "is square of a number" on A. Write R as a Subset of $A \times A$. Also, find the domain and range of R.

Sol:

$A = \{1, 2, 3, 4, \ldots, 45\}$

$R \subseteq A \times A$

R = Square of a number on A

$\therefore R = \{(1,1), (2,4), (3,9), (4,16), (5,25), (6,36)\}$

$\therefore$ Domain $= \{1, 2, 3, 4, 5, 6\}$

Range $= \{1, 4, 9, 16, 25, 36\}$

---

③ A Relation R is given by the Set $\{(x, y) / y = x + 3,\ x \in \{0, 1, 2, 3, 4, 5\}\}$. Determine its domain and range.

Sol:

$$R = \{(x, y) / y = x + 3\}$$

$$x \in \{0, 1, 2, 3, 4, 5\}$$

$$y = x + 3$$

$x = 0;\quad y = 0 + 3 = 3$

$x = 1;\quad y = 1 + 3 = 4$

$x = 2;\quad y = 2 + 3 = 5$

$x = 3;\quad y = 3 + 3 = 6$

$x = 4;\quad y = 4 + 3 = 7$

$x = 5;\quad y = 5 + 3 = 8$

$\therefore R = \{(0,3), (1,4), (2,5), (3,6), (4,7), (5,8)\}$

Domain $= \{0, 1, 2, 3, 4, 5\}$

Range $= \{3, 4, 5, 6, 7, 8\}$

---

④ Represent each of the given relations by (a) an arrow diagram

(b) a graph and

(c) a set of roster form, wherever possible.

(i) $\{(x,y) / x = 2y,\ x \in \{2,3,4,5\}$, and $y \in \{1,2,3,4\}$

Sol:

$$x = 2y$$

$$y = \frac{x}{2}$$

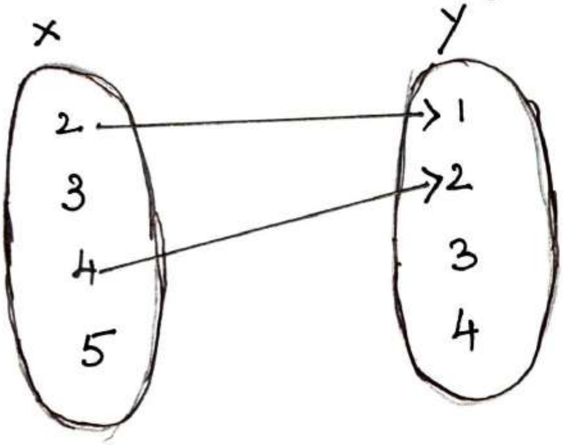
$x = 2;\quad y = \frac{\cancel{2}^{1}}{\cancel{2}} = 1 \in y$

$x = 3$; $y = \frac{3}{2} \notin y$

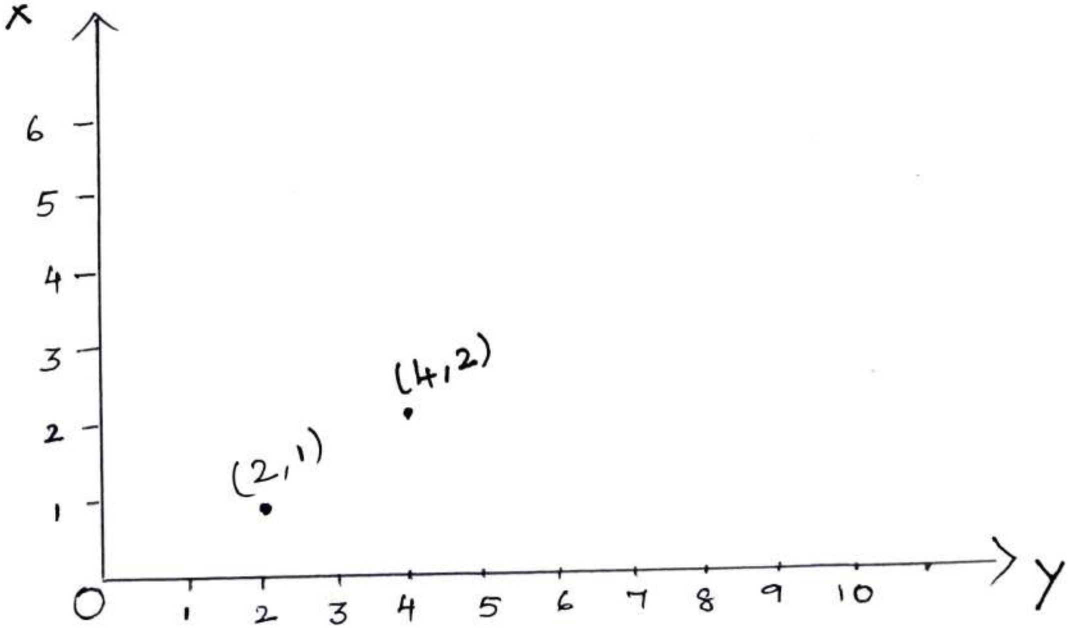
$x = 4$; $y = \frac{\cancel{4}^{2}}{\cancel{2}} = 2 \in y$

$x = 5$; $y = \frac{5}{2} \notin y.$

(a) An arrow diagram



(b) A graph



(c) A Set in roster form

$$R = \{(2,1), (4,2)\}$$

(ii) $\{(x, y) / y = x+3$, x, y are Natural numbers less than 10$\}$

Sol:

$$x = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

$$y = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

$$\boxed{y = x+3}$$

$x = 1$; $y = 1+3 = 4$

$x = 2$; $y = 2+3 = 5$

$x = 3$; $y = 3+3 = 6$

$x = 4$; $y = 4+3 = 7$

$x = 5$; $y = 5+3 = 8$

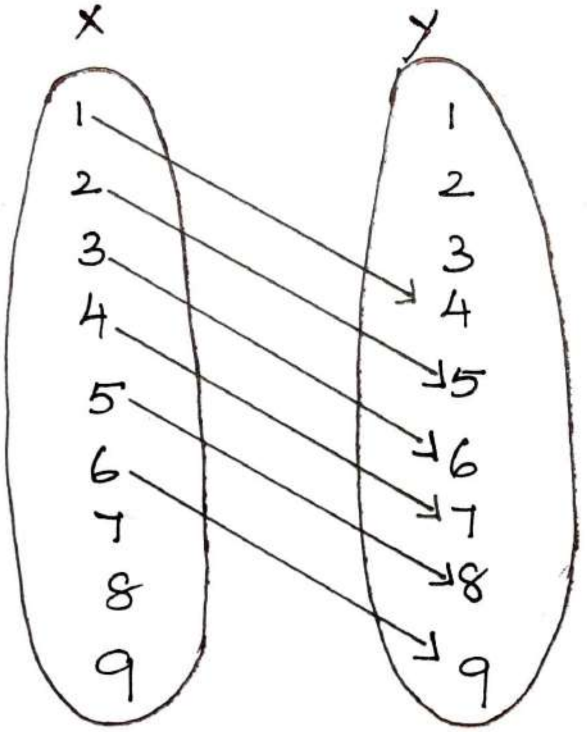
$x = 6$; $y = 6+3 = 9$

$x = 7$; $y = 7+3 = 10$

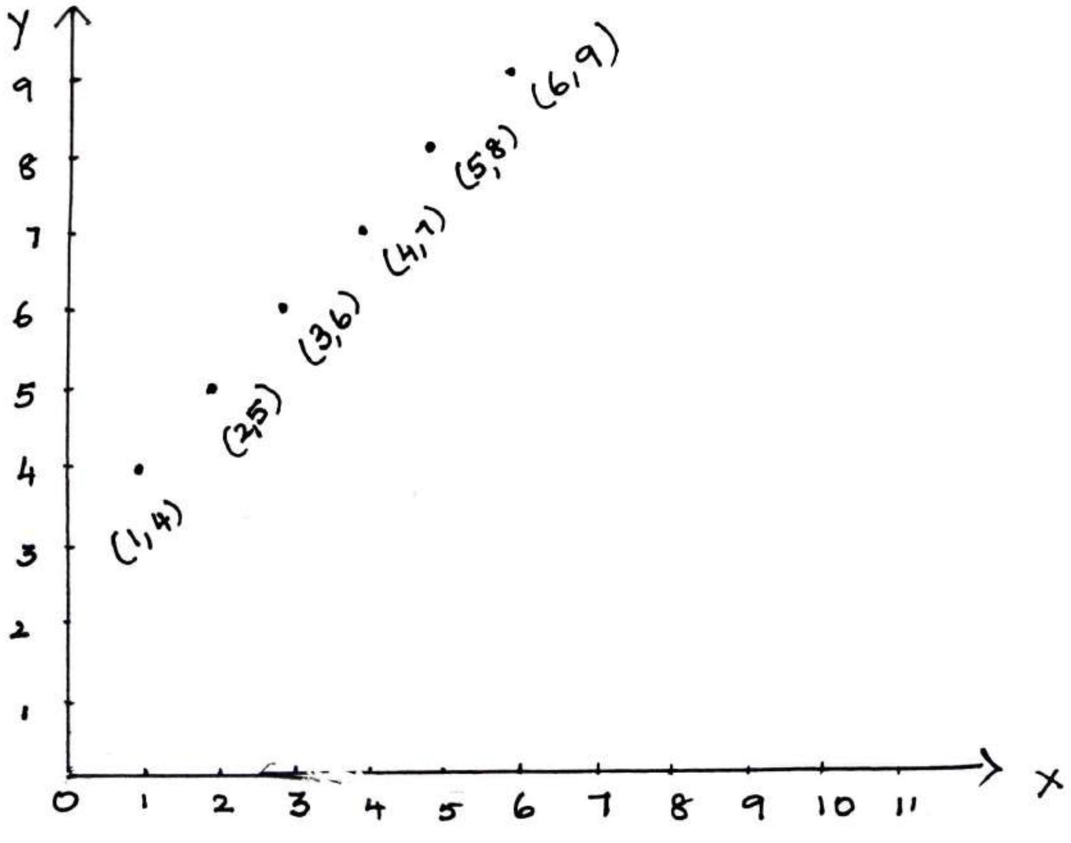
$x = 8$; $y = 8+3 = 11$

$x = 9$; $y = 9+3 = 12$

(a) An Arrow diagram



(b) A graph



(c) A Set in roster form

$R = \{(1,4),(2,5),(3,6),(4,7),(5,8),(6,9)\}$

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⑤ A company has four categories of employees given by Assistants (A), Clerks (C), Managers (M) and an Executive Officer (E). The Company Provide ₹ 10,000, ₹25,000, ₹50,000 and ₹1,00,000 as Salaries to the People who work in the categories A, C, M and E respectively. If A_1, A_2, A_3, A_4 and A_5 were Assistants; C_1, C_2, C_3, C_4, were Clerks; M_1, M_2, M_3 were managers and E_1, E_2 were Executive officers and if the relation R is defined by xRy, where x is the salary given to person y, express the relation R through an Ordered pair and an arrow diagram.

Sol :-

Assistants = A_1, A_2, A_3, A_4, A_5

Clerks = C_1, C_2, C_3, C_4

Managers = M_1, M_2, M_3

Executive Officers = E_1, E_2

Salary :-

Assistants = ₹10,000

Clerks = ₹25,000

Managers = ₹50,000

Executive Officers = ₹1,00,000

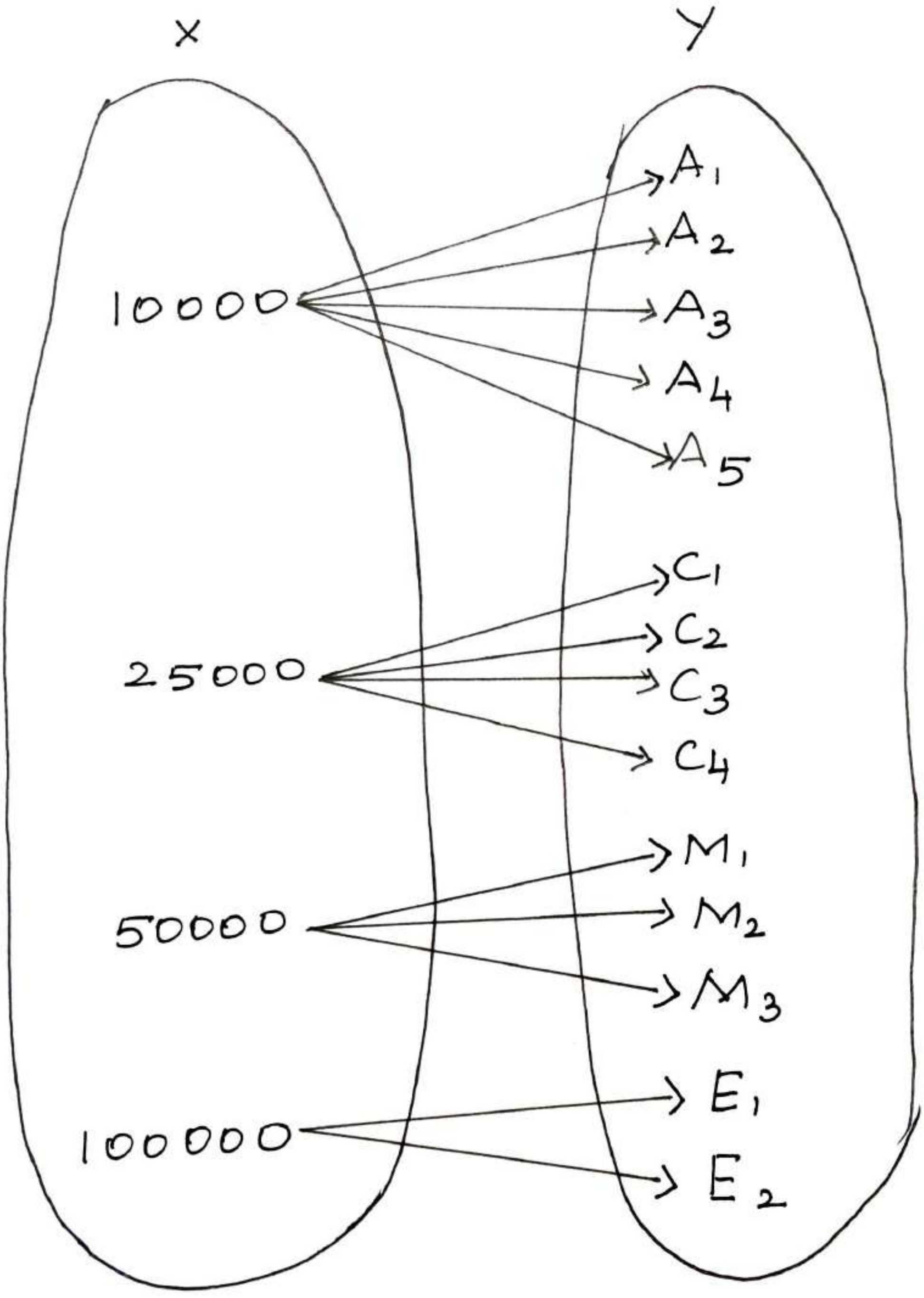
X = Salary

y = Categories of Employee.

(a) Ordered pair:

$\{(10{,}000, A_1); (10{,}000, A_2), (10{,}000, A_3), (10{,}000, A_4); (10{,}000, A_5), (25{,}000, C_1), (25000, C_2), (25000, C_3), (25000, C_4), (50000, M_1), (50000, M_2)(50000, M_3) (100000, E_1) (1{,}00000, E_2)\}$

(b) An Arrow diagram



Exercise - 1.3

① Let $f = \{(x, y) / x, y \in N \text{ and } y = 2x\}$ be a relation on N. Find the domain, co-domain and range. Is this relation a function?

Sol:-

$$x, y \in N$$

$$x = y = 1, 2, 3, 4, 5, 6 \cdots\cdots$$

$$\boxed{y = 2x}$$

$x = 1$; $y = 2(1) = 2$

$x = 2$; $y = 2(2) = 4$

$x = 3$; $y = 2(3) = 6$

So on

Domain $= \{1, 2, 3, 4, \cdots\}$

Co-Domain $= \{1, 2, 3, 4, \cdots\}$

Range $= \{2, 4, 6, 8, \cdots\}$

Yes, the relation is a function.

② Let $X = \{3, 4, 6, 8\}$. Determine whether the relation $R = \{(x, f(x)) / x \in X, f(x) = x^2+1\}$ is a function from X to N?

Sol:-

$$X = \{3, 4, 6, 8\}$$

$$R = \{(x, f(x)) / x \in X ; f(x) = x^2+1\}$$

$$\boxed{f(x) = x^2+1}$$

$x = 3$; $f(3) = 3^2+1 = 9+1 = 10$

$x = 4$; $f(4) = 4^2+1 = 16+1 = 17$

$x = 6$; $f(6) = 6^2+1 = 36+1 = 37$

$x = 8$; $f(8) = 8^2+1 = 64+1 = 65$

$R = \{(3, 10), (4, 17), (6, 37), (8, 65)\}$

Yes ; R is a function from X to N.

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③ Given the function $f : x \rightarrow x^2 - 5x + 6$; evaluate

(i) $f(-1)$

Sol:-

$$f(x) = x^2 - 5x + 6$$

$$f(-1) = (-1)^2 - 5(-1) + 6$$

$$= 1 + 5 + 6$$

$$\boxed{f(-1) = 12}$$

(ii) $f(2a)$

Sol:-

$$f(x) = x^2 - 5x + 6$$

$$f(2a) = (2a)^2 - 5(2a) + 6$$

$$\boxed{f(2a) = 4a^2 - 10a + 6}$$

(iii) $f(2)$

Sol:-

$$f(x) = x^2 - 5x + 6$$

$$f(2) = 2^2 - 5(2) + 6$$

$$= \overleftrightarrow{4 - 10 + 6}$$

$$\boxed{f(2) = 0}$$

R.W

$4+6 = 10$

$10-10 = 0$

(iv) $f(x-1)$

Sol: $f(x) = x^2 - 5x + 6$

$$f(x-1) = (x-1)^2 - 5(x-1) + 6$$

$$\boxed{(a-b)^2 = a^2 + b^2 - 2ab}$$

$$= x^2 + 1^2 - 2(x)(1) - 5x + 5 + 6$$

$$= x^2 + 1 - 2x - 5x + 11$$

$$f(x-1) = x^2 - 7x + 12$$

---

④ A graph representing the function $f(x)$ is given in Fig 1.16 it is clear that $f(9) = 2$.

(i) Find the following Values of the function

(a) $f(0)$ (b) $f(7)$

(c) $f(2)$ (d) $f(10)$

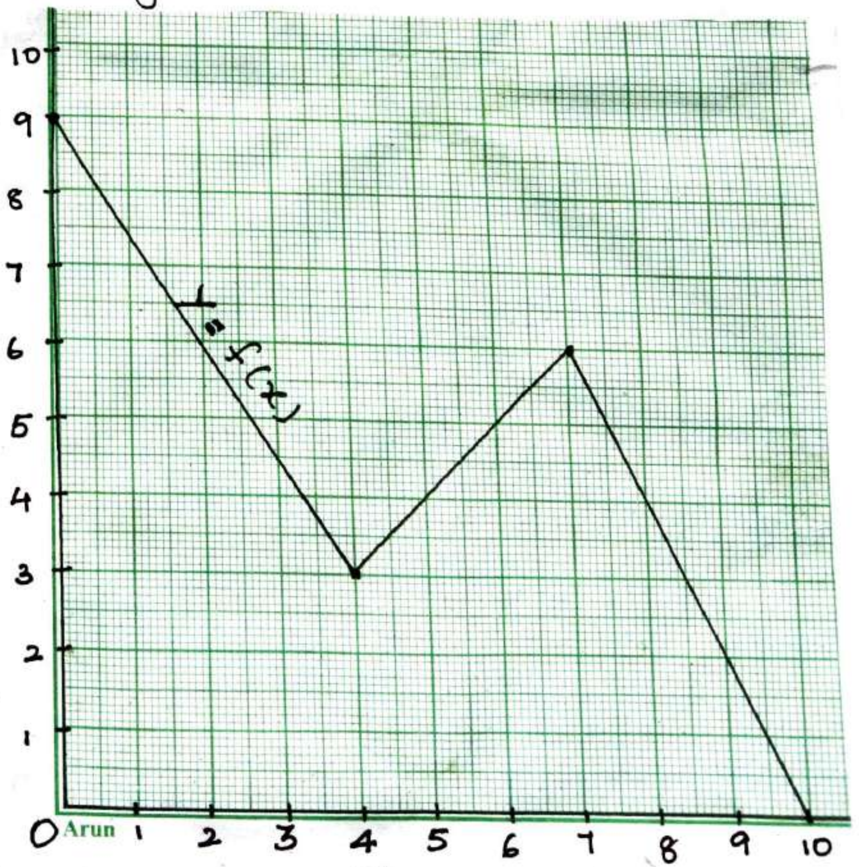


Fig 1.16

Sol :-

ⓐ $f(0) = 9$

ⓑ $f(7) = 6$

ⓒ $f(2) = 6$

ⓓ $f(10) = 0$

(ii) For what Value of x is $f(x) = 1$?

Sol :-

When $x = 9.5$,

$f(x) = 1$

(iii) Describe the following (i) Domain (ii) Range.

Sol :-

Domain $= \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

Range $= \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

(iv) What is the image of 6 under f?

Sol :- $\boxed{f(6) = 5}$

⑤ Let $f(x) = 2x+5$. If $x \neq 0$ then find $\dfrac{f(x+2) - f(2)}{x}$

Sol:-

$$f(x) = 2x+5$$

To find $\dfrac{f(x+2) - f(2)}{x}$

$$f(x+2) = 2(x+2)+5$$

$$= 2x+4+5$$

$$\boxed{f(x+2) = 2x+9}$$

$$f(2) = 2(2)+5$$

$$= 4+5$$

$$\boxed{f(2) = 9}$$

$$\frac{f(x+2)-f(2)}{x} = \frac{2x+\cancel{9}-\cancel{9}}{x}$$

$$= \frac{2\cancel{x}}{\cancel{x}}$$

$$\boxed{\frac{f(x+2)-f(2)}{x} = 2}$$

⑥ A function f is defined by

$f(x) = 2x - 3$.

(i) find $\frac{f(0) + f(1)}{2}$

Sol :— $f(x) = 2x - 3$

$$\frac{f(0) + f(1)}{2} = ?$$

$f(0) = 2(0) - 3$

$= 0 - 3$

$\boxed{f(0) = -3}$

$f(1) = 2(1) - 3$

$= 2 - 3$

$\boxed{f(1) = -1}$

$$\therefore \frac{f(0) + f(1)}{2} = \frac{-3 - 1}{2}$$

$$= \frac{-4}{2}$$

$$\boxed{\frac{f(0) + f(1)}{2} = -2}$$

(ii) Find x, Such that $f(x) = 0$

Sol :-

$$f(x) = 0$$

ie, $$2x - 3 = 0$$

$$2x = 3$$

$$\boxed{x = \frac{3}{2}}$$

(iii) Find x, Such that $f(x) = x$

Sol :- $$f(x) = x$$

ie, $$2x - 3 = x$$

$$2x - x = 3$$

$$\boxed{x = 3}$$

(iv) Find x, Such that

$$f(x) = f(1-x)$$

Sol:- $f(x) = f(1-x)$

We Know $\boxed{f(x) = 2x-3}$

Now; $f(1-x) = 2(1-x)-3$

$= 2-2x-3$

$$\boxed{f(1-x) = -2x-1}$$

$$2x-3 = -2x-1$$

$$2x+2x = -1+3$$

$$4x = 2$$

$$x = \frac{\cancel{2}^{1}}{\cancel{4}_{2}}$$

$$\boxed{x = \frac{1}{2}}$$

---

⑦ An open box is to be made from a Square piece of material, 24 cm on a side, by cutting equal squares from the corners and turning up the sides as shown. (Fig. 1.17). Express the

Volume V of the box as a function of x.

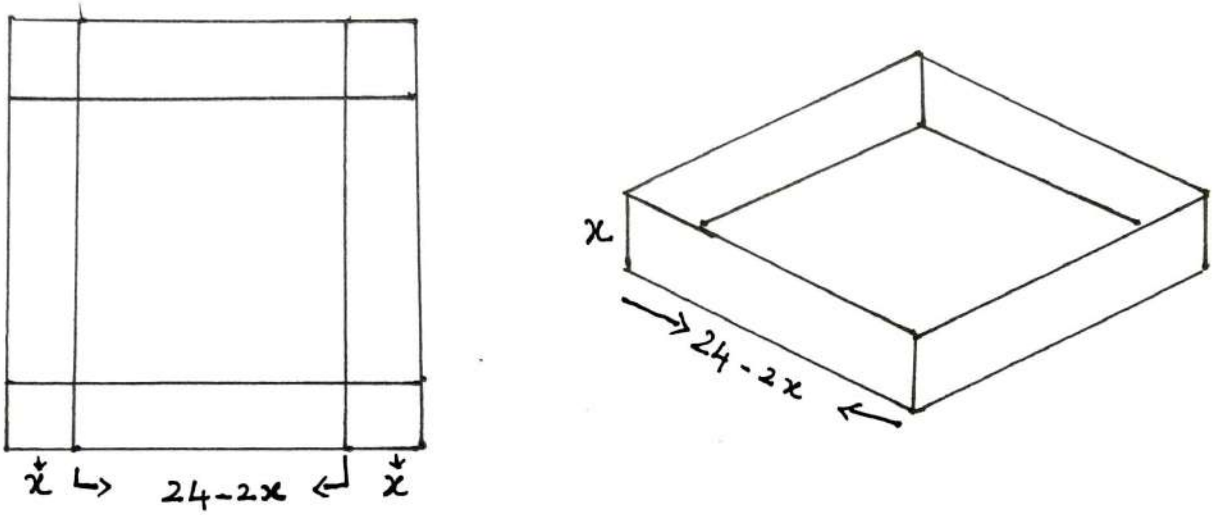


Fig 1.17

Sol:

$$l = 24-2x$$

$$b = 24-2x$$

$$h = x$$

$$V = l \times b \times h$$

$$= (24-2x)(24-2x)(x)$$

$$= (24-2x)^2 \cdot x$$

$$[(a-b)^2 = a^2+b^2-2ab]$$

$$= \left[(24)^2 + (2x)^2 - 2(24)(2x)\right] x$$

$$= \left[576 + 4x^2 - 96x\right] x$$

$$= 576x + 4x^3 - 96x^2$$

$$\therefore \boxed{V = 4x^3 - 96x^2 + 576x}$$

---

⑧ A function f is defined by $f(x) = 3-2x$.
Find x such that $f(x^2) = [f(x)]^2$

Sol:-

$$f(x^2) = [f(x)]^2$$

$$3 - 2x^2 = (3-2x)^2$$

$$\boxed{(a-b)^2 = a^2 + b^2 - 2ab}$$

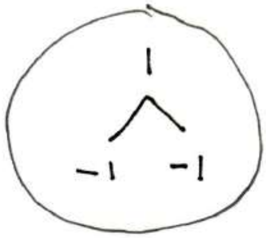
$$3 - 2x^2 = 3^2 + (2x)^2 - 2(3)(2x)$$

$$3 - 2x^2 = 9 + 4x^2 - 12x$$

$$3 - 2x^2 - 9 - 4x^2 + 12x = 0$$

$$-6x^2+12x-6=0 \quad [\div \text{by} -6]$$

$$x^2-2x+1=0$$



$$(x-1)(x-1)=0$$

$x-1=0$ | $x-1=0$

$\boxed{x=1}$ | $\boxed{x=1}$

---

⑨ A plane is flying at a speed of 500 km per hour. Express the distance 'd' travelled by the plane as function of time t in hours.

Sol:-

Speed = 500 km/hr.

Distance = d

Time = t

Distance = Speed × Time

$$\boxed{d = 500t}$$

---

(10) The data in the adjacent table depicts the length of a person forehand and their corresponding height. Based on this data, a student finds a relationship between the height (y) and the forehand length (x) as $y = ax+b$, where a, b are constants.

Length 'x' of forehand (in cm)	Height 'y' (in inches)
35	56
45	65
50	69.5
55	74

(i) Check if this relation is a function

Sol :- $Y = ax+b$

For given Length 'x'; We have corresponding height 'Y'.

Yes, This relation is a function.

(ii) Find a and b

Sol:- Let us consider any 2 x values and its corresponding y values.

$\Rightarrow$ $x = 35$, $y = 56$

$$y = ax + b$$

$$\boxed{56 = 35a + b} \longrightarrow ①$$

$\Rightarrow$ $x = 45$, $y = 65$

$$y = ax + b$$

$$\boxed{65 = 45a + b} \longrightarrow ②$$

Solve ① & ②

$$\begin{array}{rcl} 35a + \cancel{b} & = & 56 \\ (-)\quad 45a \underset{(-)}{+} \cancel{b} & = & \underset{(-)}{}65 \\ \hline \cancel{-}10a & = & \cancel{-}9 \\ \hline \end{array}$$

$$a = \frac{9}{10}$$

$$\boxed{a = 0.9}$$

Put $a = 0.9$ in ①

$$35a + b = 56$$

$$35(0.9) + b = 56$$

$$31.5 + b = 56$$

$$b = 56 - 31.5$$

$$\boxed{b = 24.5}$$

$$\begin{array}{r} \overset{4}{3}5 \\ \times 0.9 \\ \hline 31.5 \end{array}$$

$$\begin{array}{r} 5\overset{5}{\cancel{6}}.{}^{1}0 \\ (-)\,31.5 \\ \hline 24.5 \end{array}$$

(iii) Find the height of a person whose forehand length is 40 cm.

Sol :- Fore hand length $\Rightarrow x = 40$ cm

Height $\Rightarrow y = ?$

$y = ax + b$ [Subsitute $a = \frac{9}{10}$, $b = 24.5$]

$$y = \frac{9}{10}x + 24.5$$

$$y = \left(\frac{9}{\cancel{10}} \times \overset{4}{\cancel{40}}\right) + 24.5$$

$$= 36 + 24.5$$

$$\boxed{y = 60.5 \text{ inches}}$$

$$\begin{array}{r} 36.0 \\ +\,24.5 \\ \hline 60.5 \end{array}$$

(iv) Find the length of forehand of a person if the height is 53.3 inches.

Sol :-

Height ⇒ $y = 53.3$ inches.

'Forehand ⇒ $x = ?$

$$y = ax + b$$

[Subsitute $a = \frac{9}{10}$, $b = 24.5$]

$$53.3 = \frac{9}{10}x + 24.5$$

$$53.3 - 24.5 = \frac{9}{10}x$$

$$\begin{array}{r} 53.3 \\ -\,24.5 \\ \hline 28.8 \\ \hline \end{array}$$

$$28.8 = \frac{9}{10}x$$

$$\leftarrow$$

$$28.8 \times 10 = 9x$$

$$9x = 288$$

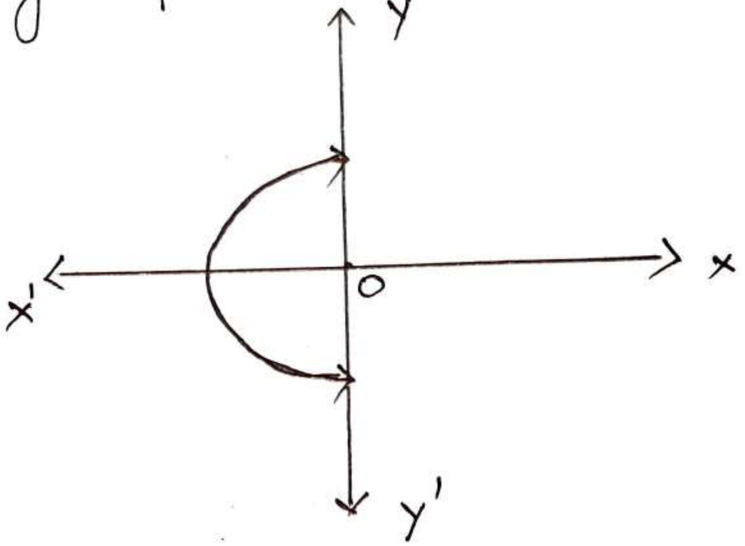
$$x = \frac{\cancel{288}^{32}}{\cancel{9}_{1}}$$

$$\boxed{x = 32 \text{ Cm}}$$

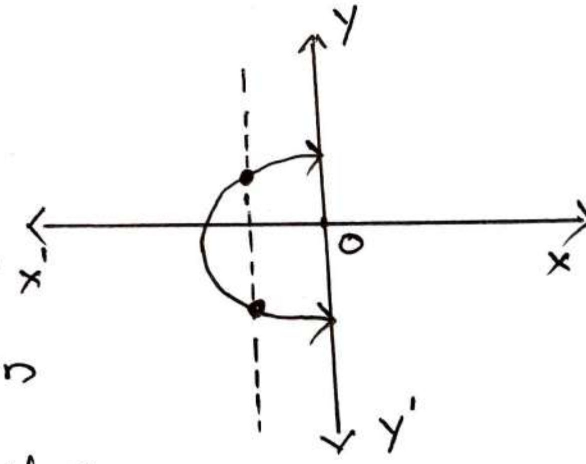
Exercise - 1.4

1. Determine whether the graph given below represent functions. Give reason for your answers concerning each group.

(i)

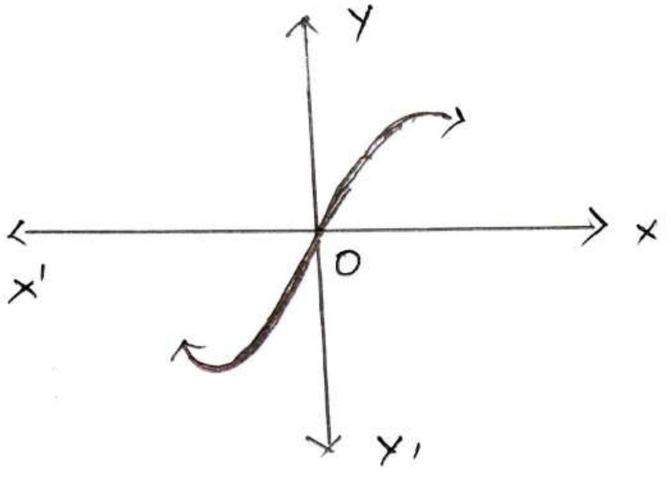


Sol:-



It is not a function.
Because the line drawn parallel to y-axis touches the curve at more than one point.

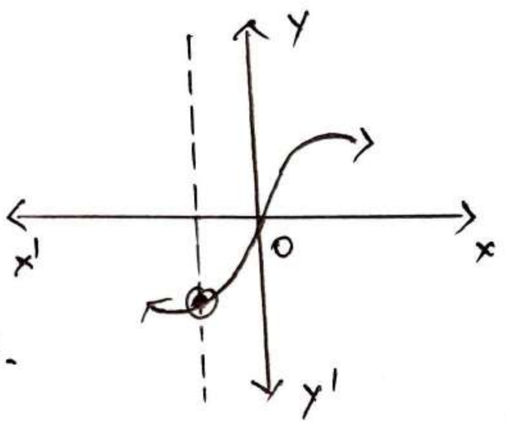
(ii)



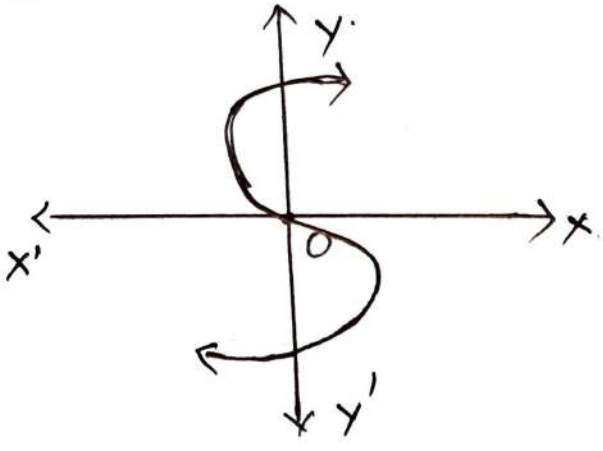
Sol :-

It is a function.

Because the line drawn parallel to y-axis touches the curve at one point.

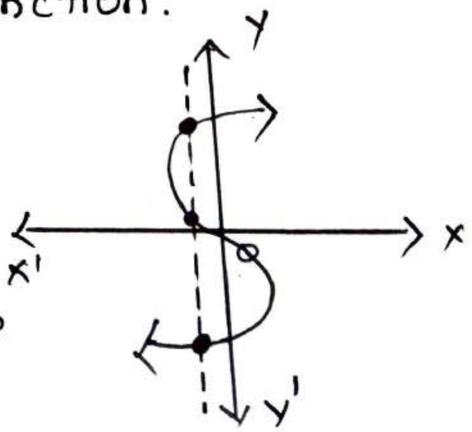


(iii)

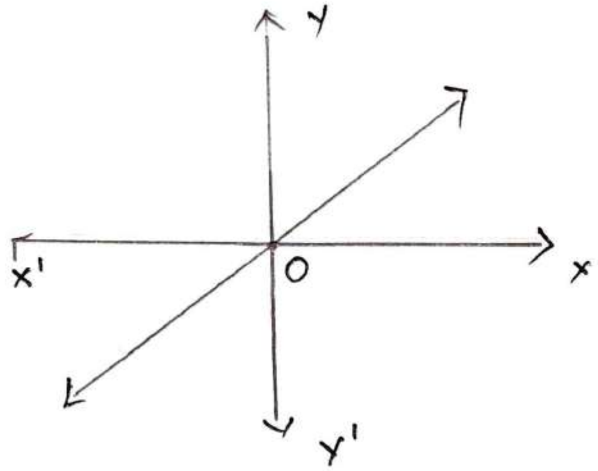


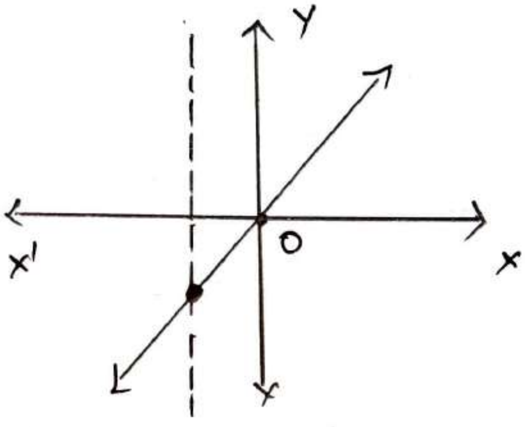
Sol :- It is not a function.

Because the line drawn parallel to y axis touches the curve at more than one point.



(iv)





Sol:-

It is a function.

Because the line drawn parallel to y axis touches the Curve at One point.

---

2. Let $f: A \rightarrow B$ be a function defined by $f(x) = \frac{x}{2} - 1$, where $A = \{2, 4, 6, 10, 12\}$ $B = \{0, 1, 2, 4, 5, 9\}$. Represent f by

(i) Set of ordered pair.

(ii) a table

(iii) an arrow diagram.

(iv) a graph.

Sol :- $f(x) = \frac{x}{2} - 1$ $\boxed{f : A \to B}$

$x = 2$; $f(2) = \frac{2}{2} - 1 = 1 - 1 = 0$

$x = 4$; $f(4) = \frac{\cancel{4}^{2}}{\cancel{2}} - 1 = 2 - 1 = 1$

$x = 6$; $f(6) = \frac{\cancel{6}^{3}}{\cancel{2}} - 1 = 3 - 1 = 2$

$x = 10$; $f(10) = \frac{\cancel{10}^{5}}{\cancel{2}} - 1 = 5 - 1 = 4$

$x = 12$; $f(12) = \frac{\cancel{12}^{6}}{\cancel{2}} - 1 = 6 - 1 = 5$

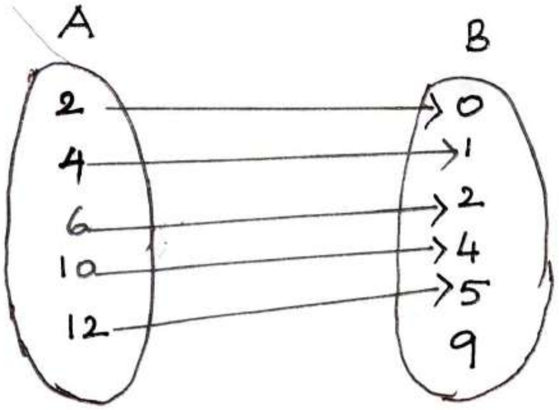
(i) Set of Ordered pair:

$f = \{(2, 0), (4, 1), (6, 2), (10, 4), (12, 5)\}$

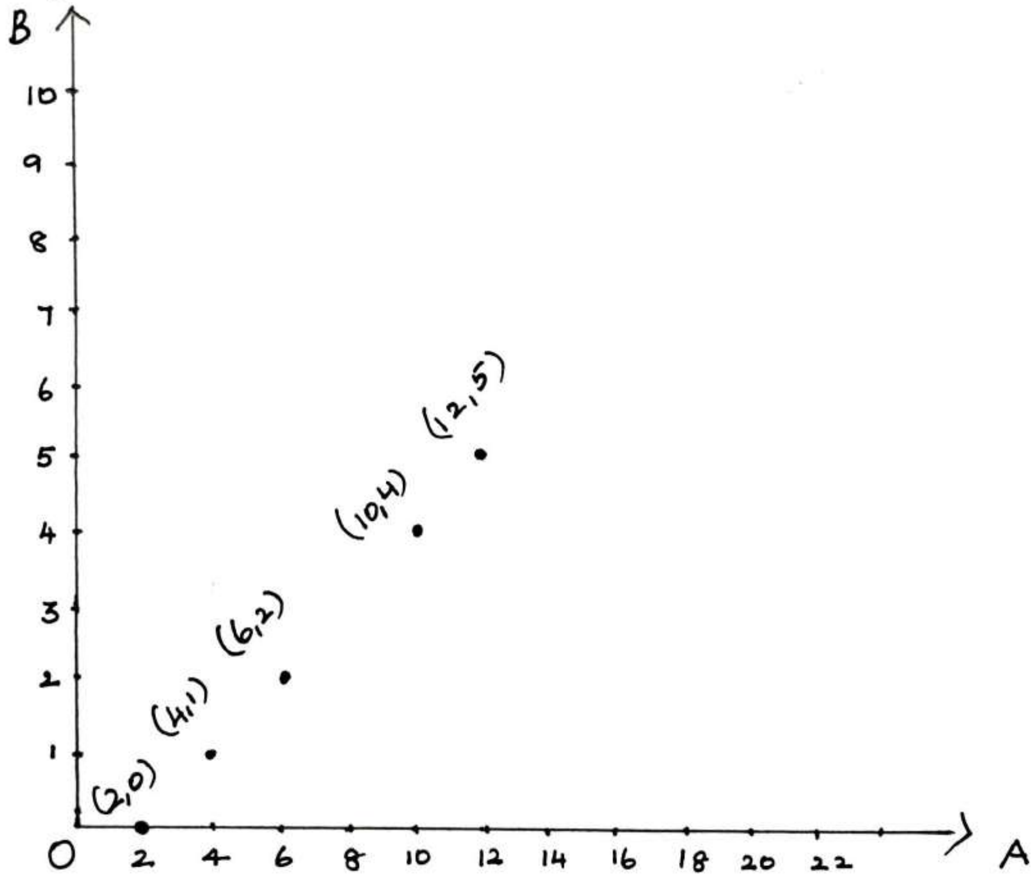
(ii) A table form:-

A	2	4	6	10	12
B	0	1	2	4	5

(iii) An arrow diagram :-



(iv) A graph



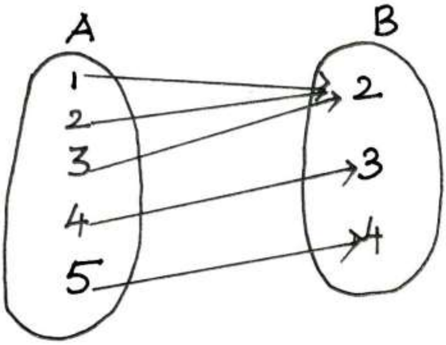
③ Represent the function

$f = \{(1,2), (2,2), (3,2), (4,3), (5,4)\}$ through

(i) An Arrow diagram (ii) A Table form

(iii) A graph.

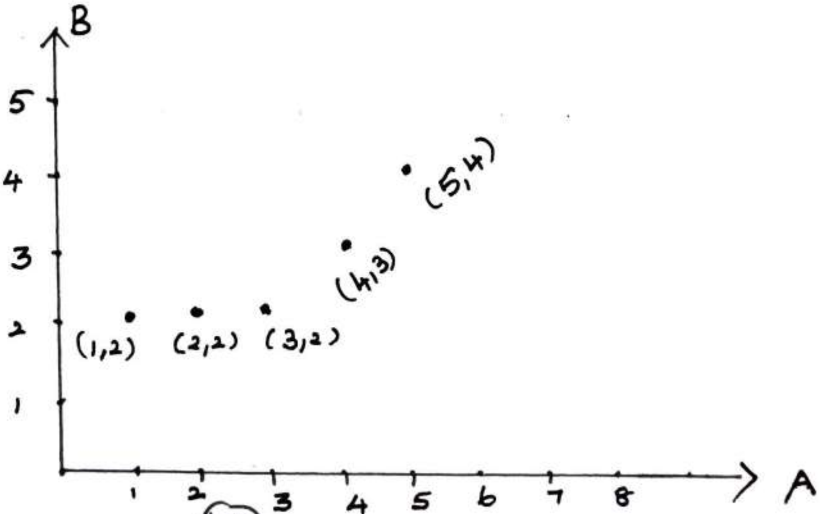
Sol :- (i) An Arrow diagram.



(ii) A Table form

A	1	2	3	4	5
B	2	2	2	3	4

(iii) A Graph



4. Show that the function $f: N \to N$ defined by $f(x) = 2x-1$ is One-One but not Onto.

Sol :-

$f: N \to N$ [$\because$ N is natural No]

$\therefore x = 1, 2, 3, \ldots$

$x=1;\quad f(1) = 2(1)-1 = 2-1 = 1$

$x=2;\quad f(2) = 2(2)-1 = 4-1 = 3$

$x=3;\quad f(3) = 2(3)-1 = 6-1 = 5$

So on.

$\therefore f = \{(1,1), (2,3), (3,5), \ldots\}$

Hence it is One-One. But not Onto.

---

(5) Show that the function $f: N \to N$ defined by $f(m) = m^2+m+3$ is One-One function.

Sol :-

$f: N \to N$ [$\because$ N is natural No]

$\therefore m = 1, 2, 3, \ldots$

$$f(m) = m^2 + m + 3$$

$m = 1$; $f(1) = 1^2 + 1 + 3 = 1 + 1 + 3 = 5$

$m = 2$; $f(2) = 2^2 + 2 + 3 = 4 + 2 + 3 = 9$

$m = 3$; $f(3) = 3^2 + 3 + 3 = 9 + 3 + 3 = 15$

and So On.

$\therefore f = \{(1,5), (2,9), (3,15), \cdots\}$

$\therefore$ Hence It is One - One function.

---

⑥ Let $A = \{1,2,3,4\}$ and $B = N$. let $f : A \to B$ be defined by $f(x) = x^3$. then,

(i) find the range of f.

(ii) identify the type of function.

Sol:- $A = \{1, 2, 3, 4\}$

$B = N = 1, 2, 3, 4, \cdots\cdots$

$f : A \to B$, $f(x) = x^3$

$x = 1$; $f(1) = 1^3 = 1$

$x = 2$; $f(2) = 2^3 = 8$

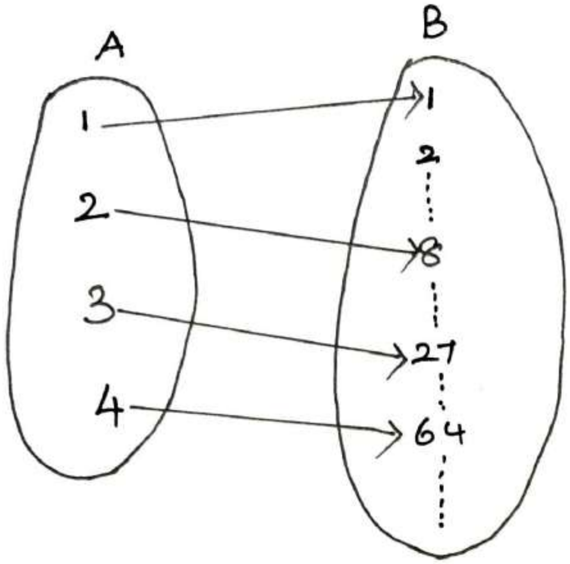
$x = 3$; $f(3) = 3^3 = 27$

$x = 4$; $f(4) = 4^3 = 64$

(i) Range of f

$f = \{1, 8, 27, 64\}$

(ii) Type of function:-



It is One - One & Into function.

---

⑦ In each of the following cases. State whether the function is bijective or not. Justify your answer.

(i) $f: R \rightarrow R$ defined by $f(x) = 2x+1$

Sol :-

$$f: R \to R$$

$$f(x) = 2x+1$$

$$\text{let } f(x) = f(y)$$

$$2x+\cancel{1} = 2y+\cancel{1}$$

$$\cancel{2}x = \cancel{2}y$$

$$\boxed{x = y}$$

$\therefore$ It is One - One function.

For every x in Domain R, We Can find $f(x)$ in co-Domain R

$\therefore$ It is Onto function.

When it is both One-One & Onto function, then the function is Bijective.

(ii) $f: R \to R$ defined by $f(x) = 3-4x^2$

Sol :-

$$f: R \to R$$

$$f(x) = 3-4x^2$$

Let $$f(x) = f(y)$$

$$\cancel{3}-4x^2 = \cancel{3}-4y^2$$

$$-\cancel{4}x^2 = -\cancel{4}y^2$$

$$\boxed{x^2 = y^2}$$

$\therefore$ For Real NO's 1 and -1 in domain the codomain is '1'

$\therefore$ It is not One-One function.

$\Rightarrow \therefore$ function is not bijective.

---

⑧ Let $A = \{-1, 1\}$ and $B = \{0, 2\}$. If the function $f: A \to B$ defined by $f(x) = ax+b$ is an Onto function ? Find a and b.

Sol :-

$$A = \{-1, 1\}$$

$$B = \{0, 2\}$$

$$f : A \rightarrow B$$

$f(x) = ax + b$

$f(-1) = 0$

$a(-1) + b = 0$

$-a + b = 0 \rightarrow$ ①

$f(x) = ax + b$

$f(1) = 2$

$a(1) + b = 2$

$a + b = 2 \rightarrow$ ②

From ①

$$-\overset{\curvearrowright}{a + b} = 0$$

$$\boxed{b = a} \rightarrow ③$$

Subsitute ③ in ②

$$a + b = 2$$

$$a + a = 2$$

$$2a = 2$$

$$a = \frac{\cancel{2}^{1}}{\cancel{2}}$$

$$\boxed{a = 1}$$

From ③ $\boxed{a = b = 1}$

⑨ If the function f is defined by $f(x) = \begin{cases} x+2 ; & x > 1 \\ 2 ; & -1 \leq x \leq 1 \\ x-1 ; & -3 < x < -1 \end{cases}$ find the

Value of (i) $f(3)$ (ii) $f(0)$

(iii) $f(-1.5)$ (iv) $f(2) + f(-2)$

Sol :-

$$\boxed{\begin{array}{lll} f(x) = x+2 & ; & x = 2, 3, 4, 5, \ldots \\ f(x) = 2 & ; & x = -1, 0, 1 \\ f(x) = x-1 & ; & x = -2 \end{array}}$$

(i) $f(3)$

$x = 3$ is in $\boxed{f(x) = x+2}$ 1^{st} interval

$\therefore f(3) = 3+2$

$$\boxed{f(3) = 5}$$

(ii) $f(0)$

$x = 0$ is in $\boxed{f(x) = 2}$ 2^{nd} interval

$$\boxed{f(0) = 2}$$

(iii) $f(-1.5)$

$x = -1.5$ is in $\boxed{f(x) = x-1}$ 3rd interval

$$f(-1.5) = -1.5 - 1$$

$$\boxed{f(-1.5) = -2.5}$$

(iv) $f(2) + f(-2)$

$x = 2$ is in $\boxed{f(x) = x+2}$ 1st interval

$$f(2) = 2+2$$

$$\boxed{f(2) = 4}$$

$x = -2$ is in $\boxed{f(x) = x-1}$ 3rd interval

$$f(-2) = -2-1$$

$$\boxed{f(-2) = -3}$$

$$\therefore f(2) + f(-2) = 4 - 3$$

$$\boxed{f(2) + f(-2) = 1}$$

⑩. A function $f: [-5,9] \to R$ is defined as follows :

$$f(x) = \begin{cases} 6x+1 ; & -5 \leq x < 2 \\ 5x^2-1 ; & 2 \leq x < 6 \\ 3x-4 ; & 6 \leq x \leq 9 \end{cases}$$

Find (i) $f(-3) + f(2)$ (ii) $f(7) - f(1)$

(iii) $2f(4) + f(8)$ (iv) $\dfrac{2f(-2) - f(6)}{f(4) + f(-2)}$

Sol :-

$f(x) = 6x+1$; $x = -5, -4, -3, -2, -1, 0, 1$

$f(x) = 5x^2-1$; $x = 2, 3, 4, 5$

$f(x) = 3x-4$; $x = 6, 7, 8, 9$

(i) $f(-3) + f(2)$

$x = -3$ is in $f(x) = 6x+1$

$x = 2$ is in $f(x) = 5x^2-1$

$$f(-3) = 6(-3)+1$$

$$= -18+1$$

$$\boxed{f(-3) = -17}$$

$$f(2) = 5(2)^2-1$$

$$= 5(4)-1$$

$$= 20-1$$

$$\boxed{f(2) = 19}$$

$$\therefore f(-3)+f(2) = -17+19$$

$$\boxed{f(-3)+f(2) = 2}$$

(ii) $f(7)-f(1)$

$x=7$ is in $f(x) = 3x-4$

$x=1$ is in $f(x) = 6x+1$

$$f(7) = 3(7)-4$$

$$= 21-4$$

$$\boxed{f(7) = 17}$$

$$f(1) = 6(1)+1$$

$$f(1) = 6+1$$

$$\boxed{f(1) = 7}$$

$$\therefore f(7)-f(1) = 17-7$$

$$\boxed{f(7)-f(1) = 10}$$

(iii) $2f(4) + f(8)$

$x = 4$ is in $f(x) = 5x^2 - 1$

$x = 8$ is in $f(x) = 3x - 4$

$$f(4) = 5(4)^2 - 1$$

$$= 5(16) - 1$$

$$= 80 - 1$$

$$f(4) = 79$$

$$\therefore 2f(4) = 2(79)$$

$$\boxed{2f(4) = 158}$$

$$f(8) = 3(8) - 4$$

$$= 24 - 4$$

$$\boxed{f(8) = 20}$$

$$\therefore 2f(4) + f(8) = 158 + 20$$

$$\boxed{2f(4) + f(8) = 178}$$

(iv) $\dfrac{2f(-2) - f(6)}{f(4) + f(-2)}$

$x = -2$ is in $f(x) = 6x+1$

$x = 6$ is in $f(x) = 3x-4$

$x = 4$ is in $f(x) = 5x^2-1$

$f(-2) = 6(-2)+1$	$f(6) = 3(6)-4$	$f(4) = 5(4)^2-1$
$= -12+1$	$= 18-4$	$= 5(16)-1$
$\boxed{f(-2) = -11}$	$\boxed{f(6) = 14}$	$= 80-1$
		$\boxed{f(4) = 79}$

$$\therefore \frac{2f(-2)-f(6)}{f(4)+f(-2)} = \frac{2(-11)-14}{79+(-11)}$$

$$= \frac{-22-14}{79-11}$$

$$\boxed{\frac{2f(-2)-f(6)}{f(4)+f(-2)} = \frac{-36}{68}}$$

(11.) The distance s an object travels under the influence of gravity in time t seconds is given by $S(t) = \frac{1}{2}gt^2 + at + b$ where, (g is the acceleration due to gravity), a, b are constants. Verify wheather the function $s(t)$ is one-one or not.

Sol :-

$$S(t) = \frac{1}{2}gt^2 + at + b$$

$t=1$; $$S(1) = \frac{1}{2}g(1)^2 + a(1) + b$$

$$S(1) = \frac{1}{2}g + a + b$$

$t=2$; $$S(2) = \frac{1}{2}g(2)^2 + a(2) + b$$

$$= \frac{1}{\cancel{2}}(\overset{2}{\cancel{4}}g) + 2a + b$$

$$S(2) = 2g + 2a + b$$

And so on.

For different values of t, there exist different images in $S(t)$.

$\Rightarrow S(t)$ is One-One.

⑫ The function 't' which maps temperature in Celsius (c) into temperature in Fahrenheit (F) is defined by $t(c) = F$ where $F = \frac{9}{5}c + 32$. Find (i) $t(0)$ (ii) $t(28)$ (iii) $t(-10)$ (iv) the value of c when $t(c) = 212$.

(v) the temperature when the Celsius value is equal to the Farenheit value.

Sol :-

$$t(c) = F$$

$$F = \frac{9}{5}c + 32$$

$$t(c) = \frac{9}{5}c + 32$$

(i) $t(0)$

$$t(0) = \frac{9}{5}(0) + 32$$

$$\boxed{t(0) = 32}$$

(ii) $t(28)$

$$t(28) = \frac{9}{\underset{1}{\cancel{5}}}(\overset{5.6}{\cancel{28}}) + 32$$

$$= 9(5.6) + 32$$

$$= 50.4 + 32$$

$$\boxed{t(28) = 82.4}$$

(iii) $t(-10)$

$$t(-10) = \frac{9}{\underset{1}{\cancel{5}}}(-\overset{2}{\cancel{10}}) + 32$$

$$= 9(-2) + 32$$

$$= -18 + 32$$

$$\boxed{t(-10) = 14}$$

(iv) $t(c) = 212$

$$\frac{9}{5}c + 32 = 212$$

$$\frac{9}{5}c = 212 - 32$$

$$\frac{9}{5}c = 180$$

$$c = \overset{20}{\cancel{180}} \times \frac{5}{\underset{1}{\cancel{9}}}$$

$$= 20 \times 5$$

$$\boxed{C = 100^\circ}$$

(v)

$C = F \rightarrow$ ①

$t(c) = F \rightarrow$ ②

From ① & ②

$$t(c) = C$$

$$\frac{9}{5}c + 32 = C$$

$$\frac{9}{5}\underrightarrow{c - c} = -32$$

$$\frac{9c - 5c}{5} = -32$$

$$\frac{4c}{5} = -32$$

$$c = -\overset{8}{\cancel{32}} \times \frac{5}{\cancel{4}_{1}}$$

$$c = -8 \times 5$$

$$\boxed{c = -40}$$

Exercise - 1.5

① Using the functions f and g given below, find fog and gof. Check whether fog = gof.

(i) $f(x) = x-6$; $g(x) = x^2$

$fog = f[g(x)]$

$= f[x^2]$

$\boxed{fog = x^2-6}$

$gof = g[f(x)]$

$= g[x-6]$

$\boxed{gof = (x-6)^2}$

$\therefore fog \neq gof$

(ii) $f(x) = \frac{2}{x}$; $g(x) = 2x^2-1$

$fog = f[g(x)]$

$= f[2x^2-1]$

$\boxed{fog = \frac{2}{2x^2-1}}$

$$gof = g[f(x)]$$

$$= g\left[\frac{2}{x}\right]$$

$$= 2\left(\frac{2}{x}\right)^2 - 1$$

$$= 2\left(\frac{4}{x^2}\right) - 1$$

$$\boxed{gof = \frac{8}{x^2} - 1}$$

$$\therefore fog \neq gof$$

(iii) $f(x) = \frac{x+6}{3}$; $g(x) = 3 - x$

$$fog = f[g(x)]$$

$$= f[3 - x]$$

$$= \frac{3 - x + 6}{3}$$

$$\boxed{fog = \frac{9 - x}{3}}$$

$$gof = g[f(x)]$$

$$= g\left[\frac{x+6}{3}\right]$$

$$= 3 - \left(\frac{x+6}{3}\right)$$

$$= \frac{9 - x - 6}{3}$$

$$\boxed{gof = \frac{3 - x}{3}}$$

$$\therefore fog \neq gof$$

(iv) $f(x) = 3+x \;;\quad g(x) = x-4$

$$fog = f[g(x)]$$
$$= f[x-4]$$
$$= 3+x-4$$
$$\boxed{fog = x-1}$$

$$gof = g[f(x)]$$
$$= g[3+x]$$
$$= 3+x-4$$
$$\boxed{gof = x-1}$$

$$\therefore fog = gof$$

(v) $f(x) = 4x^2-1 \;;\quad g(x) = 1+x.$

$$fog = f[g(x)]$$
$$= f[1+x]$$
$$= 4(1+x)^2-1$$
$$= 4[1+x^2+2x]-1$$
$$= 4+4x^2+8x-1$$
$$\boxed{fog = 4x^2+8x+3}$$

$$gof = g[f(x)]$$
$$= g[4x^2-1]$$
$$= \cancel{1}+4x^2-\cancel{1}$$
$$\boxed{gof = 4x^2}$$

$$\therefore fog \neq gof.$$

② Find the Value of k, Such that

$f \circ g = g \circ f$

(i) $f(x) = 3x+2$; $g(x) = 6x-k$

Sol:-

$$f \circ g = f[g(x)]$$
$$= f[6x-k]$$
$$= 3(6x-k)+2$$
$$\boxed{f \circ g = 18x-3k+2}$$

$$g \circ f = g[f(x)]$$
$$= g[3x+2]$$
$$= 6(3x+2)-k$$
$$\boxed{g \circ f = 18x+12-k}$$

Given $f \circ g = g \circ f$

$$\cancel{18x}-3k+2 = \cancel{18x}+12-k$$
$$-3k+2 = 12-k$$
$$2-12 = -k+3k$$
$$-10 = 2k$$
$$2k = -10$$
$$k = -\frac{10}{2}$$
$$\boxed{k = -5}$$

(ii) $f(x) = 2x - k \quad ; \quad g(x) = 4x + 5$

$$\begin{aligned} fog &= f[g(x)] \\ &= f[4x+5] \\ &= 2(4x+5) - k \end{aligned}$$

$$\boxed{fog = 8x + 10 - k}$$

$$\begin{aligned} gof &= g[f(x)] \\ &= g[2x-k] \\ &= 4(2x-k) + 5 \end{aligned}$$

$$\boxed{gof = 8x - 4k + 5}$$

Given $\quad fog = gof$

$$\cancel{8x} + 10 - k = \cancel{8x} - 4k + 5$$

$$10 - k = -4k + 5$$

$$-k + 4k = -10 + 5$$

$$3k = -5$$

$$\boxed{k = -\frac{5}{3}}$$

---

③ If $f(x) = 2x - 1$; $g(x) = \frac{x+1}{2}$, show that $fog = gof = x$

Sol :-

$$fog = f[g(x)]$$
$$= f\left[\frac{x+1}{2}\right]$$
$$= \cancel{2}\left(\frac{x+1}{\cancel{2}}\right) - 1$$
$$= x + \cancel{1} - \cancel{1}$$
$$\boxed{fog = x}$$

$$gof = g[f(x)]$$
$$= g[2x-1]$$
$$= \frac{2x - \cancel{1} + \cancel{1}}{2}$$
$$= \frac{\cancel{2}x}{\cancel{2}}$$
$$\boxed{gof = x}$$

$$\therefore fog = gof = x$$

---

④ If $f(x) = x^2 - 1$, $g(x) = x - 2$, find a, if $gof(a) = 1$

Sol :-

$$gof(a) = 1$$
$$g[f(a)] = 1$$
$$g[a^2 - 1] = 1$$
$$a^2 - 1 - 2 = 1$$

$$a^2 - 3 = 1$$
$$a^2 = 1 + 3$$
$$a^2 = 4$$
$$a = \sqrt{4}$$
$$\boxed{a = \pm 2}$$

---

⑤ Let $A, B, C \subseteq N$ and a function $f : A \to B$ be defined by $f(x) = 2x+1$ and $g : B \to C$ be defined by $g(x) = x^2$. Find the range of fog and gof.

Sol:-

$$fog = f[g(x)]$$
$$= f[x^2]$$
$$\boxed{fog = 2x^2+1}$$

Range of fog $= \{y / y = 2x^2+1 ;\ x \in N\}$.

$$\begin{aligned} gof &= g[f(x)] \\ &= g[2x+1] \\ &= (2x+1)^2 \\ &= (2x)^2+1+2(2x)(1) \\ &= 4x^2+1+4x \end{aligned}$$

$$\boxed{gof = 4x^2+4x+1}$$

Range of gof $= \{y/y = 4x^2+4x+1, x \in N\}$

---

⑥ Let $f(x) = x^2-1$ Find

(i) fof

Sol:-
$$\begin{aligned} fof &= f[f(x)] \\ &= f[x^2-1] \\ &= (x^2-1)^2-1 \\ &= (x^2)^2+1-2(x^2)(1)-1 \\ &= x^4+\cancel{1}-2x^2-\cancel{1} \end{aligned}$$

$$\boxed{fof = x^4-2x^2}$$

(ii) fofof

Sol :-

$$\begin{aligned}
fofof &= fof\left[x^2-1\right] \\
&= f\left[(x^2-1)^2-1\right] \\
&= f\left[(x^2)^2+1-2(x^2)(1)-1\right] \\
&= f\left[x^4+\cancel{1}-2x^2-\cancel{1}\right] \\
&= f\left[x^4-2x^2\right] \\
&= \left(x^4-2x^2\right)^2-1 \\
&= (x^4)^2+(2x^2)^2-2(x^4)(2x^2)-1 \\
fofof &= x^8+2x^4-4x^6-1
\end{aligned}$$

---

⑦. If $f: R \to R$ and $g: R \to R$ are defined by $f(x) = x^5$ and $g(x) = x^4$. then Check if f, g are One-One and fog is One-One ?

Sol :-

(i) To Check f is One-One

$$f(x) = x^5 \qquad \boxed{f: R \to R}$$

$x = -2$; $f(-2) = (-2)^5 = -32$

$x = -1$; $f(-1) = (-1)^5 = -1$

$x = 1$; $f(1) = (1)^5 = 1$

$\therefore$ f is One-One...

(ii) To Check g is One-One

$$g(x) = x^4$$

$x = -2$; $g(-2) = (-2)^4 = 16$

$x = -1$; $g(-1) = (-1)^4 = 1$

$x = 1$; $g(1) = (1)^4 = 1$

$\therefore$ g is not One-One.

(iii) To Check fog is One - One

$$fog = f[g(x)]$$

$$= f[x^4]$$

$$= (x^4)^5$$

$$\boxed{fog = x^{20}}$$

$x = -1$; $fog(-1) = (-1)^{20} = 1$

$x = 1$; $fog(1) = (1)^{20} = 1$

$\therefore$ fog is not One - One.

---

⑧ Consider the functions $f(x)$, $g(x)$, $h(x)$ as given below. Show that $(fog)oh = fo(goh)$ in each case

(i) $f(x) = x-1$; $g(x) = 3x+1$; $h(x) = x^2$

Sol :-

To Show $(fog)oh = fo(goh)$

LHS:- $(fog)oh$

$$fog = f[g(x)]$$
$$= f[3x+1]$$
$$= 3x+\cancel{1}-\cancel{1}$$
$$fog = 3x$$

$$(fog)oh = fog[h(x)]$$
$$= fog(x^2)$$
$$\boxed{(fog)oh = 3x^2}$$

RHS:- $fo(goh)$

$$(goh) = g[h(x)]$$
$$= g[x^2]$$
$$= 3x^2+1$$
$$goh = 3x^2+1$$

$$fo(goh) = f[goh(x)]$$
$$= f[3x^2+1]$$
$$= 3x^2+\cancel{1}-\cancel{1}$$
$$\boxed{fo(goh) = 3x^2}$$

$\therefore (fog)oh = fo(goh)$

(ii) $f(x) = x^2$; $g(x) = 2x$, $h(x) = x+4$

Sol :-

To Show :- $(fog)oh = fo(goh)$

LHS :- $(fog)oh$

$$fog = f[g(x)]$$
$$= f[2x]$$
$$= (2x)^2$$
$$fog = 4x^2$$
$$(fog)oh = (fog)[h(x)]$$
$$= fog[x+4]$$
$$= 4(x+4)^2$$
$$= 4[x^2+4^2+2(x)(4)]$$
$$= 4[x^2+16+8x]$$
$$\boxed{(fog)oh = 4x^2+64+32x}$$

RHS :- $fo(goh)$

$$goh = g[h(x)]$$
$$= g[x+4]$$
$$= 2(x+4)$$
$$goh = 2x+8$$
$$fo(goh) = f[goh(x)]$$
$$= f[2x+8]$$
$$= (2x+8)^2$$
$$= (2x)^2+8^2+2(2x)(8$$
$$\boxed{fo(goh) = 4x^2+64+32x}$$

$\therefore (fog)oh = fo(goh)$

(iii) $f(x) = x-4$; $g(x) = x^2$, $h(x) = 3x-5$

Sol :-

To Show :- $(fog)oh = fo(goh)$

L.H.S :- $(fog)oh$

$$fog = f[g(x)]$$
$$= f[x^2]$$
$$fog = x^2-4$$

$$(fog)oh = fog[h(x)]$$
$$= fog[3x-5]$$
$$= (3x-5)^2-4$$
$$= (3x)^2+5^2-2(3x)(5)-4$$
$$= 9x^2+25-30x-4$$
$$= 9x^2-30x+21$$

$$\boxed{(fog)oh = 9x^2-30x+21}$$

RHS :- $f\circ(g\circ h)$

$$g\circ h = g[h(x)]$$
$$= g[3x-5]$$
$$= (3x-5)^2$$
$$= (3x)^2 + 5^2 - 2(3x)(5)$$
$$g\circ h = 9x^2 + 25 - 30x$$

$$f\circ(g\circ h) = f[g\circ h(x)]$$
$$= f[9x^2+25-30x]$$
$$= 9x^2+25-30x-4$$
$$\boxed{f\circ(g\circ h) = 9x^2-30x+21}$$

$\therefore$

$$\boxed{(f\circ g)\circ h = f\circ(g\circ h)}$$

⑨ Let $f = \{(-1,3), (0,-1), (2,-9)\}$ be a linear function from Z into Z. find $f(x)$

Sol :-

The linear function is in the form

$$\boxed{f(x) = mx + c}$$

$$f = \{(-1,3), (0,-1), (2,-9)\}$$

Considering any two Values.

$\underline{\underline{f(-1, 3)}}$

$mx + c = f(x)$

$m(-1) + c = 3$

$\boxed{-m + c = 3} \rightarrow ①$

$\underline{\underline{f(0,-1)}}$

$mx + c = f(x)$

$m(0) + c = -1$

$c = -1$

Subsitute the Value of c in ①

$$-m - 1 = 3$$

$$-m = 3 + 1$$

$$-m = 4$$

$$\boxed{m = -4}$$

$$\therefore \boxed{f(x) = -4x - 1}$$

⑩ In electrical circuit theory, a circuit $C(t)$ is called linear circuit If it satisfies the superposition Principle given by $C(at_1+bt_2)=aC(t_1)+bC(t_2)$ where a, b are constants. Show that the circuit $C(t)=3t$ is linear.

Sol :-

To Show: $C(t)=3t$ is linear

$$
\begin{aligned}
C(at_1+bt_2) &= \overset{\curvearrowright}{3(at_1}+bt_2) \\
&= 3at_1+3bt_2 \\
&= a\,3t_1+b.3t_2 \\
&= aC(t_1)+b.C(t_2)
\end{aligned}
$$

∴ $C(t)$ satisfies the Superposition Principle.

∴ $C(t)=3t$ is linear.

CHAPTER - 2

Numbers And Sequences

Exercise - 2.1

① Find all positive integers, when divided by 3 leaves remainder 2.

Sol:- By Euclid's division lemma

$$\boxed{a = bq + r}$$

Here $\boxed{b = 3}$, $\boxed{r = 2}$

$\therefore \quad a = bq + r$

$\quad\quad a = 3q + 2$

Let $q = 0$; $a = 3(0) + 2 = 0 + 2 = 2$

$q = 1$; $a = 3(1) + 2 = 3 + 2 = 5$

$q = 2$; $a = 3(2) + 2 = 6 + 2 = 8$ and so on

$\therefore$ positive Integers are 2, 5, 8, ...

② A man has 532 flower pots. He wants to arrange them in rows such that each row contains 21 flower pots. Find the number of completed rows and how many flower pots are left over.

Sol:-

Total number of flower pots ⇒ $\boxed{a = 532}$

Number of flower pots in each row ⇒ $\boxed{b = 21}$

Number of completed rows ⇒ $q = ?$

Number of flower pots left over ⇒ $r = ?$

$$\boxed{a = bq + r}$$

$$532 = 21q + r$$

$$532 = 21(25) + 7$$

$$\begin{array}{r|l} & \overline{25} \rightarrow q \\ 21 & 532 \\ & \underline{42\downarrow} \\ & 112 \\ & \underline{105} \\ & 7 \rightarrow r \end{array}$$

∴ Number of completed rows = 25

Number of left over flower pots = 7

③ Prove that the product of Two consecutive positive integers is divisible by 2.

Sol: -

Let x and $x+1$ be 2 consecutive positive integers.

Let x be odd

then $x+1$ will be even.

If $x = 2k+1$

then $x+1 = 2k+1+1 = 2k+2$

To prove :- Product of two integer is divisible by 2.

$$(x)(x+1) = (2k+1)(2k+2)$$

$$= 4k^2+4k+2k+2$$

$$= 4k^2+6k+2$$

$$= 2(2k^2+3k+1)$$

Let
$2k^2+3k+1 = q$

$$x(x+1) = 2q+0$$

When the remainder is Zero, the product of two consecutive integers is divisible by 2

Hence Proved.

④ When the positive integers, a, b and c are divided by 13, the respective remainders are 9, 7 and 10. Show that $a+b+c$ is divisible by 13

Sol:-

By Euclid's division Lemma

$$a = bq + r$$

$$\boxed{\begin{aligned} a &= 13q_1 + 9 \\ b &= 13q_2 + 7 \\ c &= 13q_3 + 10 \end{aligned}}$$

To prove:- $a+b+c$ is divisible by 13

$$\begin{aligned} a+b+c &= 13q_1+9+13q_2+7+13q_3+10 \\ &= 13q_1+13q_2+13q_3+26 \\ &= 13(q_1+q_2+q_3+2) \end{aligned}$$

$$\boxed{a+b+c = 13q+0} \qquad \boxed{\begin{array}{c}\text{Assume} \\ q_1+q_2+q_3+2 = q\end{array}}$$

When $r=0$, then $a+b+c$ is divisible by 13

Hence Proved.

⑤ Prove that Square of any integer leaves the remainder either 0 or 1 when divided by 4.

Sol:- Let x be any integer

Case : 1

Let x = odd

$$x = 2k+1$$

To Prove:- Square of any integer leaves remainder 0 or 1, divided by 4

$$\therefore x^2 = (2k+1)^2$$
$$= (2k)^2 + 1^2 + 2(2k)(1)$$
$$= 4k^2 + 1 + 4k$$
$$= 4(k^2+k) + 1$$

Let $k^2+k = q$

$$\boxed{x^2 = 4q+1}$$

Square of an integer leaves remainder 1 when divided by 4

Case : 2

Let x = even

$x = 2k$ (or) $2k+2$

$x^2 = (2k)^2$

$x^2 = 4k^2$

$\boxed{x^2 = 4q}$ $k^2 = q$

∴ Square of an integer leaves the remainder 0 when divided by 4.

∴ When x = odd; Remainder = 1
x = even; Remainder = 0

Hence proved.

---

⑥. Use Euclid's Division Algorithm to find the Highest Common Factor (HCF) of

(i) 340 and 412 (ii) 867 and 255

(iii) 10224 and 9648 (iv) 84, 90 and 120

(i) 340 and 412

Sol :- Using Euclid's division algorithm

$$a = bq + r$$

$$412 = 340(1) + 72$$

$$\begin{array}{r} 1 \\ 340\overline{)412} \\ 340 \\ \hline 72 \end{array}$$

$$\boxed{r = 72 \neq 0}$$

$$340 = 72(4) + 52$$

$$\begin{array}{r} 4 \\ 72\overline{)340} \\ 288 \\ \hline 52 \end{array}$$

$$\boxed{r = 52 \neq 0}$$

$$72 = 52(1) + 20$$

$$\begin{array}{r} 1 \\ 52\overline{)72} \\ 52 \\ \hline 20 \end{array}$$

$$\boxed{r = 20 \neq 0}$$

$$52 = 20(2) + 12$$

$$\begin{array}{r} 2 \\ 20\overline{)52} \\ 40 \\ \hline 12 \end{array}$$

$$\boxed{r = 12 \neq 0}$$

$$20 = 12(1) + 8$$

$$\boxed{r = 8 \neq 0}$$

$$12 = 8(1) + 4$$

$$\boxed{r = 4 \neq 0}$$

$$8 = \underline{4}(2) + 0$$

$\therefore \boxed{\text{HCF is } 4}$

(ii) 867 and 255

Sol:- Using Euclid's division Algorithm

$$\boxed{a = bq + r}$$

$$867 = 255(3) + 102$$

$$\boxed{r = 102 \neq 0}$$

$$\begin{array}{r} 3 \\ 255\overline{)867} \\ -765 \\ \hline 102 \end{array}$$

$$255 = 102(2) + 51$$

$$\boxed{r = 51 \neq 0}$$

$$\begin{array}{r} 2 \\ 102\overline{)255} \\ -204 \\ \hline 51 \end{array}$$

$$102 = \underline{\underline{51}}(2) + 0 \qquad \boxed{r = 0}$$

$$\therefore \boxed{HCF = 51}$$

(iii) 10224 and 9648

Sol:- Using Euclid's division Algorithm

$$\boxed{a = bq + r}$$

$$10224 = 9648(1) + 576$$

$$\boxed{r = 576 \neq 0}$$

$$\begin{array}{r} 1 \\ 9648\overline{)10224} \\ -9648 \\ \hline 576 \end{array}$$

$9648 = 576(16) + 432$

$$\begin{array}{r} 16 \\ 576\overline{)9648} \\ 576 \\ \hline 3888 \\ 3456 \\ \hline 432 \end{array}$$

$\boxed{r = 432 \neq 0}$

$576 = 432(1) + 144$

$$\begin{array}{r} 1 \\ 432\overline{)576} \\ 432 \\ \hline 144 \end{array}$$

$\boxed{r = 144 \neq 0}$

$432 = \underline{\underline{144}}(3) + 0$

$$\begin{array}{r} 3 \\ 144\overline{)432} \\ 432 \\ \hline 0 \end{array}$$

$\boxed{r = 0}$

$\therefore$ HCF = 144

(iv) 84, 90 and 120

Sol:- Using Euclid's division Algorithm for 84, 90

$\boxed{a = bq + r}$

$90 = 84(1) + 6$

$\boxed{r = 6 \neq 0}$

$84 = \underline{\underline{6}}(14) + 0$

$$\begin{array}{r} 14 \\ 6\overline{)\,84} \\ \underline{6} \\ 24 \\ \underline{24} \\ 0 \end{array}$$

$\therefore$ HCF of 84, 90 is $\underline{\underline{6}}$

Now H.CF of 6. and 120

Using Euclid's division Algorithm

$$\boxed{a = bq + r}$$

$120 = \underline{\underline{6}}(20) + 0$

$$\begin{array}{r} 20 \\ 6\overline{)\,120} \\ \underline{12} \\ 0 \end{array}$$

$\therefore$ HCF of 6 and 120 is 6

$\therefore$ HCF of 84, 90 and 120 is 6

---

⑦ Find the largest number which divides 1230 and 1926 leaving remainder 12 in each case.

Sol:- Since remainder $\underline{\underline{12}}$ is there for both the numbers,
We have to Subtract both the numbers by 12.

$\therefore 1230 - 12 = 1218$

$1926 - 12 = 1914$

$\therefore$ HCF of 1218 and 1914, Using Euclid's division Algorithm

$$\boxed{a = bq + r}$$

$1914 = 1218(1) + 696$

$$\boxed{r = 696 \neq 0}$$

$$\begin{array}{r} 1 \\ 1218\overline{)1914} \\ 1218 \\ \hline 696 \end{array}$$

$1218 = 696(1) + 522$

$$\boxed{r = 522 \neq 0}$$

$$\begin{array}{r} 1 \\ 696\overline{)1218} \\ 696 \\ \hline 522 \end{array}$$

$696 = 522(1) + 174$

$$\boxed{r = 174 \neq 0}$$

$$\begin{array}{r} 1 \\ 522\overline{)696} \\ 522 \\ \hline 174 \end{array}$$

$522 = \underline{\underline{174}}(3) + 0$

$$\boxed{r = 0}$$

$$\begin{array}{r} 3 \\ 174\overline{)522} \\ 522 \\ \hline 0 \end{array}$$

$\therefore$ HCF of 1218 and 1914 is 174

⑧ If d is the Highest Common Factor of 32 and 60, find x and y satisfying $d = 32x + 60y$.

Sol :- HCF of 32 and 60 using Euclid's Division Algorithm

$$\boxed{a = bq + r}$$

$$60 = 32(1) + 28 \longrightarrow ①$$

$$\boxed{r = 28 \neq 0}$$

$$\begin{array}{r} 1 \\ 32\overline{)60} \\ -32 \\ \hline 28 \end{array}$$

$$32 = 28(1) + 4 \longrightarrow ②$$

$$\boxed{r = 4 \neq 0}$$

$$\begin{array}{r} 1 \\ 28\overline{)32} \\ 28 \\ \hline 4 \end{array}$$

$$28 = \underline{\underline{4}}(7) + 0$$

$$r = 0$$

$$\therefore \text{HCF} = 4$$

Given $d = \text{HCF}$

$$\boxed{d = 4}$$

$$\therefore \boxed{4 = 32x + 60y} \longrightarrow ☆$$

From ②

$$32 = 28(1) + 4$$

$$32 - 28(1) = 4 \longrightarrow \text{Ⓐ}$$

From ①

$$60 = 32(1) + 28$$

$$60 - 32(1) = 28 \longrightarrow \text{Ⓑ}$$

Subsituting Ⓑ in Ⓐ

$$32 - [60 - 32(1)] = 4$$

$$32(1) - 60 + 32(1) = 4$$

$$\therefore \quad 4 = 32(1+1) - 60$$

$$\boxed{4 = 32(2) + 60(-1)} \longrightarrow \text{Ⓒ}$$

Eqn Ⓒ and ⊛ are Similar

$$\therefore \quad 4 = 32x + 60y \longrightarrow \star$$

$$4 = 32(2) + 60(-1) \longrightarrow c$$

$$\therefore \quad \boxed{x = 2} \qquad \boxed{y = -1}$$

⑨ A Positive integer when divided by 88 gives the remainder 61. what will be the remainder when the Same number is divided by 11 ?

Sol:-

Let positive integer = a

Divisor = b = 88

remainder = r = 61

By Euclid's division Algorithm

$$\boxed{a = bq + r}$$

$$a = 88q + 61$$

To Show :- The remainder when divided by 11

$$\therefore \quad a = 88q + (55+6) \qquad \boxed{61 = 55+6}$$

$$a = 11(8q + 5) + \underline{\underline{6}} \qquad \boxed{8q+5 = q}$$

$\therefore$ Remainder = 6

⑩ Prove that two Consecutive positive integers are always Coprime.

Sol :-

Let the two Consecutive positive integers be $n, n+1$

$n = 1, 2, 3, \ldots$

If $n =$ even

then $n+1 =$ odd [Vice Versa]

$\therefore$ H.C.F of two Consecutive number $= 1$

Thus they are Coprime.

Exercise 2.2

① For what Values of natural number n, 4^n can end with the digit 6 ?

Sol :-

$$4^n = (2 \times 2)^n$$
$$= (2^2)^n$$

2 is a factor of 4

$\Rightarrow$ 4^n is always even.

$\Rightarrow$ 4^n ends with 4 and 6

$n = 1 \Rightarrow 4^1 = 4$ ‖ $n = 2 \Rightarrow 4^2 = 16$

$n = 3 \Rightarrow 4^3 = 64$ ‖ $n = 4 \Rightarrow 4^4 = 256$

when $n = 2, 4, 6$, 4^n ends with 6.

---

② If m, n are natural numbers, for what Values of m, does $2^n \times 5^m$ ends in 5 ?

Sol:-

For all Values of n, 2^n is always even.
For all Values of m, 5^m always ends with 5.

But $2^n \times 5^m$ always ends in 0.

$\therefore$ $2^n \times 5^m$ Cannot end with digit 5, for any Value of m.

---

③ Find HCF of 252525 and 363636

Sol Using Euclid's Division Algorithm.

$$\boxed{a = bq + r}$$

$363636 = 252525(1) + 111111$

$$\boxed{r = 111111 \neq 0}$$

$$\begin{array}{r|l} & 1 \\ 252525 & 363636 \\ & (-)\,252525 \\ \hline & 111111 \end{array}$$

$252525 = 111111(2) + 30303$

$$\boxed{r = 30303 \neq 0}$$

$$\begin{array}{r|l} & 2 \\ 111111 & 252525 \\ & (-)\,222222 \\ \hline & 30303 \end{array}$$

$111111 = 30303(3) + 20202$

$$\boxed{r = 20202 \neq 0}$$

$$\begin{array}{r|l} & 3 \\ 30303 & 111111 \\ & (-)\,90909 \\ \hline & 20202 \end{array}$$

$$30303 = 20202(1) + 10101$$

$$\boxed{r = 10101 \neq 0}$$

$$\begin{array}{r} 1 \\ 20202\overline{)30303} \\ (-)\,20202 \\ \hline 10101 \\ \hline \end{array}$$

$$20202 = \underline{10101}(2) + 0$$

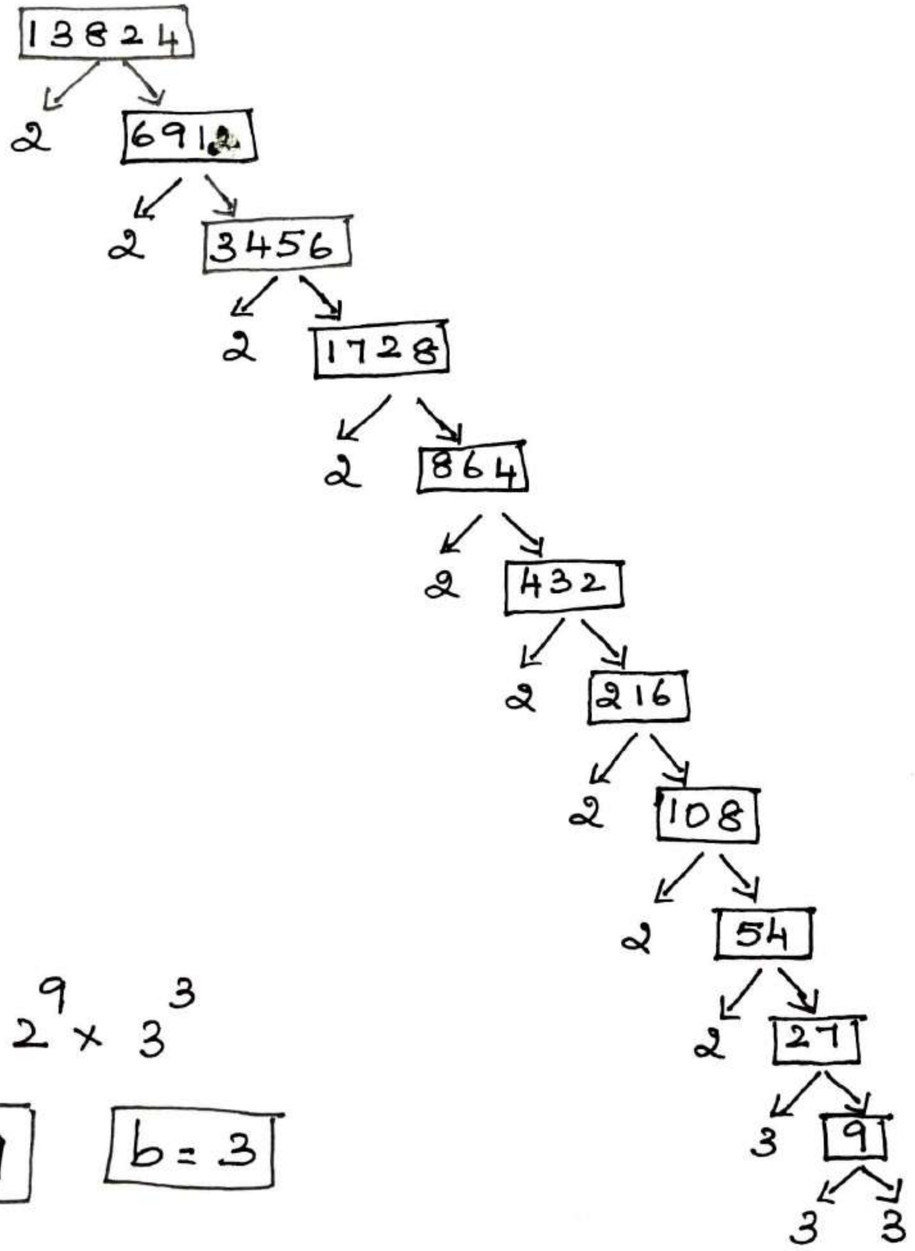
$$\boxed{r = 0}$$

∴ HCF of 252525 and 363636 is 10101

---

④ If $13824 = 2^a \times 3^b$ then find a and b.

Sol:-

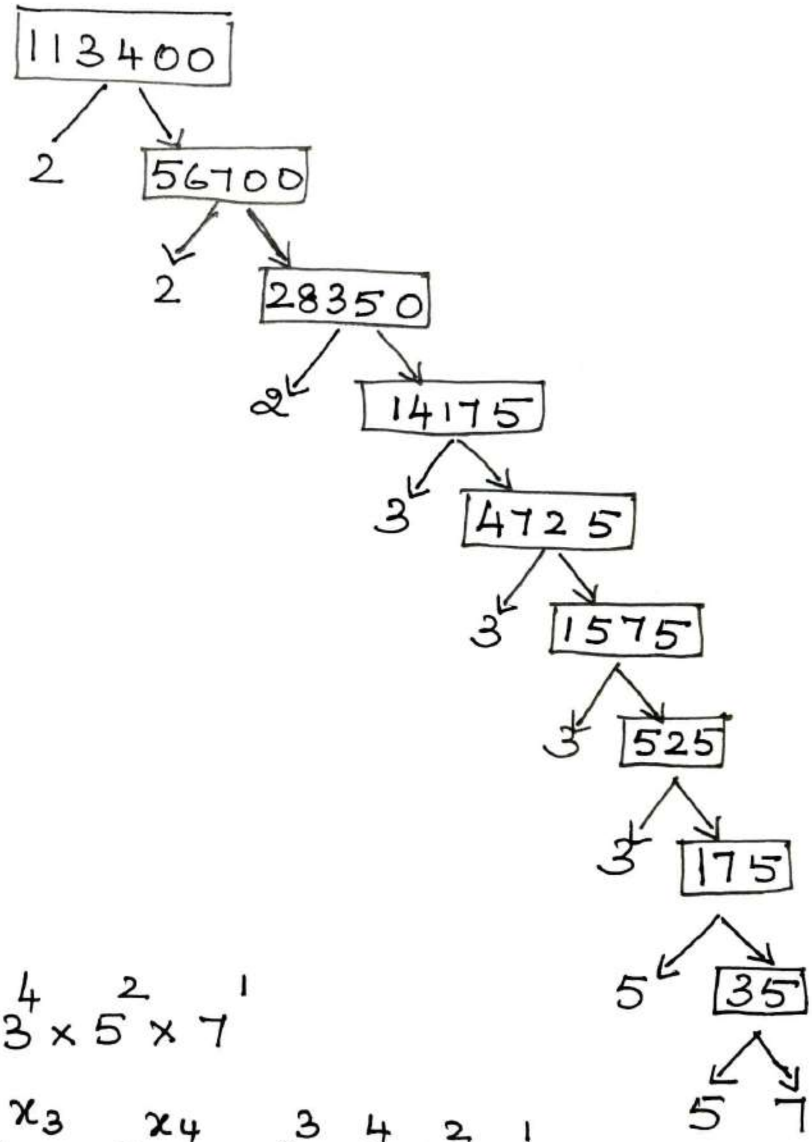


$$\therefore 13824 = 2^9 \times 3^3$$

$$\therefore \boxed{a=9} \quad \boxed{b=3}$$

⑤ If $P_1^{x_1} \times P_2^{x_2} \times P_3^{x_3} \times P_4^{x_4} = 113400$ where P_1, P_2, P_3, P_4 are primes in ascending order and x_1, x_2, x_3, x_4 are integers, find the value of P_1, P_2, P_3, P_4 and x_1, x_2, x_3, x_4.

Sol: –



$$113400 = 2^3 \times 3^4 \times 5^2 \times 7^1$$

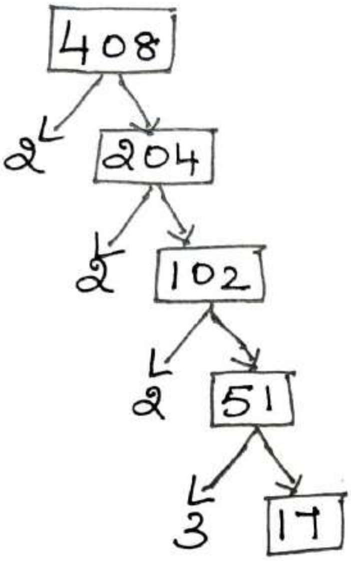
$$\therefore P_1^{x_1} \times P_2^{x_2} \times P_3^{x_3} \times P_4^{x_4} = 2^3 \times 3^4 \times 5^2 \times 7^1$$

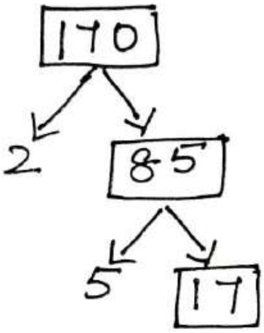
$$\therefore \boxed{P_1 = 2} \quad \boxed{P_2 = 3} \quad \boxed{P_3 = 5} \quad \boxed{P_4 = 7}$$

$$\boxed{x_1 = 3} \quad \boxed{x_2 = 4} \quad \boxed{x_3 = 2} \quad \boxed{x_4 = 1}$$

⑥ Find the LCM and HCF of 408 and 170 by applying the fundamental theorem of arithmetic.

Sol: -





$408 = 2^3 \times 3^1 \times 17^1$

$170 = 2^1 \times 5^1 \times 17^1$

$\text{LCM} = 2^3 \times 3^1 \times 17^1 \times 5$

$= 8 \times 3 \times 17 \times 5$

$\boxed{\text{LCM} = 2040}$

$\text{HCF} = 2^1 \times 17$

$= 2 \times 17$

$\boxed{\text{HCF} = 4}$

⑦ Find the greatest number consisting of 6 digits which is exactly divisible by 24, 15, 36 ?

Sol : -

Step 1 :- To find LCM of 24, 15, 36

$$\begin{array}{r|l} 2 & 24 \\ \hline 2 & 12 \\ \hline 2 & 6 \\ \hline 3 & 3 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 3 & 15 \\ \hline 5 & 5 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 2 & 36 \\ \hline 2 & 18 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

$$24 = 2^3 \times 3^1$$

$$15 = 3^1 \times 5^1$$

$$36 = 2^2 \times 3^2$$

$$\therefore \text{LCM} = 2^3 \times 3^2 \times 5$$

$$= 8 \times 9 \times 5$$

$$\therefore \boxed{\text{LCM} = 360}$$

When a number is exactly divisible by 24, 15, 36, then it must be divisible by 360

Step: 2 To find the greatest 6 digit number divisible by 360

The greatest 6 digit number = 999999

$$\begin{array}{r} 2777 \\ 360\overline{)999999} \\ (-)720\downarrow\quad \\ \hline 2799\quad \\ (-)\,2520\downarrow\ \ \\ \hline 2799\ \ \\ (-)\,2520\downarrow \\ \hline 2799 \\ 2520 \\ \hline 279 \rightarrow r \end{array}$$

$$\therefore \left.\begin{array}{l}\text{Greatest 6 digit number} \\ \text{divisible by 360}\end{array}\right\} = \begin{array}{r} 999999 \\ -\quad 279 \\ \hline \underline{999720} \end{array}$$

$$\therefore \left.\begin{array}{l}\text{The Greatest 6 digit number} \\ \text{divisible by 24, 15, 36}\end{array}\right\} = 999720$$

---

⑧ What is the Smallest number that when divided by three numbers Such as 35, 56 and 91 leaves remainder 7 in each Case ?

Sol :-

Step 1 :- To find LCM of 35, 56, 91

$$\begin{array}{r|l} 5 & 35 \\ \hline 7 & 7 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 2 & 56 \\ \hline 2 & 28 \\ \hline 2 & 14 \\ \hline 7 & 7 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 7 & 91 \\ \hline 13 & 13 \\ \hline & 1 \end{array}$$

$35 = 5^1 \times 7^1$

$56 = 2^3 \times 7^1$

$91 = 7^1 \times 13^1$

$\text{LCM} = 5 \times 7 \times 2^3 \times 13$

$= 5 \times 7 \times 8 \times 13$

$\text{LCM} = 3640$

∴ Smallest number divisible by 35, 56, 91 = 3640

∴ Smallest number divisible by 35, 56, 91 and leaves remainder 7 } = $\begin{array}{r} 3640 \\ +\ 7 \\ \hline \underline{3647} \end{array}$

⑨ Find the least number that is divisible by first ten natural numbers.

Sol:-

The least number divisible by first 10 natural number $\left.\right\}$ = LCM of first 10 natural number

$$\begin{array}{r|l}
2 & 1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7,\ 8,\ 9,\ 10 \\ \hline
2 & 1,\ 1,\ 3,\ 2,\ 5,\ 3,\ 7,\ 4,\ 9,\ 5 \\ \hline
2 & 1,\ 1,\ 3,\ 1,\ 5,\ 3,\ 7,\ 2,\ 9,\ 5 \\ \hline
3 & 1,\ 1,\ 3,\ 1,\ 5,\ 3,\ 7,\ 1,\ 9,\ 5 \\ \hline
3 & 1,\ 1,\ 1,\ 1,\ 5,\ 1,\ 7,\ 1,\ 3,\ 5 \\ \hline
5 & 1,\ 1,\ 1,\ 1,\ 5,\ 1,\ 7,\ 1,\ 1,\ 5 \\ \hline
7 & 1,\ 1,\ 1,\ 1,\ 1,\ 1,\ 7,\ 1,\ 1,\ 1 \\ \hline
 & 1,\ 1,\ 1,\ 1,\ 1,\ 1,\ 1,\ 1,\ 1,\ 1
\end{array}$$

$$\text{LCM} = 2^3 \times 3^2 \times 5 \times 7$$

$$= 8 \times 9 \times 5 \times 7$$

$$\boxed{\text{LCM} = 2520}$$

Exercise - 2.3

① Find the least positive Value of x Such that

(i) $71 \equiv x \pmod 8$

Sol:- $\boxed{R = x}$

$$\begin{array}{r} 8 \\ 8\overline{)71} \\ 64 \\ \hline 7 \rightarrow R \end{array}$$

$\therefore 71 \equiv 7 \pmod 8$

(ii) $78 + x \equiv 3 \pmod 5$

Sol:- $78 + x - 3 \equiv 0 \pmod 5$

$75 + x \equiv 0 \pmod 5$

Here $\boxed{R = 0}$, Means $(75 + x)$ is divisible by 5.

$\therefore$ Least Value of $x = 5$

iè, $75 + 5 = 80$ is divisible by 5

$\therefore \boxed{x = 5}$

(iii) $89 \equiv (x+3) \pmod 4$

Sol:-

$$89 - (x+3) \equiv 0 \pmod 4$$

$$89 - x - 3 \equiv 0 \pmod 4$$

$$86 - x \equiv 0 \pmod 4$$

$\therefore$ Here $\boxed{R=0}$

$\therefore$ $86-x$ is divisible by 4

The least Value of x must be 2

ie, $86 - 2 = 84$ is divisible by 4

$\therefore$ $\boxed{x = 2}$

(iv) $96 \equiv \frac{x}{7} \pmod 5$

Sol:-

$$96 - \frac{x}{7} \equiv 0 \pmod 5$$

Here $\boxed{R=0}$, $96 - \frac{x}{7}$ is divisible by 5

The Least Value of x must be multiple of 7

$\therefore$ If $x = 7$; $96 - \frac{7}{7} \Rightarrow 96-1 \Rightarrow 95$ is divisible by 5

$\therefore$ $\boxed{x = 7}$

(v) $5x \equiv 4 \pmod 6$

Sol :- $5x - 4 \equiv 0 \pmod 6$

Here $\boxed{R = 0}$, $5x - 4$ is divisible by 6

The least Value of x must be 2

ie, $5(2) - 4 \Rightarrow 10 - 4 \Rightarrow 6$

$\therefore$ 6 is divisible by 6.

$\therefore \boxed{x = 2}$

---

② If x is Congruent to 13 modulo 17 then $7x - 3$ is congruent to which number modulo 17?

Sol :- Given :-

$$x \equiv 13 \pmod{17}$$

$$x - 13 \equiv 0 \pmod{17}$$

$\boxed{R = 0}$, $x - 13$ is divisible by 17

$\therefore$ The least Value of x must by 30

ie, $30 - 13 = 17$, divisible by 17.

$$\boxed{x = 30}$$

To Prove:- $7x-3$ is congruent to which number modulo 17

ie, $7x-3 \equiv y \pmod{17}$

Sub $\boxed{x=30}$

$$7(30)-3 \equiv y \pmod{17}$$

$$210-3 \equiv y \pmod{17}$$

$$207 \equiv y \pmod{17}$$

$$207-y \equiv 0 \pmod{17}$$

$\boxed{R=0}$, $207-y$ is divisible by 17

The least value of y must be 3

ie, $207-3 = 204$ is divisible by 17

$$\therefore \boxed{y=3}$$

③ Solve $5x \equiv 4 \pmod{6}$

Sol:-

$$5x-4 \equiv (\bmod 6)$$

$$5x \overset{\curvearrowright}{-4 = 6n} \quad \left[n \text{ is some integer}\right]$$

$$5x = 6n+4$$

$$x = \frac{6n+4}{5}$$

$n = 1$; $x = \frac{6(1)+4}{5} = \frac{6+4}{5} = \frac{10}{5} = 2$

when $n = 2, 3, 4, 5$, the Value is not divisible by 5

$\therefore n = 6$; $x = \frac{6(6)+4}{5} = \frac{36+4}{5} = \frac{40}{5} = 8$

when $n = 7, 8, 9, 10$, the Value is not divisible by 5.

$\therefore n = 11$; $x = \frac{6(11)+4}{5} = \frac{66+4}{5} = \frac{70}{5} = 14$

$\therefore \boxed{x = 2, 8, 14, \cdots}$

④ Solve $3x - 2 \equiv 0 \pmod{11}$

Sol :-

$$3x - 2 = 11n$$

$$3x = 11n + 2$$

$$x = \frac{11n+2}{3}$$

$n = 1$; $x = \frac{11(1)+2}{3} = \frac{11+2}{3} = \frac{13}{3}$

$n = 2$; $x = \frac{11(2)+2}{3} = \frac{22+2}{3} = \frac{24}{3} = \underline{\underline{8}}$

when $n = 3, 4$, the Value is not divisible by 3.

$n = 5$; $x = \frac{11(5)+2}{3} = \frac{55+2}{3} = \frac{57}{3} = \underline{\underline{19}}$

when $n = 6, 7$, the Value is not divisible by 3.

$n = 8$; $x = \frac{11(8)+2}{3} = \frac{88+2}{3} = \frac{90}{3} = \underline{\underline{30}}$

$\therefore$ $\boxed{x = 8, 19, 30, \ldots\ldots}$

⑤ What is the time 100 hours after 7.am?

Sol:- Let time after 100 hours = x

modulus = 24

ie, $100 \equiv x \pmod{24}$

$100 - x \equiv 0 \pmod{24}$

$$24\overline{)100}^{\,4}$$
96
4→R

$\boxed{R=0}$; $100 - x$ is divisible by 24

The least value of $x = 4$

ie, $100 - 4 = 96$ is divisible by 24

$\therefore \boxed{x = 4}$

$\therefore$ The time 100hrs after 7.am = 7 + 4

= 11am

---

⑥ What is the time 15 hours before 11 Pm ?

Sol:- Let the time before 15 hours = x

modulus = 12

[Reason 1:- 15 is not divisible by 24

Reason 2:- 11 o' clock comes Once in 12 hrs] ie, $15 \equiv x \pmod{12}$

$$15 - x \equiv 0 \pmod{12}$$

$\boxed{R=0}$, $15 - x$ is divisible by 12

The least value of $x = 3$

$$\begin{array}{r} 1 \\ 12\overline{)15} \\ \underline{12} \\ 3 \rightarrow R \end{array}$$

$\therefore$ $15 - 3 = 12$ is divisible by 12

$\therefore$ $\boxed{x = 3}$

$\therefore$ The time 15 hours before 11pm $= 11 - 3$

$= 8$ pm

---

⑦ Today is Tuesday. My uncle will come after 45 days. In which day my uncle will be coming.

Sol:- Let the day uncle comes $= x$

Modulus = No of days in a week

modulus $= 7$

$\therefore$ $45 \equiv x \pmod{7}$

$$\begin{array}{r} 6 \\ 7\overline{)45} \\ \underline{42} \\ 3 \rightarrow R \end{array}$$

$$\therefore \boxed{R = x = 3}$$

$$45 \equiv 3 \pmod{7}$$

∴ Three days after Tuesday is Friday

---

⑧ Prove that $2^n + 6 \times 9^n$ is always divisible by 7 for any positive integer n.

Sol :- Since $(2^n + 6 \times 9^n)$ is divisible by 7

$$\boxed{R = 0}$$

$$\therefore 2^n + 6 \times 9^n \equiv 0 \pmod{7}$$

$$2^n + 54^n \equiv 0 \pmod{7}$$

$$(2 + 54)^n \equiv 0 \pmod{7}$$

$$(56)^n = 7n$$

For any positive integer n, 56 is divisible by 7.

---

⑨ Find the remainder when 2^{81} is divided by 17.

Sol :-

$$2^{81} \equiv x \pmod{17} \longrightarrow Ⓐ$$

$$\boxed{2^{81} = (2^9)^9}$$

$2^9 = 512$

$$17\overline{)512}$$
3
51↓
02

$$2^9 \equiv x \pmod{17}$$

$$512 \equiv 2 \pmod{17}$$

$$\therefore \boxed{2^9 \equiv 2 \pmod{17}} \longrightarrow ①$$

From A

$$2^{81} \equiv x \pmod{17}$$

$$(2^9)^9 \equiv 2^9 \pmod{17}$$

Put the Value of ①

$$2^{81} \equiv 2 \pmod{17}$$

$$\therefore \boxed{x = 2} \qquad \therefore \boxed{\text{Remainder} = 2}$$

⑩ The duration of flight travel from Chennai to London through British Airlines is approximately 11 hours. The airplane begins its journey on Sunday at 23:30 hours. If the time at Chennai is four and half hours ahead to that of London's time, then find the time at London, when will the flight lands at London Airport.

Sol :-

Total Travelling Time
from CHENNAI To LONDON = 11 hrs

Time at CHENNAI To That
of LONDON'S Time = $4\frac{1}{2}$ hrs ahead
= 4:30 hrs

Depature From Chennai
Airport On Sunday = 23:30 hrs

∴ London Time ⇒

$$\begin{array}{r} = \ 23{:}30 \\ - \ 4{\cdot}30 \\ \hline 19{\cdot}00 \text{ hrs} \end{array}$$

Arrived at London Airport after 11 hrs (Indian Time) $\Bigg\} = \begin{array}{r} 23.30 \\ +\,11.00 \\ \hline 34.30 \\ \hline \end{array}$

We know, A day has 24 hrs

$\therefore$ Indian Time $= \begin{array}{r} 34.30 \\ -\,24.00 \\ \hline 10.30 \\ \hline \end{array}$

ie, Arrived at London Airport after 11 hrs (Indian Time) $\Big\}$ = 10.30 am on Monday

---

(or)

Arrived at London Airport after 11 hrs (London Time) $\Bigg\} = \begin{array}{r} 19.00 \\ +\,11.00 \\ \hline 30.00 \\ \hline \end{array}$ hrs

London Time $= \begin{array}{r} 30.00 \\ -\,24.00 \\ \hline 6.00 \\ \hline \end{array}$

$\therefore$ Arrived at London Airport after 11 hrs (London Time) $\Big\}$ = 6.00 am on Monday

Exercise 2.4

① Find the next three terms of following Sequence.

(i) 8, 24, 72, ...

Sol:-

$$8 \underset{\times 3}{\curvearrowright} 24 \underset{\times 3}{\curvearrowright} 72, \ldots\ldots$$

$\therefore 72 \times 3 = 216$

$216 \times 3 = 648$

$648 \times 3 = 1944$

$\therefore$ The next three term are, 216, 648, 1944

(ii) 5, 1, −3, ...

Sol:-

$$5 \underset{(-4)}{\curvearrowright} 1 \underset{(-4)}{\curvearrowright} -3, \ldots\ldots$$

$\therefore -3-4 = -7$

$-7-4 = -11$

$-11-4 = -15$

Next three terms are −7, −11, −15

(iii) $\frac{1}{4}, \frac{2}{9}, \frac{3}{16}, \ldots\ldots$

Sol The numerator is increased by 1

Denominator is in the form of $2^2, 3^2, \ldots$

ie,

$$\begin{tikzcd} \frac{1}{4} \arrow[r, bend left, "+1"] \arrow[r, bend right] & \frac{2}{9} \arrow[r, bend left, "+1"] \arrow[r, bend right] & \frac{3}{16}, \ldots \\ 2^2 & 3^2 & 4^2 \end{tikzcd}$$

4^{th} term $= \frac{3+1}{5^2} = \frac{4}{25}$

5^{th} term $= \frac{4+1}{6^2} = \frac{5}{36}$

6^{th} term $= \frac{5+1}{7^2} = \frac{6}{49}$

The next three terms are $\frac{4}{25}, \frac{5}{36}, \frac{6}{49}$

---

② Find the first four terms of the sequences whose n^{th} terms are given by

(i) $a_n = n^3 - 2$

Sol:- $a_n = n^3 - 2$

$n=1$; $a_1 = 1^3 - 2 = 1-2 = -1$

$n=2$; $a_2 = 2^3 - 2 = 8-2 = 6$

$n=3$; $a_3 = 3^3 - 2 = 27-2 = 25$

$n=4$; $a_4 = 4^3 - 2 = 64-2 = 62$

∴ The first four terms are

$-1, 6, 25, 62$

(ii) $a_n = (-1)^{n+1} n(n+1)$

Sol:- $a_n = (-1)^{n+1} \cdot n\ (n+1)$

$n=1$; $a_1 = (-1)^{1+1} \cdot 1\ (1+1) = (-1)^2 \cdot 1(2) = 2$

$n=2$; $a_2 = (-1)^{2+1} \cdot 2\ (2+1) = (-1)^3 \cdot 2(3) = -6$

$n=3$; $a_3 = (-1)^{3+1} \cdot 3\ (3+1) = (-1)^4 \cdot 3(4) = 12$

$n=4$; $a_4 = (-1)^{4+1} \cdot 4\ (4+1) = (-1)^5 \cdot 4(5) = -20$

∴ The first four terms are

$2, -6, 12, -20$

(iii) $a_n = 2n^2 - 6$

Sol:- $a_n = 2n^2 - 6$

$n=1$; $a_1 = 2(1)^2 - 6 = 2(1) - 6 = 2 - 6 = -4$

$n=2$; $a_2 = 2(2)^2 - 6 = 2(4) - 6 = 8 - 6 = 2$

$n=3$; $a_3 = 2(3)^2 - 6 = 2(9) - 6 = 18 - 6 = 12$

$n=4$; $a_4 = 2(4)^2 - 6 = 2(16) - 6 = 32 - 6 = 26$

∴ The first four terms are

$-4, 2, 12, 26$

---

③ Find the n^{th} term of the following Sequences

(i) $2, 5, 10, 17, \ldots$

Sol:- $2, 5, 10, 17, \ldots$

$2 = 1^2 + 1$

$5 = 2^2 + 1$

$10 = 3^2 + 1$

$17 = 4^2 + 1$

∴ n^{th} term $= n^2 + 1$

(ii) $0, \frac{1}{2}, \frac{2}{3}, \ldots\ldots$

Sol :- $0 = \frac{1-1}{1}$

$\frac{1}{2} = \frac{2-1}{2}$

$\frac{2}{3} = \frac{3-1}{3}$

$\therefore$ nth term $= \frac{n-1}{n}$

(iii) $3, 8, 13, 18, \ldots\ldots$

Sol :- $3 = 5(1) - 2$

$8 = 5(2) - 2$

$13 = 5(3) - 2$

$18 = 5(4) - 2$

$\therefore$ nth term $= 5n-2$

---

④ Find the indicated terms of the Sequences whose nth terms are given by

(i) $a_n = \frac{5n}{n+2}$; a_6 and a_{13}

Sol :- $a_n = \frac{5n}{n+2}$

$$a_6 = \frac{5(6)}{6+2} = \frac{\overset{15}{\cancel{30}}}{\underset{4}{\cancel{8}}} = \frac{15}{4}$$

$$a_{13} = \frac{5(13)}{13+2} = \frac{\overset{13}{\cancel{65}}}{\underset{3}{\cancel{15}}} = \frac{13}{3}$$

$\therefore \boxed{a_6 = \frac{15}{4}}$ $\boxed{a_{13} = \frac{13}{3}}$

(ii) $a_n = -(n^2-4)$; a_4 and a_{11}

Sol :- $a_n = -(n^2-4)$

$$a_4 = -(4^2-4) = -(16-4) = -12$$

$$a_{11} = -(11^2-4) = -(121-4) = -117$$

$\therefore \boxed{a_4 = -12}$ $\boxed{a_{11} = -117}$

⑤ Find a_8 and a_{15} whose n^{th} term is

$$a_n = \begin{cases} \frac{n^2-1}{n+3} ; & n \text{ is even} ; n \in N \\ \frac{n^2}{2n+1} ; & n \text{ is odd} ; n \in N \end{cases}$$

Sol:-

$\underset{\sim\sim}{a_8}$ [8 is even number]

$$\therefore \quad a_n = \frac{n^2-1}{n+3}$$

$$\boxed{n = 8} \quad a_8 = \frac{8^2-1}{8+3} = \frac{64-1}{11} = \frac{63}{11}$$

$\underset{\sim\sim}{a_{15}}$ [15 is odd number]

$$a_n = \frac{n^2}{2n+1}$$

$$\boxed{n = 15} \quad a_{15} = \frac{15^2}{2(15)+1} = \frac{225}{30+1} = \frac{225}{31}$$

$$\therefore \quad \boxed{a_8 = \frac{63}{11}} \qquad \boxed{a_{15} = \frac{225}{31}}$$

---

⑥. If $a_1 = 1$; $a_2 = 1$ and $a_n = 2a_{n-1} + a_{n-2}$ $n \geq 3$ $n \in N$, then find the first six terms of the sequence.

Sol :- $a_1 = 1$

$a_2 = 1$

$a_n = 2a_{n-1} + a_{n-2}$

$n \geq 3 \Rightarrow n = 3, 4, 5, 6$

To find :- First six terms

$$a_n = 2a_{n-1} + a_{n-2}$$

$n = 3$; $a_3 = 2a_{3-1} + a_{3-2}$

$= 2a_2 + a_1$

$= 2(1) + 1$

$= 2 + 1$

$$\boxed{a_3 = 3}$$

$n = 4$; $a_4 = 2a_{4-1} + a_{4-2}$

$= 2a_3 + a_2$

$= 2(3) + 1$

$= 6 + 1$

$$\boxed{a_4 = 7}$$

$n = 5$; $a_5 = 2a_{5-1} + a_{5-2}$

$= 2a_4 + a_3$

$= 2(7) + 3$

$= 14 + 3$

$\boxed{a_5 = 17}$

$n = 6$; $a_6 = 2a_{6-1} + a_{6-2}$

$= 2a_5 + a_4$

$= 2(17) + 7$

$= 34 + 7$

$\boxed{a_6 = 41}$

$\therefore$ The first six terms are

1, 1, 3, 7, 17, 41

Arithmetic Progression

1. Format of A.P

$a, a+d, a+2d, a+3d, \ldots\ldots$

a = First term

d = common difference.

2. n^{th} term of AP

$$t_n = a + (n-1)d.$$

3. Last term

l = last term

$$\boxed{l = a + (n-1)d}$$

or

$$\boxed{n = \left(\frac{l-a}{d}\right) + 1}$$

4. Three Consecutive terms of AP

$a-d, a, a+d.$

Exercise 2.5

① Check whether the following Sequences are in A.P.

(i) $a-3, a-5, a-7, \cdots$

Sol:- If the Common difference are equal, then the Sequences is in AP

ie, $t_2 - t_1 = t_3 - t_2 = \cdots\cdots$

$$t_2 - t_1 = (a-5) - (a-3)$$

$$= \cancel{a} - 5 - \cancel{a} + 3$$

$$\boxed{t_2 - t_1 = -2}$$

$$t_3 - t_2 = (a-7) - (a-5)$$

$$= \cancel{a} - 7 - \cancel{a} + 5$$

$$\boxed{t_3 - t_2 = -2}$$

∵ The difference are equal.

∴ It is in A.p

(ii) $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots$

Sol:-

$$t_2 - t_1 = t_3 - t_2 = t_4 - t_3$$

$$\frac{1}{3} - \frac{1}{2} = \frac{1}{4} - \frac{1}{3} = \frac{1}{5} - \frac{1}{4}$$

$$\frac{2-3}{6} = \frac{3-4}{12} = \frac{4-5}{20}$$

$$\frac{-1}{6} \neq \frac{-1}{12} \neq \frac{-1}{20}$$

∴ It is not in A.P

(iii) 9, 13, 17, 21, 25

Sol:-

$$t_2 - t_1 = t_3 - t_2 = t_4 - t_3 = t_5 - t_4$$

$$13 - 9 = 17 - 13 = 21 - 17 = 25 - 21$$

$$4 = 4 = 4 = 4$$

∴ It is in A.P

iv) $\frac{-1}{3}, 0, \frac{1}{3}, \frac{2}{3}, \ldots$

Sol :-

$$t_2 - t_1 = t_3 - t_2 = t_4 - t_3$$

$$0 - \left(\frac{-1}{3}\right) = \left(\frac{1}{3} - 0\right) = \frac{2}{3} - \frac{1}{3}$$

$$0 + \frac{1}{3} = \frac{1}{3} = \frac{2-1}{3}$$

$$\frac{1}{3} = \frac{1}{3} = \frac{1}{3}$$

$\therefore$ It is in A.P.

(v) $1, -1, 1, -1, 1, -1$

Sol :- $t_2 - t_1 = t_3 - t_2 = t_4 - t_3 = t_5 - t_4 = t_6 - t_5$

$$(-1-1) = (1-(-1)) = (-1-1) = (1-(-1)) = (-1-1)$$

$$-2 = (1+1) = (-2) = (1+1) = (-2)$$

$$-2 \neq 2 \neq -2 \neq 2 \neq -2$$

$\therefore$ It is not in A.P.

---

② Find term a and common difference d are given below. Find the corresponding A.P.

(i) $a = 5$, $d = 6$

Sol :- $A.P = a, a+d, a+2d, a+3d, \ldots\ldots$

$5, (5+6), (5+2(6)), (5+3(6)), \ldots\ldots$

$5, 11, (5+12), (5+18), \ldots\ldots$

$5, 11, 17, 23, \ldots\ldots$

(ii) $a = 7$, $d = -5$

Sol :- $A.P. = a, a+d, a+2d, a+3d, \ldots$

$7, (7-5), (7+2(-5)), (7+3(-5)) \ldots\ldots$

$7, 2, (7-10), (7-15), \ldots\ldots$

$7, 2, -3, -8, \ldots\ldots$

(iii) $a = \frac{3}{4}$, $d = \frac{1}{2}$

Sol :- $A.P = a, a+d, a+2d, a+3d, \ldots\ldots$

$\frac{3}{4}, \left(\frac{3}{4}+\frac{1}{2}\right), \left(\frac{3}{4}+2\left(\frac{1}{2}\right)\right); \left(\frac{3}{4}+3\left(\frac{1}{2}\right)\right), \ldots\ldots$

$\frac{3}{4}, \left(\frac{3+2}{4}\right), \left(\frac{3}{4}+\frac{2}{2}\right), \left(\frac{3}{4}+\frac{3}{2}\right), \ldots\ldots$

$\frac{3}{4}, \frac{5}{4}, \left(\frac{3+4}{4}\right), \left(\frac{3+6}{4}\right), \ldots\ldots$

$\frac{3}{4}, \frac{5}{4}, \frac{7}{4}, \frac{9}{4}, \ldots\ldots$

---

③ Find the first term and common difference of the Arithmetic progressions whose terms are given below

(i) $t_n = -3 + 2n$

Sol :- $t_1 = -3 + 2(1) = -3 + 2$

$\boxed{t_1 = -1}$

$t_2 = -3 + 2(2) = -3 + 4$

$\boxed{t_2 = 1}$

$t_3 = -3 + 2(3) = -3 + 6$

$\boxed{t_3 = 3}$

$\therefore$ A.P. $\Rightarrow -1, 1, 3, \ldots\ldots$

First term $\Rightarrow \boxed{a = -1}$

Common difference $\Rightarrow d = 1 - (-1) = 1 + 1$

$\boxed{d = 2}$

(ii) $t_n = 4 - 7n$

Sol:- $t_n = 4 - 7n$

$t_1 = 4 - 7(1) = 4 - 7 = -3$

$\boxed{t_1 = -3}$

$t_2 = 4 - 7(2) = 4 - 14$

$\boxed{t_2 = -10}$

$t_3 = 4 - 7(3) = 4 - 21$

$\boxed{t_3 = -17}$

$\therefore$ A.P. $\Rightarrow -3, -10, -17, \ldots\ldots$

First term $\Rightarrow \boxed{a = -3}$

Common difference $\Rightarrow d = -10 - (-3)$

$= -10 + 3$

$\boxed{d = -7}$

---

④ Find the 19th term of an AP

$-11, -15, -19, \ldots\ldots$

Sol:- A.P. $\Rightarrow -11, -15, -19, \ldots\ldots$

$\boxed{a = -11}$

$d = -15 - (-11)$

$d = -15 + 11$

$\boxed{d = -4}$

$\boxed{n = 19}$ → (Given :- 19th term)

$\therefore \boxed{t_n = a + (n-1)d}$

$\therefore t_{19} = -11 + (19-1)(-4)$

$= -11 + 18(-4)$

$= -11 - 72$

$\boxed{t_{19} = -83}$

---

(5) Which term of an A.p 16, 11, 6, 1.... is -54 ?

Sol :- Given :- AP ⇒ 16, 11, 6, 1, ... is -54

$\boxed{a = 16}$

$d = 11 - 16$

$\boxed{d = -5}$

$\boxed{t_n = -54}$ $\left[\text{is} -54 = t_n \right]$

To find :- $n = ?$ [Which term = n]

$$t_n = -54$$

$$a + (n-1)d = -54$$

$$\overrightarrow{16 + (n-1)(-5) = -54}$$

$$(n-1)(-5) = -54 - 16$$

$$(n-1)\underrightarrow{(-5) = -70}$$

$$(n-1) = \frac{\cancel{+70}^{14}}{\cancel{+5}_{1}}$$

$$n \overrightarrow{-1 = 14}$$

$$n = 14 + 1$$

$$\boxed{n = 15}$$

---

⑥ Find the middle term (s) of an A.P. 9, 15, 21, 27, ..., 183

Sol :- Given :- A.P. => 9, 15, 21, 27, ..., (183)

$$\boxed{a = 9}$$

$$d = 15 - 9$$

$$\boxed{d = 6}$$

$$\boxed{l = 183}$$

(l is last term after comma)

To find :- Middle term(s)

To find middle term, We must find number of terms (n)

$$n = \left(\frac{l-a}{d}\right)+1$$

$$= \left(\frac{183-9}{6}\right)+1$$

$$= \left(\frac{\overset{29}{\cancel{174}}}{\cancel{6}}\right)+1$$

$$= 29+1$$

$$\boxed{n = 30}$$

$\therefore$ Middle term must be 15 and 16

$a=9$, $d=6$, $n=15$

$$t_n = a+(n-1)d$$

$$t_{15} = 9+(15-1)6$$

$$= 9+14(6)$$

$$= 9+84$$

$$\boxed{t_{15} = 93}$$

$a=9$, $d=6$, $n=16$

$$t_n = a+(n-1)d$$

$$t_{16} = 9+(16-1)6$$

$$= 9+15(6)$$

$$= 9+90$$

$$\boxed{t_{16} = 99}$$

⑦ If nine times ninth term is equal to fifteen times fifteenth term, show that six times twenty fourth term is zero.

Sol :- Given :

9 times, 9th term = 15 times, 15th term

To prove :- 6 times, 24th term = 0

ie, $\boxed{6t_{24} = 0}$ $t_{24} = a + 23d$

9 times, 9th term = 15 times 15th term

$9\,t_9 = 15\,t_{15}$ $\left[\begin{array}{l} \text{times} = \text{multiply} \\ \text{term} = t_n \end{array}\right]$

$$9[a+(9-1)d] = 15[a+(15-1)d]$$

$$9[a+8d] = 15[a+14d]$$

$$9a + 72d = 15a + 210d$$

$$0 = 15a + 210d - 9a - 72d$$

$$0 = 6a + 138d$$

$$0 = 6[a + 23d]$$

$$\boxed{6t_{24} = 0}$$

Hence proved.

⑧ If $3+k$, $18-k$, $5k+1$ are in A.P then find k.

Sol :- To find :- $\boxed{k = ?}$

Given ; $3+k$, $18-k$, $5k+1$ are in A.P.

If it is in A.P. then

$$(18-k)-(3+k) = (5k+1)-(18-k)$$

$$18-k-3-k = 5k+1-18+k$$

$$15-2k = 6k-17$$

$$15+17 = 6k+2k$$

$$32 = 8k$$

$$8k = 32$$

$$k = \frac{32}{8}$$

$$\boxed{k = 4}$$

---

⑨ Find x, y, z, given that the numbers $x, 10, y, 24, z$ are in A.P. ?

Sol :- To find :- x, y and z

Given :- $x, 10, y, 24, z$ are in A.P.

$$\Rightarrow 10 - x = y - 10 = 24 - y = z - 24$$

$$10 - x = y - 10$$

$$10 + 10 = y + x$$

$$20 = x + y$$

$$\boxed{x + y = 20} \rightarrow ①$$

$$y - 10 = 24 - y$$

$$y + y = 24 + 10$$

$$2y = 34$$

$$y = \frac{\overset{17}{\cancel{34}}}{\cancel{2}}$$

$$\boxed{y = 17}$$

$$24 - y = z - 24$$

$$24 + 24 = z + y$$

$$48 = y + z$$

$$\boxed{y + z = 48} \rightarrow ②$$

put the Value of $y = 17$ in ① & ②

$$x + y = 20$$

$$x + 17 = 20$$

$$x = 20 - 17$$

$$\boxed{x = 3}$$

$$y + z = 48$$

$$17 + z = 48$$

$$z = 48 - 17$$

$$\boxed{z = 31}$$

$$\therefore x = 3, \; y = 17, \; z = 31$$

⑩ In a theatre, there are 20 Seats in the front row and 30 rows were allotted. Each Successive row contains two additional Seats than its front row. How many Seats are there in the last row?

Sol:- row 1 = $a = 20$

row 2 = $a+d = 20+2 = 22$

[Each Successive row = 2 + front row]

row 3 = $a+2d = 22+2 = 24$

So on

Number of rows = $n = 30$

AP ⇒ 20, 22, 24, ...

$a = 20$

$d = 22 - 20$

$\boxed{d = 2}$

$\boxed{n = 30}$

To Find :- No of Seats in last row.

ie, t_{30}

$$t_n = a + (n-1)d$$

$$t_{30} = 20 + (30-1)2$$

$$= 20 + 29(2)$$

$$= 20 + 58$$

$$\boxed{t_{30} = 78}$$

$\therefore$ No of Seats in last row = 78

---

⑪ The Sum of three Consecutive terms that are in A.P is 27. and their Product is 288. Find the three terms

Sol :-

Let the three Consecutive terms in AP $\Rightarrow$ $a-d,\ a,\ a+d$

Sum = 27

$$a-\cancel{d}+a+a+\cancel{d} = 27$$

$$3a = 27$$

$$a = \frac{\cancel{27}^{9}}{\cancel{3}_{1}}$$

$$\boxed{a = 9}$$

$$\text{Product} = 288$$

$$(a-d)\,a\,(a+d) = 288$$

$$\boxed{(a-b)(a+b) = a^2-b^2}$$

$$(a^2-d^2)a = 288$$

$$\boxed{\text{put } a=9}$$

$$(9^2-d^2)\,9 = 288$$

$$81-d^2 = \frac{\cancel{288}\ 32}{\cancel{9}_{1}}$$

$$81-d^2 = 32$$

$$81-32 = d^2$$

$$49 = d^2$$

$$d^2 = 49$$

$$d = \sqrt{49}$$

$$\boxed{d = \pm 7}$$

$\boxed{a=9}, \boxed{d=7}$	$\boxed{a=9}, \boxed{d=-7}$
$a-d = 9-7 = 2$	$a-d = 9-(-7) = 9+7 = 16$
$a = 9$	$a = 9$
$a+d = 9+7 = 16$	$a+d = 9-7 = 2$

$\therefore$ The three terms :- 2, 9, 16

---

⑫ The ratio of 6th and 8th term of an AP is 7:9. Find the ratio of 9th term to 13th term

Sol :- To prove :- $\boxed{t_9 : t_{13} = ?}$

Given :- $t_6 : t_8 = 7 : 9$

$$\frac{t_6}{t_8} = \frac{7}{9}$$

$$\frac{a+(6-1)d}{a+(8-1)d} = \frac{7}{9}$$

$$\frac{a+5d}{a+7d} = \frac{7}{9}$$

$$9(a+5d) = 7(a+7d)$$

$$9a+45d = 7a+49d$$

$$9a - 7a = 49d - 45d$$

$$\underrightarrow{2a} = 4d$$

$$a = \frac{4d}{2}$$

$$\boxed{a = 2d} \longrightarrow ①$$

To find :- $t_9 : t_{13}$

$$\frac{t_9}{t_{13}} = \frac{a + (9-1)d}{a + (13-1)d}$$

$$= \frac{a + 8d}{a + 12d}$$ put value of ①

$$= \frac{2d + 8d}{2d + 12d}$$

$$= \frac{\overset{5}{\cancel{10}}\cancel{d}}{\underset{7}{\cancel{14}}\cancel{d}}$$

$$= \frac{5}{7}$$

$$\therefore \boxed{t_9 : t_{13} = 5 : 7}$$

⑬ In a winter Season let us take the temperature of Ooty from Monday to Friday to be in A.P. The Sum of temperatures from Monday to wednesday is 0°c and the Sum of the temperature from Wednesday to Friday is 18°c. Find the temperature On each of the five days

Sol :-

Temperature of ooty from Monday to Friday be in AP.

ie, $\underset{(M)}{a}, \underset{(T)}{a+d}, \underset{(W)}{a+2d}, \underset{(Th)}{a+3d}, \underset{(Fr)}{a+4d}$

$$\left.\begin{matrix}\text{Sum of Temperature from} \\ \text{Monday to Wednesday}\end{matrix}\right\} = 0$$

ie, $a + a+d + a+2d = 0$

$$3a + 3d = 0$$

$$3(a+d) = 0$$

$$a + d = 0$$

$$\therefore \boxed{a = -d} \longrightarrow ①$$

$$\left.\begin{array}{l}\text{Sum of Temperature from}\\ \text{Wednesday to Friday}\end{array}\right\} = 18$$

$$a + 2d + a + 3d + a + 4d = 18$$

$$3a + 9d = 18$$

$$3(a + 3d) = 18$$

$$a + 3d = \frac{\overset{6}{\cancel{18}}}{\underset{1}{\cancel{3}}}$$

$$a + 3d = 6$$

$\boxed{\text{Put } a = -d}$

$$-d + 3d = 6$$

$$2d = 6$$

$$d = \frac{6}{2}$$

$$\boxed{d = 3}$$

$$\therefore \boxed{a = -3}$$

∴ Temperature of five days.

Monday = $\boxed{a = -3}$

Tuesday $= a+d = -3+3 = 0$

Wednesday $= a+2d = -3+2(3) = -3+6 = 3$

Thursday $= a+3d = -3+3(3) = -3+9 = 6$

Friday $= a+4d = -3+4(3) = -3+12 = 9$

$\therefore \boxed{-3, 0, 3, 6, 9}$

---

⑭ Priya earned ₹15000 in the first month. There after her Salary increased by ₹ 1500 per year. Her expenses are ₹13,000 during the first year and the expenses increases by ₹ 900 per year. How long will it take for her to Save ₹20,000 per month.

Sol:-

	INCOME	EXPENSES	SAVING
1^{st} YEAR	15,000	13,000	15000 - 13000 = 2000
2^{nd} YEAR	15000 + 1500 = 16500	13000 + 900 = 13900	16500 - 13900 = 2600
3^{rd} YEAR	16500 + 1500 = 18000	13900 + 900 = 14800	18000 - 14800 = 3200

The Savings are in the form of A.P

2000, 2600, 3200, ----.

To Find :- Number of years to Save ₹ 20000 per month.

ie $t_n = 20000$

$\boxed{a = 2000}$

$d = 2600 - 2000$

$\boxed{d = 600}$

$t_n = 20000$

$a + (n-1)d = 20000$

$2000 \overrightarrow{+ (n-1)600 =} 20000$

$(n-1)600 = 20000 - 2000$

$(n-1)\underrightarrow{600 =} 18000$

$n-1 = \dfrac{\overset{30}{\cancel{18000}}}{\underset{1}{\cancel{600}}}$

$n-1 = 30$

$n = 30+1$

$\boxed{n = 31}$

∴ No of years to Save ₹ 20000 per month = 31 years.

Sum of n terms of an A.P

1. $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

2. $S_n = \frac{n}{2}(a + l)$

Exercise 2.6

① Find the Sum of the following

(i) 3, 7, 11 ...... upto 40 terms

Sol:- $a = 3$

$d = 7 - 3 = 4$

$n = 40$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_{40} = \frac{\overset{20}{\cancel{40}}}{\cancel{2}}\left[2(3) + (40-1)4\right]$$

$$= 20\left[6 + 39(4)\right]$$

(ii) 102, 97, 92, ... upto 27 Terms

Sol:- $a = 102$

$d = 97 - 102 = -5$

$n = 27$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_{27} = \frac{27}{2}\left[2(102) + (27-1)(-5)\right]$$

$$= \frac{27}{2}\left[204 + 26(-5)\right]$$

$$\begin{array}{r} {}^{3} \\ 26 \\ \times\ 5 \\ \hline 130 \end{array}$$

$$= \frac{27}{2}\left[204 - 130\right]$$

$$\begin{array}{r} 204 \\ -130 \\ \hline 74 \end{array}$$

$$= \frac{27}{\cancel{2}_{1}}\left[\overset{37}{\cancel{74}}\right]$$

$$= 27 \times 37$$

$$\begin{array}{r} {}^{4} \\ 27 \\ \times\ 37 \\ \hline 189 \\ 81 \\ \hline 999 \end{array}$$

$$\boxed{S_{27} = 999}$$

(iii) $6 + 13 + 20 + \cdots + 97$

Sol:- $a = 6$

$d = 13 - 6 = 7$

$l = 97$ [Value after '+' sign is l]

$$n = \left(\frac{l - a}{d}\right) + 1$$

$$= \left(\frac{97 - 6}{7}\right) + 1$$

$$= \left(\frac{\overset{13}{\cancel{91}}}{\underset{1}{\cancel{7}}}\right) + 1$$

$$\boxed{n = 14}$$

$$S_n = \frac{n}{2}(a + l)$$

$$S_{14} = \frac{\overset{7}{\cancel{14}}}{\underset{1}{\cancel{2}}}(6 + 97)$$

$$= 7(103)$$

$$\boxed{S_{14} = 721}$$

---

② How many consecutive odd integers beginning with 5 will sum to 480 ?

Sol :-

Consecutive odd integers beginning with 5

$5, 7, 9, 11, \ldots$

$a = 5$

$d = 7 - 5 = 2$

To Find :- $n = ?$ [How many $= n$]

Given :- Sum $= 480$

$S_n = 480$

$$\frac{n}{2}[2a + (n-1)d] = 480$$

$$\frac{n}{2}[2(5) + (n-1)(2)] = 480$$

$$\frac{n}{2}[10 + 2n - 2] = 480$$

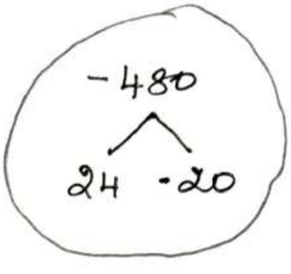
$$\frac{n}{2}[8 + 2n] = 480$$

$$\frac{n}{\cancel{2}} \times \cancel{2}[4 + n] = 480$$

$$n(4 + n) = 480$$

$$4n + n^2 = 480$$

$$n^2 + 4n - 480 = 0$$



$$(n+24)(n-20) = 0$$

$n + 24 = 0$ || $n - 20 = 0$

$n = -24$ || $\boxed{n = 20}$

[Which is impossible]

$$\therefore \boxed{n = 20}$$

---

③ Find the Sum of first 28 terms of an A.P whose n^{th} term is $4n-3$.

Sol:- $\boxed{n = 28}$ $\boxed{t_n = 4n-3}$

First term $\Rightarrow a \Rightarrow t_1 = 4(1) - 3 = 4 - 3 = 1$

$$\boxed{a = 1}$$

last term $\Rightarrow l \Rightarrow t_{28} = 4(28) - 3 = 112 - 3 = 109$

$$\boxed{l = 109}$$

$$\therefore S_n = \frac{n}{2}(a + l)$$

$$S_{28} = \frac{\overset{14}{\cancel{28}}}{\underset{1}{\cancel{2}}}\left[1+109\right]$$

$$= 14 \times 110$$

$$\boxed{S_{28} = 1540}$$

$$\begin{array}{r} 11 \\ 14 \\ \hline 44 \\ 11 \\ \hline 154 \end{array}$$

---

④ The Sum of first n terms of a Certain Series is given as $2n^2 - 3n$. Show that the Series is an A.P.

Sol:-

$$S_n = 2n^2 - 3n$$

$n = 1$; $S_1 = 2(1)^2 - 3(1) = 2(1) - 3 = 2 - 3 = -1$

$n = 2$; $S_2 = 2(2)^2 - 3(2) = 2(4) - 6 = 8 - 6 = 2$

$n = 3$; $S_3 = 2(3)^2 - 3(3) = 2(9) - 9 = 18 - 9 = 9$

$n = 4$; $S_4 = 2(4)^2 - 3(4) = 2(16) - 12 = 32 - 12 = 20$

$\therefore S_1 = \boxed{t_1 = -1} \rightarrow$ ①

$S_2 = t_1 + t_2 = 2 \rightarrow$ ②

$S_3 = t_1 + t_2 + t_3 = 9 \rightarrow$ ③

$S_4 = t_1 + t_2 + t_3 + t_4 = 20 \rightarrow$ ④

Put ① in ②

$$-1 + t_2 = 2$$

$$t_2 = 2+1$$

$$\boxed{t_2 = 3}$$

Put ② in ③

$$2 + t_3 = 9$$

$$t_3 = 9-2$$

$$\boxed{t_3 = 7}$$

Put ③ in ④

$$9 + t_4 = 20$$

$$t_4 = 20-9$$

$$\boxed{t_4 = 11}$$

$\therefore$ A.P $\Rightarrow$ $t_1, t_2, t_3, t_4, \ldots$

$= -1, 3, 7, 11, \ldots$

$\therefore$ The Series is an A.P.

⑤ The 104^{th} term and 4^{th} term of an A.P are 125 and 0. Find the Sum of first 35 terms

Sol:- Given

$$\boxed{t_{104} = 125}$$

$$a + (104-1)d = 125$$

$$a + 103d = 125 \longrightarrow ①$$

$$\boxed{t_4 = 0}$$

$$a + (4-1)d = 0$$

$$a + 3d = 0$$

$$\boxed{a = -3d} \longrightarrow ②$$

put $a = -3d$ in ①

$$-3d + 103d = 125$$

$$100d = 125$$

$$d = \frac{\cancel{125}^{5}}{\cancel{100}_{4}}$$

$$\boxed{d = \frac{5}{4}}$$

Put $d = \frac{5}{4}$ in ②

$$a = -3\left(\frac{5}{4}\right)$$

$$\boxed{a = \frac{-15}{4}}$$

To find :- $S_{35} = ?$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_{35} = \frac{35}{2}\left[2\left(\frac{-15}{4}\right) + (35-1)\left(\frac{5}{4}\right)\right]$$

$$= \frac{35}{2}\left[\frac{-30}{4} + 34\left(\frac{5}{4}\right)\right]$$

$$= \frac{35}{2}\left[\frac{-30}{4} + \frac{170}{4}\right]$$

$$= \frac{35}{2}\left[\frac{-30+170}{4}\right]$$

$$\begin{array}{r} 170 \\ -\;30 \\ \hline 140 \end{array}$$

$$= \frac{35}{\cancel{2}_{1}} \times \frac{\overset{35}{\cancel{\overset{70}{\cancel{140}}}}}{\cancel{4}_{2}}$$

$$= \frac{35 \times 35}{2}$$

$$\begin{array}{r} {}^{2} \\ 35 \\ 35 \\ \hline 175 \\ 105 \\ \hline 1225 \end{array}$$

$$= \frac{1225}{2}$$

$$\boxed{S_{35} = 612.5}$$

$$\begin{array}{r} 612.5 \\ 2\,\overline{)\,1225} \\ -12\downarrow \\ \hline 2 \\ 2\downarrow \\ \hline .5 \\ 4 \\ \hline 10 \\ 10 \\ \hline 0 \end{array}$$

⑥ Find the Sum of all odd positive integers less than 450.

Sol:-

Odd positive integers less than 450

$1+3+5+7+9+ \cdots\cdots + 449$

$a = 1$

$d = 3-1 = 2$

$l = 449$ [Number after +499 is l]

$$\therefore n = \left[\frac{l-a}{d}\right] + 1$$

$$= \left(\frac{449-1}{2}\right) + 1$$

$$= \left(\frac{\overset{224}{\cancel{448}}}{\underset{1}{\cancel{2}}}\right) + 1$$

$n = 224 + 1$

$$\boxed{n = 225}$$

$$\therefore S_n = \frac{n}{2}(a+l)$$

$$S_{225} = \frac{225}{2}(1+449)$$

$$= \frac{225}{\cancel{2}_{1}}\left(\overset{225}{\cancel{450}}\right)$$

$$\boxed{S_{225} = 50625}$$

$$\begin{array}{r} \overset{1}{2}\overset{\overset{1}{2}}{2}5 \\ 225 \\ \hline 1125 \\ 450 \\ 450 \\ \hline 50625 \\ \hline \end{array}$$

---

⑦ Find the sum of all natural numbers between 602 and 902 which are not divisible by 4.

Sol:-

Natural numbers between 602 and 902 are

603, 604, 605, ..., 901

$a = 603$

$d = 604 - 603 = 1$

$l = 901$

$$n = \left(\frac{l-a}{d}\right) + 1$$

$$= \left(\frac{901-603}{1}\right) + 1$$

$= 298 + 1$

$\boxed{n = 299}$

$S_n = \frac{n}{2}(a + l)$

$S_{299} = \frac{299}{2}(603 + 901)$

$= \frac{299}{\cancel{2}_{1}} \times \overset{752}{\cancel{1504}}$

$\boxed{S_{299} = 224848}$

$$\begin{array}{r} 901 \\ 603 \\ \hline 1504 \end{array}$$

$$\begin{array}{r} {}^{46}\,{}^{46} \\ 299 \\ 752 \\ \hline 598 \\ 1495 \\ 2093 \\ \hline 224848 \end{array}$$

Natural numbers between 602 and 902

Divisible by 4

$604 + 608 + 612 + \cdots + 900$

$a = 604$

$d = 608 - 604 = 4$

$l = 900$

$n = \left(\frac{l - a}{d}\right) + 1$

$$n = \left(\frac{900-604}{4}\right)+1$$

$$= \left(\frac{\overset{74}{\cancel{296}}}{\underset{1}{\cancel{4}}}\right)+1$$

$$= 74+1$$

$$\boxed{n = 75}$$

$$S_n = \frac{n}{2}(a+l)$$

$$S_{75} = \frac{75}{2}(604+900)$$

$$= \frac{75}{\underset{1}{\cancel{2}}} \times \overset{752}{\cancel{1504}}$$

$$\begin{array}{r} {}^{3}_{2}\ {}^{1}_{1} \\ 752 \\ \times\ 75 \\ \hline 3760 \\ 5264 \\ \hline 56\overset{}{4}00 \end{array}$$

$$\boxed{S_{75} = 56400}$$

Sum of Natural Number Not Divisible by 4 $\Big\}$ = Sum of all Natural No − Sum of Natural NO Divisible by 4

$$= 224848 - 56400$$

$$= 168448$$

---

⑧ Raghu wish to buy a laptop. He Can buy it by paying ₹40,000 Cash Or by giving it in 10 installments as ₹4800 in first month, ₹4750 in the Second month, ₹4700 in the third month and so on. If he pays the money in this fashion, find.

(i) total amount paid in 10 installments

Sol:-

1st installment = ₹4800

2nd installment = ₹4750

3rd installment = ₹4700

and so on

It is in A.P

$\therefore$ 4800, 4750, 4700, ----

$a = 4800$

$d = 4750 - 4800 = -50$

Total installment $\Rightarrow n = 10$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_{10} = \frac{\overset{5}{\cancel{10}}}{\underset{1}{\cancel{2}}}\left[2(4800) + (10-1)(-50)\right]$$

$$= 5\left[9600 + 9(-50)\right]$$

$$= 5\left[9600 - 450\right]$$

$$= 5\left[9150\right]$$

$$\boxed{S_{10} = 45750}$$

∴ Amount paid in 10 installments

= ₹45750

(ii) How much extra amount that he has to pay than the cost ?

Sol:-

Extra Amount = Installment Amount - Actual Amount

= 45750 - 40000

∴ $\boxed{\text{Extra Amount} = ₹5750}$

⑨ A man repays a loan of ₹65,000 by paying ₹400 in the first month and then increasing the payment by ₹300 every month. How long will it take for him to clear the loan?

Sol :-

Total loan amount = ₹65000

Amount paid on 1^{st} month = ₹400

Amount paid on 2^{nd} month = 400 + 300 = ₹700

Amount paid on 3^{rd} month = 700 + 300 = ₹1000

$\therefore$ 400, 700, 1000, ...

It is in A.P

$a = 400$

$d = 700 - 400 = 300$

$n = ?$ [How long = n]

$S_n = 65000$

$$\frac{n}{2}\left[2a + (n-1)d\right] = 65000$$

$$\frac{n}{2}\left[2(400)+(n-1)(300)\right]=65000$$

$$n\left[\overparen{800+300n-300}\right]=65000\times 2$$

$$n\left[500+300n\right]=130000$$

$$500n+300n^2-130000=0$$

$$300n^2+500n-130000=0$$

[÷ by 100]

$$3n^2+5n-1300=0$$

$$\boxed{n=\frac{-b\pm\sqrt{b^2-4ac}}{2a}}$$

$a=3$, $b=5$, $c=-1300$

$$\therefore\ n=\frac{-5\pm\sqrt{5^2-4(3)(-1300)}}{2(3)}$$

$$=\frac{-5\pm\sqrt{25+15600}}{6}$$

$$\begin{array}{r} 12 \\ 13 \\ \hline 36 \\ 12 \\ \hline 15600 \end{array}$$

$$= \frac{-5 \pm \sqrt{15625}}{6}$$

$$= \frac{-5 \pm 125}{6}$$

$$\begin{array}{r|l} & 125 \\ 1 & \overline{1\,56\,25} \\ & 1 \\ \hline 22 & 056 \\ & (-)\,44 \\ \hline 245 & 1225 \\ & (-)\,1225 \\ \hline & 0 \end{array}$$

$\therefore n = \frac{-5+125}{6}$ | $n = \frac{-5-125}{6}$

$n = \frac{\overset{20}{\cancel{120}}}{\cancel{6}}$ | $n = \frac{-130}{6}$

$\boxed{n = 20}$ | [Omitting −ve Value]

∴ He take 20 months to clear loan.

⑩ A brick staircase has a total of 30 steps. The bottom step requires 100 bricks. Each successive step requires two bricks less than the previous step.

(i) How many bricks are required for the top most step?

Sol :- Total Steps $\Rightarrow n = 30$

$100, (100-2), \ldots$

$100, 98, 96, \ldots$

$a = 100$

$d = 98 - 100 = -2$

$t_n = a + (n-1)d$

$t_{30} = 100 + (30-1)(-2)$

$= 100 + 29(-2)$

$= 100 - 58$

$$\boxed{t_{30} = 42}$$

$\therefore$ Top most Step requires 42 Bricks.

(ii) How many bricks are required to build the stair case ?

Sol :- $S_n = \frac{n}{2}[2a + (n-1)d]$

$$S_{30} = \frac{\overset{15}{\cancel{30}}}{\underset{1}{\cancel{2}}}[2(100) + (30-1)(-2)]$$

$= 15[200 + 29(-2)]$

$= 15[200 - 58]$

$= 15[142]$

$$\boxed{S_{30} = 2130}$$

$$\begin{array}{r} \overset{2\,1}{1}42 \\ \times 15 \\ \hline 710 \\ 142 \\ \hline 2130 \\ \hline \end{array}$$

$\therefore$ Total Bricks required to Build Stair Case $= 2130$

---

⑪ If $S_1, S_2, S_3, \dots, S_m$ are the Sums of n terms of m A.P.'s whose first terms are $1, 2, 3, \dots, m$ and whose Common difference are $1, 3, 5, \dots (2m-1)$ respectively, then Show that

$$S_1 + S_2 + S_3 + \dots + S_m = \frac{1}{2} mn(mn+1)$$

Sol: - First terms $\Rightarrow 1, 2, 3, \dots, m$

Common Difference $\Rightarrow 1, 3, 5, \dots, (2m-1)$

Sum of n terms of m A.P's are

$S_1, S_2, S_3, \dots, S_m$.

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$a = 1, \; d = 1 \Rightarrow S_1 = \frac{n}{2}\left[2(1) + (n-1)(1)\right]$

$$= \frac{n}{2}\left[2 + n - 1\right]$$

$$\boxed{S_1 = \frac{n}{2}\left[n+1\right]}$$

$a = 2, \; d = 3 \Rightarrow S_2 = \frac{n}{2}\left[2(2) + (n-1)3\right]$

$$= \frac{n}{2}\left[4 + 3n - 3\right]$$

$$\boxed{S_2 = \frac{n}{2}\left[3n+1\right]}$$

$a = 3; \; d = 5 \Rightarrow S_3 = \frac{n}{2}\left[2(3) + (n-1)5\right]$

$$= \frac{n}{2}\left[6 + 5n - 5\right]$$

$$\boxed{S_3 = \frac{n}{2}\left[5n+1\right]}$$

$$a = m, \quad d = (2m-1) \Rightarrow S_m = \frac{n}{2}\left[2(m) + (n-1)(2m-1)\right]$$

$$= \frac{n}{2}\left[\cancel{2m} + 2mn - \cancel{2m} - n + 1\right]$$

$$\boxed{S_m = \frac{n}{2}\left[2mn - n + 1\right]}$$

$$\therefore S_1 + S_2 + S_3 + \cdots + S_m = \frac{n}{2}(n+1) + \frac{n}{2}(3n+1)$$

$$+ \frac{n}{2}(5n+1) + \cdots + \frac{n}{2}(2mn - n + 1)$$

$$= \frac{n}{2}\left[n+1 + 3n+1 + 5n+1 + \cdots + \underbrace{2mn - n}_{} + 1\right]$$

$$= \frac{n}{2}\left[n+1 + 3n+1 + 5n+1 + \cdots + (2m-1)n+1\right]$$

$$= \frac{n}{2}\left[n\left[1+3+5+\cdots+(2m-1)\right] + m\right]$$

$$\boxed{a = 1, \ d = 3-1 = 2, \ n = m}$$

$$= \frac{n}{2}\left[n \cdot \left[\frac{m}{2}\left[2(1) + (m-1)2\right]\right] + m\right]$$

$$= \frac{n}{2}\left[n \cdot \left[\frac{m}{2}\left(\cancel{2} + 2m - \cancel{2}\right)\right] + m\right]$$

$$= \frac{n}{2}\left[n \cdot \left[\frac{m}{\cancel{2}}\left(\cancel{2}m\right)\right] + m\right]$$

$$= \frac{n}{2}\left[n \cdot \left[m^2\right] + m\right]$$

$$= \frac{1}{2}\left[n^2 m^2 + mn\right]$$

$$= \frac{1}{2} mn\left[mn + 1\right]$$

$$\therefore S_1 + S_2 + S_3 + \cdots + S_m = \frac{1}{2} mn\left[mn+1\right]$$

---

(12) Find the Sum $\left[\frac{a-b}{a+b} + \frac{3a-2b}{a+b} + \frac{5a-3b}{a+b} + \cdots \text{ to 12 terms}\right]$

Sol :→

$$a = \frac{a-b}{a+b}$$

$$d = \frac{3a-2b}{a+b} - \frac{a-b}{a+b}$$

$$\frac{3a-2b-a+b}{a+b}$$

$$d = \frac{2a-b}{a+b}$$

$$n = 12$$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_{12} = \frac{\overset{6}{\cancel{12}}}{\cancel{2}}\left[2\left(\frac{a-b}{a+b}\right) + (12-1)\left(\frac{2a-b}{a+b}\right)\right]$$

$$= 6\left[\frac{2a-2b}{a+b} + 11\left(\frac{2a-b}{a+b}\right)\right]$$

$$= 6\left[\frac{2a-2b}{a+b} + \frac{22a-11b}{a+b}\right]$$

$$= 6\left[\frac{2a-2b+22a-11b}{a+b}\right]$$

$$\boxed{S_{12} = 6\left[\frac{24a-13b}{a+b}\right]}$$

Geometric Progression
[G.P]

1. Format of G.P

$a, ar, ar^2, ar^3, \ldots$

a = first term

r = Common ratio

2. n^{th} term of G.P

$$t_n = ar^{n-1}$$

3. Three Consecutive terms of G.P

$$\frac{a}{r}, a, ar.$$

Exercise 2.7

1. Which of the following Sequences are in G.P ?

(i) 3, 9, 27, 81, ...

Sol $$\frac{\cancel{9}^{3}}{\cancel{3}_{1}} = \frac{\cancel{27}^{3}}{\cancel{9}_{1}} = \frac{\cancel{81}^{3}}{\cancel{27}_{1}}$$

$$\boxed{r = 3}$$

If the Common ratio are Same then it is in G.p.

(ii) 4, 44, 444, 4444, ...

Sol

$$\frac{\cancel{44}^{11}}{\cancel{4}_{1}} = \frac{\cancel{444}^{111}}{\cancel{44}_{11}} = \frac{\cancel{4444}^{1111}}{\cancel{444}_{111}}$$

$\therefore 11 \neq 111 \neq 1111$

$\therefore$ It is not in G.p.

(iii) $0.5,\ 0.05,\ 0.005,\ \ldots\ldots$

Sol :-

$$\frac{0.05}{0.5}\times\frac{100}{100} = \frac{0.005}{0.05}\times\frac{1000}{1000}$$ [To Remove Decimal point]

$$\frac{\overset{1}{\cancel{5}}}{\underset{10}{\cancel{50}}} = \frac{\overset{1}{\cancel{5}}}{\cancel{50}_{10}}$$

$$\therefore \boxed{r = \frac{1}{10}}$$

It is in G.P.

(iv) $\frac{1}{3},\ \frac{1}{6},\ \frac{1}{12},\ \ldots\ldots$

Sol $$\frac{\frac{1}{6}}{\frac{1}{3}} = \frac{\frac{1}{12}}{\frac{1}{6}}$$ [Reciprocate]

$$\frac{1}{\underset{2}{\cancel{6}}}\times\frac{\overset{1}{\cancel{3}}}{1} = \frac{1}{\cancel{12}_{2}}\times\frac{\overset{1}{\cancel{6}}}{1}$$

$$\frac{1}{2} = \frac{1}{2}$$

$$\therefore r = \frac{1}{2}$$

It is in G.P

(v) $1, -5, 25, -125, \ldots\ldots$

Sol:- $$\frac{-5}{1} = \frac{\overset{5}{\cancel{25}}}{-\cancel{5}} = \frac{-\overset{5}{\cancel{125}}}{\cancel{25}}$$

$$-5 = -5 = -5$$

$$\therefore \boxed{r = -5}$$

$\therefore$ It is in G.P

(vi) $120, 60, 30, 18, \ldots$

Sol:- $$\frac{\overset{1}{\cancel{60}}}{\underset{2}{\cancel{120}}} = \frac{\overset{1}{\cancel{30}}}{\underset{2}{\cancel{60}}} = \frac{\overset{3}{\cancel{\overset{6}{\cancel{18}}}}}{\underset{5}{\cancel{\underset{10}{\cancel{30}}}}}, \ldots$$

$$\therefore \frac{1}{2} = \frac{1}{2} \neq \frac{3}{5}$$

$\therefore$ It is not in Gp

(vii) $16, 4, 1, \frac{1}{4}, \ldots\ldots$

Sol:- $$\frac{\overset{1}{\cancel{4}}}{\underset{4}{\cancel{16}}} = \frac{1}{4} = \frac{\frac{1}{4}}{1}$$

$$\frac{1}{4} = \frac{1}{4} = \frac{1}{4}$$

$$r = \frac{1}{4}$$

$\therefore$ It is in G.P.

② Write the first 3 terms of the G.P whose first term and the common ratio are given below:

(i) $a = 6$, $r = 3$

Sol

The first 3 terms of GP

a, ar, ar^2

$\Rightarrow 6, (6\times 3), (6\times 3^2)$

$6, 18, (6\times 9)$

$\boxed{6, 18, 54}$

(ii) $a = \sqrt{2}$, $r = \sqrt{2}$

Sol 3 terms of G.P

a, ar, ar^2

$\Rightarrow \sqrt{2}, (\sqrt{2}\times\sqrt{2}), (\sqrt{2}\times\sqrt{2}^2)$

$\sqrt{2}, (2), (\sqrt{2}\times 2)$

$\boxed{\sqrt{2}\times\sqrt{2}=\sqrt{2}^2 = 2}$

$\boxed{\sqrt{2}, 2, 2\sqrt{2}}$

(iii) $a = 1000, \quad r = \frac{2}{5}$

Sol:- Three terms of GP

a, ar, ar^2

$\Rightarrow \quad 1000, \left(\overset{200}{\cancel{1000}} \times \frac{2}{\underset{1}{\cancel{5}}}\right), \left(1000 \times \left(\frac{2}{5}\right)^2\right)$

$1000, (200 \times 2), \left(\overset{40}{\cancel{1000}} \times \frac{4}{\cancel{25}}\right)$

$\boxed{1000, 400, 160}$

---

③ In a G.P, 729, 243, 81, ... find t_7.

Sol:-

729, 243, 81, ...

$\boxed{a = 729}$

$r = \frac{\overset{1}{\cancel{243}}}{\underset{3}{\cancel{729}}}$; $\boxed{r = \frac{1}{3}}$

$$\begin{array}{r} \overset{1}{243} \\ \times\ 3 \\ \hline 729 \\ \hline \end{array}$$

$n = 7$

$\boxed{t_n = a \cdot r^{n-1}}$

$t_7 = 729 \cdot \left(\frac{1}{3}\right)^{7-1}$

$$= 729 \cdot \left(\frac{1}{3}\right)^6$$

$$= \overset{1}{\cancel{729}} \times \frac{1}{\cancel{729}}$$

$$\boxed{\begin{aligned} 3^6 &= 3\times3\times3\times3\times3\times3 \\ &= 729 \end{aligned}}$$

$$\boxed{t_7 = 1}$$

---

④ Find x So that $x+6$, $x+12$, and $x+15$ are Consecutive term of Geometric Progression.

Sol

$x+6$, $x+12$, $x+15$ are in G.P

$$\frac{x+12}{x+6} = \frac{x+15}{x+12}$$

$$(x+12)(x+12) = (x+15)(x+6)$$

$$\cancel{x^2}+12x+12x+144 = \cancel{x^2}+6x+15x+90$$

$$24x+144 = 21x+90$$

$$24x-21x = 90-144$$

$$\begin{array}{r} -144 \\ 90 \\ \hline -54 \end{array}$$

$$3x = -54$$

$$\boxed{x = \frac{-54}{3}}$$

⑤ Find the number of terms in the following G.P

(i) 4, 8, 16, ..., 8192?

Sol:-

$n = ?$ [no of terms = n = ?]

$a = 4$

$r = \frac{8}{4} = 2$

$t_n = 8192$

$a \cdot r^{n-1} = 8192$

$4 \cdot (2)^{n-1} = 8192$

$$2^{n-1} = \frac{\overset{2048}{\cancel{8192}}}{\underset{1}{\cancel{4}}}$$

$$\begin{array}{r|l} 2 & 2048 \\ 2 & 1024 \\ 2 & 512 \\ 2 & 256 \\ 2 & 128 \\ 2 & 64 \\ 2 & 32 \\ 2 & 16 \\ 2 & 8 \\ 2 & 4 \\ 2 & 2 \\ & 1 \end{array}$$

$2^{n-1} = 2048$

$2^{n-1} = 2^{11}$

[When base are Same Powers are equal]

$n-1 = 11$

$n = 11+1$

$$\boxed{n = 12}$$

(ii) $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \ldots\ldots \frac{1}{2187}$

Sol:-

$$n = ?$$

$$a = \frac{1}{3}$$

$$r = \frac{\frac{1}{9}}{\frac{1}{3}} \Rightarrow \frac{1}{\cancel{9}_{3}} \times \frac{\cancel{3}}{1}$$

$$\boxed{r = \frac{1}{3}}$$

$$t_n = \frac{1}{2187}$$

$$a \cdot r^{n-1} = \frac{1}{2187}$$

$$\frac{1}{3} \cdot \left(\frac{1}{3}\right)^{n-1} = \frac{1}{2187}$$

$$\left(\frac{1}{3}\right)^{n-1} = \frac{1}{\underset{729}{\cancel{2187}}} \times \frac{\overset{1}{\cancel{3}}}{1}$$

$$\begin{array}{r|l} 3 & 729 \\ \hline 3 & 243 \\ \hline 3 & 81 \\ \hline 3 & 27 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

$$\left(\frac{1}{3}\right)^{n-1} = \frac{1}{729}$$

$$\left(\frac{1}{3}\right)^{n-1} = \left(\frac{1}{3}\right)^{6}$$

$$n - 1 = 6$$

$$n = 6 + 1$$

$$\boxed{n = 7}$$

---

⑥ In a G.P. the 9th term is 32805 and 6th term is 1215. Find the 12th term.

Sol :-

9th term $\Rightarrow t_9 = 32805$

6th term $\Rightarrow t_6 = 1215$

To Find 12th term $\Rightarrow t_{12} = ?$

$$t_9 = 32805$$

$$a \cdot r^{9-1} = 32805$$

$$a \cdot r^8 = 32805 \longrightarrow ①$$

$$t_6 = 1215$$

$$a \cdot r^{6-1} = 1215$$

$$a \cdot r^5 = 1215 \longrightarrow ②$$

Divide ① by ②

$$\frac{\cancel{a} \cdot r^8}{\cancel{a} \cdot r^5} = \frac{32805}{1215}$$

$$\begin{array}{r} 27 \\ 1215 \overline{)\,32805} \\ -2430 \\ \hline 8505 \\ -8505 \\ \hline 0 \end{array}$$

$$r^{8-5} = 27$$

$$r^{3} = 3^{3}$$

[When powers are Same base are equal.]

$$\boxed{r = 3}$$

Put $r = 3$ in ②

$$a \cdot r^{5} = 1215$$

$$a \cdot 3^{5} = 1215$$

$$a \cdot = \frac{\cancel{1215}\ \cancel{405}\ \cancel{135}\ \cancel{45}\ \cancel{15}\ 5}{\cancel{3}_{1} \times \cancel{3}_{1} \times \cancel{3}_{1} \times \cancel{3}_{1} \times \cancel{3}_{1}}$$

$$\boxed{a = 5}$$

$$\therefore \quad t_{12} = a \cdot r^{n-1}$$

$$t_{12} = 5 \cdot (3)^{12-1}$$

$$\boxed{t_{12} = 5(3)^{11}} \text{ (or)}$$

$$= 5 \times 19683$$

$$\boxed{t_{12} = 98415}$$

⑦ Find the 10th term of a G.P whose 8th term is 768. and the Common ratio is 2.

Sol! -

8th term ⇒ $t_8 = 768$

Common ratio ⇒ $r = 2$

To find! - 10th term ⇒ $t_{10} = ?$

$$t_8 = 768$$

$$a \cdot r^{8-1} = 768$$

$$a \cdot (2)^7 = 768$$

$$\boxed{2^7 = 128}$$

$$a = \frac{\cancel{768}\ 6}{\cancel{128}}$$

$$\boxed{a = 6}$$

$$\boxed{\begin{array}{r} 6 \\ 128 \overline{)768} \\ -768 \\ \hline 0 \end{array}}$$

$$\therefore t_{10} = a \cdot r^{10-1}$$

$$= 6 \cdot (2)^9$$

$$\boxed{2^9 = 512}$$

$$= 6 \cdot (512)$$

$$\boxed{t_{10} = 3072}$$

⑧ If a, b, c are in A.P, then show that $3^a, 3^b, 3^c$ are in G.P.

Sol :-

Given :- a, b, c are in A.P

$\Rightarrow b - a = c - b$

Let us consider, $\boxed{b-a=c-b=x} \longrightarrow$ ①

To Show :- $3^a, 3^b, 3^c$ are in GP

$$\Rightarrow \frac{3^b}{3^a} = \frac{3^c}{3^b}$$

$$3^{b-a} = 3^{c-b} \quad [\text{using ①}]$$

$$3^x = 3^x$$

$$\therefore x = x$$

$\Rightarrow 3^a, 3^b, 3^c$ are in G.P.

---

⑨ In a G.P the product of three consecutive terms is 27 and the sum of the product of 2 terms taken at a time is $57/2$. Find the three terms

Sol:-

Three Consecutive terms in G.P.

$\frac{a}{r}, a, ar$

$$\text{Product} = 27$$

$$\frac{a}{\cancel{r}} \times a \times a\cancel{r} = 27$$

$$a^3 = 27$$

$$a^3 = 3^3$$

$$\boxed{a = 3}$$

Sum of their product $= \frac{57}{2}$

$$\left(\frac{a}{r} \times a\right) + (a \times ar) + \left(a\cancel{r} \times \frac{a}{\cancel{r}}\right) = \frac{57}{2}$$

$$\frac{a^2}{r} + a^2 r + a^2 = \frac{57}{2}$$

$$a^2\left[\frac{1}{r} + r + 1\right] = \frac{57}{2}$$

$$a^2\left[\frac{1 + r^2 + r}{r}\right] \xrightarrow{} \frac{57}{2}$$

$$a^2\left[r^2 + r + 1\right] = \frac{57r}{2}$$

Put $a = 3$

$$3^2\left[r^2+r+1\right] = \frac{57r}{2}$$

$$r^2+r+1 = \frac{\overset{19}{\cancel{57}}r}{2} \times \frac{1}{3\times\cancel{3}},$$

$$r^2+r+1 = \frac{19r}{6}$$

$$6(r^2+r+1) = 19r$$

$$6r^2+6r+6-19r = 0$$

$$6r^2-13r+6 = 0$$

$$(2r-3)(3r-2) = 0$$

$$36$$

$$\frac{\overset{3}{-\cancel{9}}}{\underset{2}{\cancel{6}r}} \quad \frac{\overset{2}{-\cancel{4}}}{\underset{3}{\cancel{6}r}}$$

$$2r-3=0$$

$$2r = 3$$

$$\boxed{r = \frac{3}{2}}$$

$a=3$, $r = \frac{3}{2}$

$$\frac{a}{r} = \frac{3}{\frac{3}{2}} = \cancel{3}\times\frac{2}{\cancel{3}} = 2$$

$$a = 3$$

$$ar = 3\times\frac{3}{2} = \frac{9}{2}$$

$$3r-2=0$$

$$3r = 2$$

$$\boxed{r = \frac{2}{3}}$$

$a = 3$; $r = \frac{2}{3}$

$$\frac{a}{r} = \frac{3}{\frac{2}{3}} = 3\times\frac{3}{2} = \frac{9}{2}$$

$$a = 3$$

$$ar = \cancel{3}\times\frac{2}{\cancel{3}} = 2$$

⑩ A man joined a company as Assistant Manager. The company gave him a Starting Salary of ₹60,000 and agreed to increase his Salary 5% annually. What will be his Salary after 5 years?

Sol:-

Salary = ₹60,000

Annual increment = 5%

1st year increased Amount:- $60\cancel{00}0 \times \frac{5}{1\cancel{00}} = ₹3000$	Salary at the end of 1st year $60000 + 3000 = ₹63000$
2nd year increased Amount:- $630\cancel{00} \times \frac{5}{1\cancel{00}} = ₹3150$	Salary at the end of 2nd year $63000 + 3150 = ₹66150$
3rd year increased Amount:- $6615\cancel{0} \times \frac{5}{10\cancel{0}} = 661.5 \times 5$ $= ₹3307.5$	Salary at the end of 3rd year $66150 + 3307.5$ $= ₹69457.5$

4th year increased Amount :- $69457.5 \times \frac{5}{100}$ $= 694.575 \times 5$ = ₹3472.875	Salary at the end of 4th year. $69457.5 + 3472.87$ = ₹72930.37
5th year increased Amount :- $72930.37 \times \frac{5}{100}$ 729.3037×5 ₹3646.5185	Salary at the end of 5th year. $72930.37 + 3646.52$ ⇒ ₹76576.89

∴ Salary at the end of 5th year

⇒ ₹76577

---

(11) Sivamani is attending an interview for a job and the company gave two offers to him.

Offer A: ₹20,000 to start with followed by a guaranteed annual increase of 6% for first 5 years

Offer B :- ₹22,000 to start with followed by a guaranteed annual increase of 3% for the first 5 years What is his salary in the 4th year with respect to offer A and B?

Sol :-

OFFER A

Salary = ₹20000
Increased Amount } = 6%

1st year increased amount :- $200\cancel{00} \times \frac{6}{1\cancel{00}} = ₹1200$	Salary at the end of 1st year :- $20000 + 1200$ $= ₹21200$
2nd year increased amount :- $212\cancel{00} \times \frac{6}{1\cancel{00}} = ₹1272$	Salary at the end of 2nd year $21200 + 1272$ $= ₹22472.$
3rd year increased amount :- $22472 \times \frac{6}{100} = 224.72 \times 6$ $= ₹1348.32$	Salary at the end of 3rd year. $22472 + 1348.32$ $= ₹23820.32$

Salary of 4th year = ₹23820

OFFER B

Salary = ₹22000

Increased Amount = 3%

1st year increased Amount :- $220\cancel{00} \times \frac{3}{1\cancel{00}}$ = ₹660	Salary at the end of 1st year :- 22000 + 660 ⇒ ₹22660
2nd year increased Amount :- $2266\cancel{0} \times \frac{3}{10\cancel{0}} = 226.6 \times 3$ = ₹679.8	Salary at the end of 2nd year :- 22660 + 679.8 ⇒ ₹23339.8
3rd year increased Amount :- $23339.8 \times \frac{3}{100} = 233.398 \times 3$ = ₹700.194	Salary at the end of 3rd year :- 23339.8 + 700.19 = ₹24040.06

Salary of 4th year = ₹24040

∴ Salary with respect to Offer A = ₹23820

Salary with respect to offer B = ₹24040

⑫ If a, b, c are three Consecutive terms of an A.P and x, y, z are the Consecutive terms of a G.P then prove that $x^{b-c} \times y^{c-a} \times z^{a-b} = 1$

Sol :-

$$\left.\begin{array}{l}\text{3 consecutive terms} \\ \text{of A.P.}\end{array}\right\} = a-d,\ a,\ a+d$$

Given :- $$\left.\begin{array}{l}\text{3 consecutive} \\ \text{terms of A.P}\end{array}\right\} = a,\ b,\ c$$

$$\Rightarrow \boxed{\begin{aligned} a &= a-d \\ b &= a \\ c &= a+d \end{aligned}} \longrightarrow \textcircled{1}$$

$$\left.\begin{array}{l}\text{3 consecutive terms} \\ \text{of G.P}\end{array}\right\} = \frac{a}{r},\ a,\ ar$$

Given :- $$\left.\begin{array}{l}\text{3 consecutive} \\ \text{terms of G.P}\end{array}\right\} = x,\ y,\ z$$

$$\Rightarrow \boxed{\begin{aligned} x &= \frac{a}{r} \\ y &= a \\ z &= ar \end{aligned}} \longrightarrow \textcircled{2}$$

To prove :-

$$x^{b-c} \times y^{c-a} \times z^{a-b} = 1$$

Subsituting ① and ②

$$\left(\frac{a}{r}\right)^{a-(a+d)} \times (a)^{(a+d)-(a-d)} \times (ar)^{\cancel{a}-d-\cancel{a}} = 1$$

$$\left(\frac{a}{r}\right)^{a-\cancel{a}-d} \times a^{\cancel{a}+d-\cancel{a}+d} \times (ar)^{-d} = 1$$

$$\left(\frac{a}{r}\right)^{-d} \times (a)^{2d} \times (ar)^{-d} = 1$$

$$\frac{a^{-d}}{r^{-d}}) \times (a)^{2d} \times a^{-d} \cdot r^{-d} = 1$$

$$\frac{\cancel{r^{d}}}{a^{d}} \times (a)^{2d} \times \frac{1}{a^{d}} \times \frac{1}{\cancel{r^{d}}} = 1$$

$$\frac{\cancel{a^{2d}}}{\cancel{a^{2d}}} = 1$$

$$1 = 1$$

Hence proved.

Sum to n terms of G.P

$$S_n = \frac{a(r^n-1)}{r-1} \quad \boxed{r>1}$$

$$S_n = \frac{a(1-r^n)}{1-r} \quad \boxed{r<1}$$

Exercise 2.8

(1.) Find the Sum of first n terms of the G.P.

(i) $5, -3, \frac{9}{5}, -\frac{27}{25}, \ldots$

Sol :-

$a = 5$

$r = \frac{-3}{5} < 1$

$$\therefore \quad S_n = \frac{a(1-r^n)}{1-r}$$

$$S_n = \frac{5\left(1-(-3/5)^n\right)}{1-(-3/5)}$$

$$= \frac{5\left(1-\left(-\frac{3}{5}\right)^{n}\right)}{1+\frac{3}{5}}$$

$$= \frac{5\left(1-\left(-\frac{3}{5}\right)^{n}\right)}{\frac{5+3}{5}}$$

$$= \frac{5\left(1-\left(-\frac{3}{5}\right)^{n}\right)}{\frac{8}{5}}$$

$$= 5\times\frac{5}{8}\left(1-\left(-\frac{3}{5}\right)^{n}\right)$$

$$S_n = \frac{25}{8}\left[1-\left(-\frac{3}{5}\right)^{n}\right]$$

(ii) 256, 64, 16, ...

Sol:-

$a = 256$

$r = \frac{\cancel{64}^{1}}{\cancel{256}_{4}} = \frac{1}{4}$ $\boxed{r < 1}$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_n = \frac{256\left(1-\left(\frac{1}{4}\right)^n\right)}{1-\frac{1}{4}}$$

$$= \frac{256\left(1-\left(\frac{1}{4}\right)^n\right)}{\frac{4-1}{4}}$$

$$= \frac{256\left(1-\left(\frac{1}{4}\right)^n\right)}{\frac{3}{4}}$$

$$= 256 \times \frac{4}{3}\left(1-\left(\frac{1}{4}\right)^n\right)$$

$$S_n = \frac{1024}{3}\left(1-\left(\frac{1}{4}\right)^n\right)$$

---

② Find the Sum of first Six terms of the G.P 5, 15, 45, ...

Sol :-

$a = 5$

$r = \frac{15}{5} = 3 \quad (r > 1)$

$n = 6$

$$S_n = \frac{a(r^n - 1)}{r - 1} \qquad (r > 1)$$

$$S_6 = \frac{5(3^6 - 1)}{3 - 1}$$

$$= \frac{5(729 - 1)}{2}$$

$$= 5 \times \frac{\cancel{728}^{364}}{\cancel{2}_1}$$

$$= 5 \times 364$$

$$\boxed{S_6 = 1820}$$

---

③ Find the first term of the G.P whose common ratio 5 and whose sum to first 6 terms is 46872

Sol :-

$a = ?$

$r = 5 \qquad (r > 1)$

$S_6 = 46872$

$$S_n = \frac{a(r^n - 1)}{r - 1} \qquad (r > 1)$$

$$S_6 = 46872$$

$$\frac{a(r^n-1)}{r-1} = 46872$$

$$\frac{a(5^6-1)}{5-1} = 46872$$

$$\boxed{5^6 = 15625}$$

$$\frac{a(15625-1)}{4} = 46872$$

$$a\left(\frac{\overset{3906}{\cancel{15624}}}{\underset{1}{\cancel{4}}}\right) = 46872$$

$$a = \frac{46872}{3906}$$

$$\begin{array}{r} 12 \\ 3906\,\overline{)\,46872} \\ -\,3906 \\ \hline 7812 \\ -\,7812 \\ \hline 0 \end{array}$$

$$\boxed{a = 12}$$

∴ First term = 12

---

④ Find the Sum to Infinity of

(i) $9 + 3 + 1 + \cdots\cdots$

$a = 9$

$r = \frac{3}{9} = \frac{1}{3}$

Sum to infinity

$$S_\infty = \frac{a}{1-r}$$

$$S_\infty = \frac{9}{1-\frac{1}{3}}$$

$$= \frac{9}{\frac{3-1}{3}}$$

$$= \frac{9}{\frac{2}{3}}$$

$$= 9 \times \frac{3}{2}$$

$$\boxed{S_\infty = \frac{27}{2}}$$

(ii) $21 + 14 + \frac{28}{3} + \ldots$

Sol:- $a = 21$

$$r = \frac{\cancel{14}^{2}}{\cancel{21}_{3}} = \frac{2}{3}$$

$$S_\infty = \frac{a}{1-r}$$

$$= \frac{21}{1-\frac{2}{3}}$$

$$= \frac{21}{\frac{3-2}{3}}$$

$$= \frac{21}{\frac{1}{3}}$$

$$= 21 \times \frac{3}{1}$$

$$\boxed{S_\infty = 63}$$

---

⑤ If the first term of an infinite G.P is 8 and its sum of infinity is $\frac{32}{3}$. then find the common ratio.

Sol:-

$a = 8$
$r = ?$
$S_\infty = \frac{32}{3}$

$$\frac{a}{1-r} = \frac{32}{3}$$

$$\frac{8}{1-r} = \frac{32}{3}$$

$$\frac{\overset{1}{\cancel{8}} \times 3}{\underset{4}{\cancel{32}}} = 1-r$$

$$\frac{3}{4} = 1-r$$

$$r = 1 - \frac{3}{4}$$

$$r = \frac{4-3}{4}$$

$$\boxed{r = \frac{1}{4}}$$

---

(6) Find the Sum to n terms of the Series

(i) 0.4 + 0.44 + 0.444 + ... to n terms

Sol :- 0.4 + 0.44 + 0.444 + ... + n terms

4 [0.1 + 0.11 + 0.111 + ... + n terms]

Multiply and divide by 9.

$$\frac{4}{9}\left[0.9+0.99+0.999+\cdots+n \text{ terms}\right]$$

$$\frac{4}{9}\left[(1-0.1)+(1-0.01)+(1-0.001)+\cdots\right]$$

$$\frac{4}{9}\left[(1+1+1+\cdots+n)-(0.1+0.01+0.001+\cdots+n)\right]$$

$$\frac{4}{9}\left[n-\left(\frac{1}{10}+\frac{1}{100}+\frac{1}{1000}+\cdots+n\right)\right]$$

$$a=\frac{1}{10}$$

$$r=\frac{1}{10\cancel{0}}\times\frac{1\cancel{0}}{1}=\frac{1}{10}<1$$

$$\boxed{S_n=\frac{a(1-r^n)}{1-r}}$$

$$S_n=\frac{4}{9}\left[n-\frac{\frac{1}{10}\left(1-\left(\frac{1}{10}\right)^n\right)}{1-\frac{1}{10}}\right]$$

$$=\frac{4}{9}\left[n-\frac{\frac{1}{1\cancel{0}}\left[1-\left(\frac{1}{10}\right)^n\right]}{\frac{10-1}{1\cancel{0}}}\right]$$

$$\boxed{S_n=\frac{4}{9}\left[n-\frac{1}{9}\left(1-\left(\frac{1}{10}\right)^n\right)\right]}$$

(ii) $3 + 33 + 333 + \dots$ to n terms

Sol:-

$$3 + 33 + 333 + \dots + n \text{ terms}$$

$$3[1 + 11 + 111 + \dots + n]$$

Multiply and divide by 9

$$\frac{\overset{1}{\cancel{3}}}{\underset{3}{\cancel{9}}}[9 + 99 + 999 + \dots + n]$$

$$\frac{1}{3}[(10-1) + (100-1) + (1000-1) + \dots + n]$$

$$\frac{1}{3}[(10 + 100 + 1000 + \dots + n) - (1+1+\dots+n)]$$

$$a = 10$$

$$r = \frac{10\cancel{0}}{1\cancel{0}} = 10 > 1$$

$$S_n = \frac{a(r^n - 1)}{r-1}$$

$$S_n = \frac{1}{3}\left[\frac{10(10^n - 1)}{10-1} - n\right]$$

$$\boxed{S_n = \frac{1}{3}\left[\frac{10(10^n - 1)}{9} - n\right]}$$

⑦ Find the Sum of the Geometric Series $3+6+12+\cdots+1536$

Sol :-

$$3+6+12+\cdots+1536$$

$$a = 3$$

$$r = \frac{\cancel{6}^{2}}{\cancel{3}_{1}} = 2$$

$$n = ?$$

$$t_n = 1536$$

$$a \cdot r^{n-1} = 1536$$

$$3(2)^{n-1} = 1536$$

$$2^{n-1} = \frac{\cancel{1536}^{512}}{\cancel{3}_{1}}$$

$$\begin{array}{r|l} 2 & 512 \\ \hline 2 & 256 \\ \hline 2 & 128 \\ \hline 2 & 64 \\ \hline 2 & 32 \\ \hline 2 & 16 \\ \hline 2 & 8 \\ \hline 2 & 4 \\ \hline & 2 \end{array}$$

$$2^{n-1} = 512$$

$$2^{n-1} = 2^{9}$$

$$n-1 = 9$$

$$n = 9+1$$

$$\boxed{n = 10}$$

$$\therefore S_n = \frac{a(r^n-1)}{r-1} \qquad (r>1)$$

$$S_{10} = \frac{3(2^{10}-1)}{2-1} \qquad \boxed{2^{10} = 1024}$$

$$= \frac{3(1024-1)}{1}$$

$$= 3 \times 1023$$

$$\boxed{S_{10} = 3069}$$

---

⑧ Kumar writes a letter to four of his friends. He asks each One of them to Copy the letter and mail to four different persons with the instruction that they Continue the process Similarly. Assuming that the process is unaltered and it costs ₹2 to mail One letter, find the amount Spent On postage when 8th set of letters is mailed.

Sol :-

$\left.\begin{array}{l}\text{No of letters Sent} \\ \text{in first Set}\end{array}\right\}$ = 4 letters by 1 person

$= 4 \times 1 = 4$

Second Set = 4 letters by 4 person

$= 4 \times 4 = 16$

Third Set = 4 letters by 16 person

$= 4 \times 16 = 64$

So on till 8 Set of letters

$\therefore$ This forms G.P

$\Rightarrow \quad 4 + 16 + 64 + \cdots\cdots 8^{th}$ set

ie, $4 + 4^2 + 4^3 + \cdots\cdots + 4^8$

$a = 4$

$r = \frac{4^2}{4} = \boxed{4 > 1}$

$n = 8$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_8 = \frac{4(4^8-1)}{4-1}$$

$$\boxed{4^8 = 65536}$$

$$= \frac{4(65536-1)}{3}$$

$$= 4 \times \frac{65535}{3}$$

$$= 4 \times 21845$$

$$\boxed{S_8 = 87380}$$

$$\begin{array}{r} 21845 \\ 3\overline{)65535} \\ -6 \\ \hline 5 \\ -3 \\ \hline 25 \\ -24 \\ \hline 13 \\ -12 \\ \hline 15 \\ -15 \\ \hline 0 \end{array}$$

Given that each mail cost = ₹2

∴ Cost of 87380 mails = 87380×2

= ₹174760

---

⑨ Find the rational form of the number. $0.\overline{123}$

Sol:- Let $x = 0.\overline{123} \longrightarrow$ ①

$x = 0.123123123......$

Mutiply by 1000 on both sides

$1000x = 123.123123 \ldots \longrightarrow$ ②

Subtract ② - ①

$$1000x - x = 123.\overline{123}\ldots - 0.\overline{123}$$

$$999x = 123$$

$$\begin{array}{r} 123.\overline{123} \\ (-)\ 0.\overline{123} \\ \hline 123.0 \end{array}$$

$$x = \frac{\cancel{123}^{41}}{\underset{333}{\cancel{999}}}$$

$$\therefore \boxed{x = \frac{41}{333}}$$

---

⑩ If $S_n = (x+y) + (x^2+xy+y^2) + (x^3+x^2y+xy^2+y^3) + \cdots$ n term

then prove that $(x-y)S_n = \left[\frac{x^2(x^n-1)}{x-1} - y^2\frac{(y^n-1)}{y-1}\right]$

Sol :-

$$S_n = (x+y) + (x^2+xy+y^2) + (x^3+x^2y+xy^2+y^3) + \cdots \to ①$$

[Multiply ① by $\underset{\sim}{x}$ on both side]

$$xS_n = x(x+y) + x(x^2+xy+y^2) + x(x^3+x^2y+xy^2+y^3) + \cdots$$

$$= (x^2+xy) + (x^3+x^2y+xy^2) + (x^4+x^3y+x^2y^2+xy^3) + \cdots \to ②$$

[Multiply ① by $\underset{\sim}{y}$ on both side]

$$yS_n = y(x+y) + y(x^2+xy+y^2) + y(x^3+x^2y+xy^2+y^3) + \cdots$$

$$= (yx+y^2) + (yx^2+xy^2+y^3) + (yx^3+x^2y^2+xy^3+y^4) + \cdots \longrightarrow ③$$

[Subtract ② - ③]

$$xS_n - yS_n = \left[(x^2+xy) + (x^3+x^2y+xy^2) + (x^4+x^3y+x^2y^2+xy^3) + \cdots\right]$$

$$- \left[(yx+y^2) + (yx^2+xy^2+y^3) + (yx^3+x^2y^2+xy^3+y^4) + \cdots\right]$$

$$= \left[(x^2+xy) - (yx+y^2)\right] + \left[(x^3+x^2y+xy^2) - (yx^2+xy^2+y^3)\right] + \left[(x^4+x^3y+x^2y^2+xy^3) - (yx^3+x^2y^2+xy^3+y^4)\cdots\right]$$

$$= \left[x^2 + \cancel{xy} - \cancel{yx} - y^2 + x^3 + \cancel{x^2y} + \cancel{xy^2} - \cancel{yx^2} - \cancel{xy^2} - y^3 + x^4 + \cancel{x^3y} + \cancel{x^2y^2} + \cancel{xy^3} - \cancel{yx^3} - \cancel{x^2y^2} - \cancel{xy^3} - y^4 \cdots\right]$$

$$xS_n - yS_n = x^2 - y^2 + x^3 - y^3 + x^4 - y^4 + \cdots$$

$$= (x^2 + x^3 + x^4 + \cdots) - (y^2 + y^3 + y^4 + \cdots)$$

$$= x^2(1 + x + x^2 + \cdots) - y^2(1 + y + y^2 + \cdots)$$

$a = 1$	$a = y$
$r = \frac{x}{1} = x > 1$	$r = \frac{y}{1} = y > 1$
$S_n = \frac{a(r^n - 1)}{r - 1}$	$S_n = \frac{a(r^n - 1)}{r - 1}$

$$= x^2\left[\frac{1(x^n - 1)}{x - 1}\right] - y^2\left[\frac{1(y^n - 1)}{y - 1}\right]$$

$$\boxed{\therefore (x - y)S_n = \frac{x^2(x^n - 1)}{x - 1} - \frac{y^2(y^n - 1)}{y - 1}}$$

$\therefore$ Hence Proved.

Formulae

1. Sum of first n Natural number

$$1+2+3+\cdots+n = \frac{n(n+1)}{2}$$

2. Sum of first n odd Natural number

$$1+3+5+\cdots = n^2$$

3. Sum of first n Square of Natural number

$$1^2+2^2+3^2+\cdots+n^2 = \frac{n(n+1)(2n+1)}{6}$$

4. Sum of first n Cube of Natural number

$$1^3+2^3+3^3+\cdots+n^3 = \left[\frac{n(n+1)}{2}\right]^2$$

Exercise 2.9

1. Find the Sum of the following Series.

(i) $1+2+3+\cdots+60$

Sol:-

Sum of first n natural number

$$n = 60$$

$$S_n = \frac{n(n+1)}{2}$$

$$S_{60} = \frac{\overset{30}{\cancel{60}}(60+1)}{\cancel{2}_{1}}$$

$$= 30 \times 61$$

$$\boxed{S_{60} = 1830}$$

(ii) $3+6+9+\cdots+96$

Sol:-

$$3+6+9+\cdots+96$$

$$3[1+2+3+\cdots+32]$$

Sum of first n natural number.

$$n = 32$$

$$S_n = \frac{n(n+1)}{2}$$

$$S_{32} = 3\left[\frac{\overset{16}{\cancel{32}}(32+1)}{\cancel{2}_{1}}\right]$$

$$= 3\left[16 \times 33\right]$$

$$= 3 \times 528$$

$$\boxed{S_{32} = 1584}$$

$$\begin{array}{r} 16 \\ 33 \\ \hline 48 \\ 48 \\ \hline 528 \\ \hline 5\overset{2}{2}8 \times 3 \\ \hline 1584 \end{array}$$

(iii) $51 + 52 + 53 + \cdots\cdots + 92$

Sol:-

The Series Should Start from 1.

$$51 + 52 + 53 + \cdots\cdots + 92 = (1+2+3+\cdots\cdots+92) - (1+2+3+\cdots\cdots+50)$$

Sum of first n natural number

$$= S_{92} - S_{50}$$

$$= \frac{\overset{46}{\cancel{92}}(92+1)}{\cancel{2}_{1}} - \frac{\overset{25}{\cancel{50}}(50+1)}{\cancel{2}}$$

$$= 46(93) - 25(51)$$

$$= 4278 - 1275$$

$$= 3003$$

$$\begin{array}{r} \overset{2}{2}5 \\ \times\ 51 \\ \hline 25 \\ 125 \\ \hline 1275 \end{array} \qquad \begin{array}{r} \overset{1}{9}3 \\ \times\ 46 \\ \hline 558 \\ 372 \\ \hline 4278 \end{array}$$

(iv) $1+4+9+\cdots+225$

Sol:-

$$1+4+9+\cdots+225$$

$$1^2+2^2+3^2+\cdots+15^2$$

Sum of first n square of natural No

$$\boxed{n=15}$$

$$S_n = \frac{n(n+1)(2n+1)}{6}$$

$$S_{15} = \frac{15(15+1)(2(15)+1)}{6}$$

$$= \frac{\overset{5}{\cancel{15}} \times \overset{8}{\cancel{16}} \times 31}{\cancel{6}\ \cancel{2}\ 1}$$

$$= 40 \times 31$$

$$\boxed{S_{15} = 1240}$$

(v) $6^2+7^2+8^2+\cdots\cdots+21^2$

Sol:-

$$6^2+7^2+8^2+\cdots\cdots+21^2$$

The Series Should Start from 1

$$6^2+7^2+8^2+\cdots\cdots+21^2 = (1^2+2^2+3^2+\cdots\cdots+21^2)-(1^2+2^2+\cdots+5^2)$$

$$\boxed{n=21} \qquad \boxed{n=5}$$

The Sum of Square of first n natural number

$$= S_{21} - S_5$$

$$= \frac{21(21+1)(2(21)+1)}{6} - \frac{5(5+1)(2(5)+1)}{6}$$

$$= \frac{\overset{7}{\cancel{21}}\times\overset{11}{\cancel{22}}\times 43}{\underset{\cancel{2}}{\cancel{6}}} - \frac{5\times\cancel{6}\times 11}{\cancel{6}}$$

77
×43
231
308
3311

$$= 3311 - 55$$

$$6^2+7^2+\cdots\cdots+21^2 = 3256$$

(vi) $10^3+11^3+12^3+\cdots+20^3$

Sol:-

$$10^3+11^3+12^3+\cdots+20^3$$

The Series Should Start from 1

$$10^3+11^3+12^3+\cdots+20^3 = (1^3+2^3+\cdots+20^3)-(1^3+2^3+\cdots\cdots+9^3)$$

The Sum of cube of first n natural number

$= S_{20} - S_9$

$= \left[\frac{20(20+1)}{2}\right]^2 - \left[\frac{9(9+1)}{2}\right]^2$

$= \left[\frac{\overset{10}{\cancel{20}} \times 21}{\underset{1}{\cancel{2}}}\right]^2 - \left[\frac{9 \times \overset{5}{\cancel{10}}}{\cancel{2}}\right]^2$

$= (10 \times 21)^2 - (45)^2$

$= (210)^2 - (45)^2$

$= 44100 - 2025$

$10^3 + 11^3 + \cdots + 20^3 = 42075$

$\boxed{\left[\frac{n(n+1)}{2}\right]^2}$

$$\begin{array}{r} 210 \\ 210 \\ \hline 21 \\ 42 \\ \hline 44100 \end{array}$$

$$\begin{array}{r} 45 \\ 45 \\ \hline 225 \\ 180 \\ \hline 2025 \end{array}$$

$$\begin{array}{r} 44100 \\ 2025 \\ \hline 42075 \end{array}$$

(vii) $1 + 3 + 5 + \cdots + 71$

Sol:- $1 + 3 + 5 + \cdots + 71 \Rightarrow$ It is in AP

In case of odd number

$\boxed{71 = l}$

$a = 1$

$d = 3 - 1 = 2$

$$n = \left(\frac{l-a}{d}\right)+1$$

$$= \left(\frac{71-1}{2}\right)+1$$

$$= \frac{\overset{35}{\cancel{70}}}{\underset{1}{\cancel{2}}}+1$$

$$\boxed{n = 36}$$

∴ Sum of first n **Odd** natural number.

$$S_n = n^2$$

$$S_{36} = 36^2$$

$$\boxed{S_{36} = 1296}$$

$$\begin{array}{r} 36 \\ 36 \\ \hline 216 \\ 108 \\ \hline 1296 \\ \hline \end{array}$$

---

② If $1+2+3+\cdots+k = 325$, then find $1^3+2^3+3^3+\cdots+k^3$

Sol $1+2+3+\cdots+k = 325$

$$\frac{k(k+1)}{2} = 325$$

To find :- $1^3+2^3+3^3+\cdots+k^3 = \left[\frac{k(k+1)}{2}\right]^2$

$$= (325)^2$$

$$1^3 + 2^3 + 3^3 + \cdots + k^3 = 105625$$

$$\begin{array}{r} 3\overset{2}{2}5 \\ 325 \\ \hline 1625 \\ 650 \\ 975 \\ \hline 105625 \end{array}$$

---

③ If $1^3 + 2^3 + 3^3 + \cdots + k^3 = 44100$ then find $1 + 2 + 3 + \cdots + k$.

Sol :-

$$1^3 + 2^3 + 3^3 + \cdots + k^3 = 44100$$

$$\left[\frac{k(k+1)}{2}\right]^2 = 44100$$

$$\therefore 1 + 2 + 3 + \cdots + k \Rightarrow \frac{k(k+1)}{2} = \sqrt{44100}$$

$$= \sqrt{441 \times 100}$$

$$= \sqrt{21 \times 21 \times 10 \times 10}$$

$$\Rightarrow \frac{k(k+1)}{2} = 21 \times 10$$

$$\therefore 1 + 2 + 3 + \cdots + k = 210$$

---

④ How many terms of the series $1^3 + 2^3 + 3^3 + \cdots\cdots$ should be taken to get the sum 14400?

Sol :-

To find $n = ?$ [How many $= n$]

$$1^3 + 2^3 + 3^3 + \cdots + n^3 = 14400$$

$$\left[\frac{n(n+1)}{2}\right]^2 = 14400$$

$$\frac{n(n+1)}{2} = \sqrt{14400}$$

$$\frac{n^2+n}{2} = \sqrt{144 \times 100}$$

$$\frac{n^2+n}{2} = \sqrt{12 \times 12 \times 10 \times 10}$$

$$\frac{n^2+n}{2} = 12 \times 10$$

$$n^2+n = 120 \times 2$$

$$n^2+n = 240$$

-240
-15 16

$$n^2+n-240 = 0$$

$$(n-15)(n+16) = 0$$

$n - 15 = 0$ | $n + 16 = 0$

$\boxed{n = 15}$ | $\boxed{n = -16}$ [n cannot be negative]

$$\therefore \boxed{n = 15}$$

⑤ The Sum of the cubes of the first n natural numbers is 2025. then find the Value of n.

Sol :-

Sum of the cubes of first n natural No } = 2025

$$\left[\frac{n(n+1)}{2}\right]^2 = 2025$$

$$\frac{n(n+1)}{2} = \sqrt{2025}$$

$$\begin{array}{r|l} & 45 \\ 4 & 2025 \\ & 16\downarrow\downarrow \\ 85 & 425 \\ & 425 \\ & 0 \end{array}$$

$$\frac{n^2+n}{2} = 45$$

$$n^2+n = 45 \times 2$$

$$n^2+n = 90$$

$$n^2+n-90 = 0$$

$$\begin{array}{c} -90 \\ -9 \quad 10 \end{array}$$

$$(n-9)(n+10) = 0$$

$$n-9 = 0 \quad \Big\| \quad n+10 = 0$$

$$\boxed{n = 9} \quad \Big\| \quad \boxed{n = -10}$$

n cannot be -ve

$$\therefore \boxed{n = 9}$$

⑥ Rekha has 15 square colour papers of sizes 10cm, 11cm, 12cm, ..., 24 cm. How much area can be decorated with these colour papers.

Sol :-

$$10^2+11^2+12^2+\cdots+24^2$$

The Series Should Start from 1

$$10^2+11^2+\cdots+24^2 = \left(1^2+2^2+3^2+\cdots+24^2\right)-\left(1^2+2^2+\cdots+9^2\right)$$

$$\boxed{n=24} \qquad \boxed{n=9}$$

Sum of square of first n natural No.

$$= S_{24} - S_9$$

$$= \frac{n(n+1)(2n+1)}{6} - \frac{n(n+1)(2n+1)}{6}$$

$$= \frac{24(24+1)(2(24)+1)}{6} - \frac{9(9+1)(2(9)+1)}{6}$$

$$= \frac{\overset{4}{\cancel{24}}\times 25\times 49}{\cancel{6}} - \frac{\overset{3}{\cancel{9}}\times \overset{5}{\cancel{10}}\times 19}{\cancel{6}\,\cancel{2}}$$

$$\begin{array}{r} {}^{4}\;\;\\ 15\\ 19\\ \hline 135\\ 15\;\\ \hline 285 \end{array}$$

$$= 100\times 49 - 15\times 19$$

$$= 4900 - 285$$

$$10^2+11^2+\cdots+24^2 = 4615\ \text{cm}^2$$

⑦ Find the sum of the Series

$(2^3-1^3)+(4^3-3^3)+(6^3-5^3)+\dots$ to

(i) n terms.

Sol:- $(2^3-1^3)+(4^3-3^3)+(6^3-5^3)+\dots$

$$\Rightarrow \left[2^3+4^3+6^3+\dots+(2n)^3\right]-\left[1^3+3^3+5^3+\dots+(2n-1)^3\right]$$

$\Rightarrow$ nth term of cube of even No $=(2n)^3$

nth term of odd no cube $=(2n-1)^3$

$$S_n = \sum(2n)^3 - \sum(2n-1)^3$$

$$\boxed{a^3-b^3=(a-b)^3+3ab(a-b)}$$

$$=\sum\left[\left[2n-(2n-1)\right]^3+3(2n)(2n-1)\left[2n-(2n-1)\right]\right]$$

$$=\sum\left[\left[\cancel{2n}-\cancel{2n}+1\right]^3+6n(2n-1)\left[\cancel{2n}-\cancel{2n}+1\right]\right]$$

$$=\sum\left[(1)^3+12n^2-6n(1)\right]$$

$$=\sum 1+\sum 12n^2-\sum 6n$$

$$= n+12\sum n^2-6\sum n$$

$$= n+\overset{2}{\cancel{12}}\left[\frac{n(n+1)(2n+1)}{\cancel{6}}\right]-\overset{3}{\cancel{6}}\left[\frac{n(n+1)}{\cancel{2}}\right]$$

$\sum 1 = n$

$\sum n^2$ = Sum of Square of Natural No

$\sum n$ = Sum of natural No

$$= n + 2\left(n(n+1)(2n+1)\right) - 3\left(n(n+1)\right)$$

$$= n + 2\overset{\curvearrowright}{(n^2+n)(2n+1)} - 3(n^2+n)$$

$$= n + 2(2n^3 + \underbrace{n^2 + 2n^2} + n) - 3n^2 - 3n$$

$$= n + \overset{\curvearrowright}{2(2n^3 + 3n^2 + n)} - 3n^2 - 3n$$

$$= \overset{\curvearrowright}{n + 4n^3 + 6n^2 + 2n} - 3n^2 - 3n$$

$$= \cancel{3n} + 4n^3 + \underbrace{6n^2 - 3n^2} - \cancel{3n}$$

$$= 4n^3 + 3n^2$$

$\therefore$ Sum of n terms $= 4n^3 + 3n^2$

(ii) 8 terms

Sol:- Sum to 8 terms $= 4(8)^3 + 3(8^2)$

$$= 4(512) + 3(64)$$

$$= 2048 + 192$$

$$= 2240$$

CHAPTER - 3

ALGEBRA

Exercise 3.1

① Solve the following system of linear equation in three Variables.

(i) $x+y+z=5$; $2x-y+z=9$; $x-2y+3z=16$

Sol:-

$$x+y+z=5 \longrightarrow ①$$
$$2x-y+z=9 \longrightarrow ②$$
$$x-2y+3z=16 \longrightarrow ③$$

Add ① & ②

$$\begin{array}{l} x+\cancel{y}+z=5 \\ 2x-\cancel{y}+z=9 \\ \hline 3x \quad +2z=14 \\ \hline \end{array} \longrightarrow ④$$

Solve ② & ③

$$2x-y+z=9 \longrightarrow [x^{ly} \text{ by } 2]$$
$$x-2y+3z=16$$

$$\begin{array}{rcrcrcl} 4x & - & \cancel{2y} & + & 2z & = & 18 \\ x & - & \cancel{2y} & + & 3z & = & 16 \\ (-) & & (+) & & (-) & & (-) \\ \hline 3x & & & - & z & = & 2 \\ \hline \end{array} \longrightarrow ⑤$$

Solve ④ + ⑤

$$\begin{array}{rcrcl} \cancel{3x} & + & 2z & = & 14 \\ \cancel{3x} & - & z & = & 2 \\ (-) & & (+) & & (-) \\ \hline & & 3z & = & 12 \\ \hline \end{array}$$

$$z = \frac{\cancel{12}}{\cancel{3}}$$

$$\boxed{z = 4}$$

Put $z=4$ in ⑤

$$3x - 4 = 2$$

$$3x = 2+4$$

$$3x = 6$$

$$x = \frac{\cancel{6}^{2}}{\cancel{3}}$$

$$\boxed{x = 2}$$

Put $x = 2$, $z = 4$ in ①

$$2 + y + 4 = 5$$
$$6 + y = 5$$
$$y = 5 - 6$$
$$\boxed{y = -1}$$

To check

$x + y + z = 5$
$2 - 1 + 4 = 5$
$6 - 1 = 5$
$5 = 5$

$$\boxed{\begin{aligned} x &= 2 \\ y &= -1 \\ z &= 4 \end{aligned}}$$

(ii) $\frac{1}{x} - \frac{2}{y} + 4 = 0$; $\frac{1}{y} - \frac{1}{z} + 1 = 0$; $\frac{2}{z} + \frac{3}{x} = 14$

Sol:- $\frac{1}{x} - \frac{2}{y} + 4 = 0 \longrightarrow$ ①

$\frac{1}{y} - \frac{1}{z} + 1 = 0 \rightarrow$ ②

$\frac{2}{z} + \frac{3}{x} = 14 \longrightarrow$ ③

Put $\frac{1}{x} = a$; $\frac{1}{y} = b$, $\frac{1}{z} = c$

$\Rightarrow$ $a - 2b = -4 \longrightarrow$ ④

$b - c = -1 \longrightarrow$ ⑤

$2c + 3a = 14 \longrightarrow$ ⑥

Solve ④ & ⑤

$$a - 2b = -4$$
$$-c + b = -1 \longrightarrow (x'ly \text{ by } 2)$$

$$\Rightarrow \begin{array}{r} a - 2\cancel{b} = -4 \\ -2c + \cancel{2}b = -2 \\ \hline a - 2c = -6 \\ \hline \end{array} \longrightarrow ⑦$$

Add ⑥ & ⑦

$$\begin{array}{r} 3a + 2\cancel{c} = 14 \\ a - \cancel{2}c = -6 \\ \hline 4a \quad = 8 \\ \hline \end{array}$$

$$a = \frac{\cancel{8}^{2}}{\cancel{4}}$$

$$\boxed{a = 2}$$

Put a=2 in ⑦

$$2 - 2c = -6$$
$$-2c = -6 - 2$$
$$\cancel{-}2c = \cancel{-}8$$
$$c = \frac{\cancel{8}^{4}}{\cancel{2}}$$
$$\boxed{c = 4}$$

Put c = 4 in ⑤

$$b - \overrightarrow{4 = -1}$$

$$b = -1 + 4$$

$$\boxed{b = 3}$$

$$a = 2 \implies \frac{1}{x} = a \Rightarrow \frac{1}{x} = 2 \Rightarrow \boxed{\frac{1}{2} = x}$$

$$b = 3 \implies \frac{1}{y} = b \Rightarrow \frac{1}{y} = 3 \Rightarrow \boxed{\frac{1}{3} = y}$$

$$c = 4 \implies \frac{1}{z} = c \Rightarrow \frac{1}{z} = 4 \Rightarrow \boxed{\frac{1}{4} = z}$$

$$\therefore \boxed{\begin{aligned} x &= \frac{1}{2} \\ y &= \frac{1}{3} \\ z &= \frac{1}{4} \end{aligned}}$$

(iii) $x + 20 = \frac{3y}{2} + 10 = 2z + 5 = 110 - (y + z)$

Sol:-

$$x + 20 = \frac{3y}{\xleftarrow{}\,2\,\xrightarrow{}} + 10$$

$$2(x + 20) = 3y + 20$$

$$2x + 40 - 3y - 20 = 0$$

$$2x - 3y + 20 = 0$$

$$\boxed{2x - 3y = -20} \longrightarrow ①$$

$$\frac{3y}{2}+10 = 2z+5$$

$$3y+20 = 2(2z+5)$$

$$3y+20 = 4z+10$$

$$3y-4z = 10-20$$

$$\boxed{3y-4z = -10} \longrightarrow ②$$

$$2z+5 = 110-(y+z)$$

$$2z+5 = 110-y-z$$

$$2z+y+z = 110-5$$

$$\boxed{3z+y = 105} \longrightarrow ③$$

Add ① & ②

$$\begin{aligned} 2x - \cancel{3y} &= -20 \\ \cancel{3y} - 4z &= -10 \\ \hline 2x-4z &= -30 \end{aligned} \longrightarrow ④$$

Solve ② & ③

$$3y - 4z = -10$$
$$y + 3z = 105 \longrightarrow (\times^{ly} \text{ by } 3)$$

$$\Rightarrow \quad \begin{aligned} \cancel{3y} - 4z &= -10 \\ \cancel{3y} + 9z &= 315 \\ \overset{(-)}{} \quad \overset{(-)}{} &\quad \overset{(-)}{} \end{aligned}$$

$$\cancel{-}13z = \cancel{-}325$$

$$z = \frac{\overset{25}{\cancel{325}}}{\cancel{13}},$$

$$\boxed{z = 25}$$

Put $z = 25$ in ② + ④

$$3y - 4z = -10$$
$$3y - 4(25) = -10$$
$$3y - 100 = -10$$
$$3y = -10 + 100$$
$$3y = 90$$
$$y = \frac{\overset{30}{\cancel{90}}}{\cancel{3}}$$
$$\boxed{y = 30}$$

$$2x - 4z = -30$$
$$2x - 4(25) = -30$$
$$2x - 100 = -30$$
$$2x = -30 + 100$$
$$2x = 70$$
$$x = \frac{\overset{35}{\cancel{70}}}{\cancel{2}}$$
$$\boxed{x = 35}$$

$x = 35$ $y = 30$ $z = 25$

② Discuss the nature of Solutions of the following system of equations:

(i) $x+2y-z=6;\ -3x-2y+5z=-12,\ x-2z=3$

Sol :-

$$x+2y-z=6 \longrightarrow ①$$

$$-3x-2y+5z=-12 \longrightarrow ②$$

$$x-2z=3 \longrightarrow ③$$

Add ① & ②

$$\begin{array}{r} x+\cancel{2y}-z=6 \\ -3x-\cancel{2y}+5z=-12 \\ \hline -2x \qquad +4z=-6 \\ \hline \end{array} \longrightarrow [\div \text{ by } 2]$$

$$-x+2z=-3 \longrightarrow ④$$

Add ③ & ④

$$\begin{array}{r} \cancel{x}-\cancel{2z}=3 \\ -\cancel{x}+\cancel{2z}=-3 \\ \hline 0=0 \\ \hline \end{array}$$

When we arrive $0=0$

The System has infinite number of Solution.

(ii) $2y + z = 3(-x+1)$

$-x + 3y - z = -4$

$3x + 2y + z = -\frac{1}{2}$

Sol:-

$2y + z = -3x + 3$

$3x + 2y + z = 3 \longrightarrow ①$

$-x + 3y - z = -4 \longrightarrow ②$

$3x + 2y + z = \frac{-1}{2} \Rightarrow 6x + 4y + 2z = -1 \longrightarrow ③$

Add ① & ②

$$\begin{array}{r} 3x + 2y + \cancel{z} = 3 \\ -x + 3y - \cancel{z} = -4 \\ \hline 2x + 5y \quad = -1 \\ \hline \end{array} \longrightarrow ④$$

Solve ② & ③

$-x + 3y - z = -4 \longrightarrow$ (x'ly by 2)

$6x + 4y + 2z = -1$

$$\begin{array}{rcl} -2x + 6y - \cancel{2z} & = & -8 \\ 6x + 4y + \cancel{2z} & = & -1 \\ \hline 4x + 10y & = & -9 \\ \hline \end{array} \longrightarrow ⑤$$

Solve ④ & ⑤

$$2x + 5y = -1 \longrightarrow (\times^{l}y \text{ by } 2)$$
$$4x + 10y = -9$$

$$\Rightarrow \quad \begin{array}{rcl} \cancel{4x} + \cancel{10y} & = & -2 \\ 4x + 10y & = & -9 \\ (-) \quad (-) & & (+) \\ \hline 0 & = & 7 \\ \hline \end{array}$$

We arrive at contradiction $0 \neq 7$

$\therefore$ System has no solution.

(iii) $\dfrac{y+z}{4} = \dfrac{z+x}{3} = \dfrac{x+y}{2}$; $x+y+z = 27$

Sol:-

$$\frac{y+z}{4} = \frac{z+x}{3} \Rightarrow 3(y+z) = 4(z+x)$$
$$3y + 3z = 4z + 4x$$
$$3y + 3z - 4z - 4x = 0$$
$$-4x + 3y - z = 0 \longrightarrow ①$$

$$\frac{z+x}{3} = \frac{x+y}{2} \Rightarrow 2(z+x) = 3(x+y)$$

$$2z+2x = 3x+3y$$

$$2z+\overbrace{2x-3x}-3y = 0$$

$$-x-3y+2z = 0 \rightarrow ②$$

$$\Rightarrow x+y+z = 27 \longrightarrow ③$$

Add ① & ②

$$\begin{array}{r} -4x+\cancel{3y}-z = 0 \\ -x-\cancel{3y}+2z = 0 \\ \hline -5x \quad +z = 0 \end{array} \rightarrow ④$$

Solve ② & ③

$$-x-3y+2z = 0$$

$$x+y+z = 27 \rightarrow (\times^{ly} \text{ by } 3)$$

$$\Rightarrow \begin{array}{r} -x-\cancel{3y}+2z = 0 \\ 3x+\cancel{3y}+3z = 81 \\ \hline 2x \quad +5z = 81 \end{array} \rightarrow ⑤$$

Solve ④ & ⑤

$-5x + z = 0 \longrightarrow$ (x'y by 5)

$2x + 5z = 81$

$$\Rightarrow \quad -25x + 5\cancel{z} = 0$$

$$2x + 5\cancel{z} = 81$$

$$(-) \quad (-) \quad (-)$$

$$+27x = +81$$

$$\Rightarrow \quad x = \frac{\cancel{81}^{\,3}}{\cancel{27}}$$

$$\boxed{x = 3}$$

Put $x = 3$ in ④

$$-5(3) + z = 0$$

$$-15 + z = 0$$

$$\boxed{z = 15}$$

Put $x = 3$, $z = 15$ in ③

$$x + y + z = 27$$

$$3 + y + 15 = 27$$

$$y + 18 = 27$$

$$y = 27 - 18$$

$$\boxed{y = 9}$$

$x = 3$, $y = 9$, $z = 15$ ∴ System has Unique Solution.

③ Vani, her father and her grand father have an average age of 53. One-half of her grand father's age plus One-third of her father's age plus One-fourth of Vanis age is 65. Four years ago if Vani's grandfather was four times as old as Vani then how old are they now.

Sol :-

Let Age of Vani $= x$

Age of father $= y$

Age of grand father $= z$

Average of their Age $= 53$

ie, $\frac{x+y+z}{3} = 53$

$$\boxed{x+y+z = 159} \longrightarrow ①$$

(One-half of grand father age) + (One-third of father's age) + (One fourth of Vani ag) = 65

$$\frac{1^{\times 6}}{2_{\times 6}}z + \frac{1^{\times 4}}{3_{\times 4}}y + \frac{1^{\times 3}}{4_{\times 3}}x = 65$$

$$\frac{6z + 4y + 3x}{12} = 65$$

$$\boxed{3x + 4y + 6z = 780} \longrightarrow ②$$

$$\begin{array}{r|l} 2 & 2, 3, 4 \\ \hline 2 & 1, 3, 2 \\ \hline 3 & 1, 3, 1 \\ \hline & 1, 1, 1 \end{array}$$

$2 \times 2 \times 3 = 12$

$$\begin{array}{r} 65 \\ \times\ 12 \\ \hline 130 \\ 65\ \ \\ \hline 780 \\ \hline \end{array}$$

4 years Ago

Grandfather age was 4 times Vani's age

$$(z-4) = 4(x-4)$$

$$z - 4 = 4x - 16$$

$$0 = 4x - 16 - z + 4$$

$$0 = 4x - z - 12$$

$$12 = 4x - z$$

$$\boxed{4x - z = 12} \longrightarrow ③$$

Solve ① & ②

$$x + y + z = 159 \longrightarrow (x^{l}y \text{ by } 4)$$

$$3x + 4y + 6z = 780$$

$$\Rightarrow \quad \begin{array}{rcrcrcl} 4x &+& \cancel{4y} &+& 4z &=& 636 \\ (-)\,3x &\underset{(-)}{+}& 4y &\underset{(-)}{+}& 6z &\underset{(-)}{=}& 780 \\ \hline x & & & - & 2z &=& -144 \end{array} \longrightarrow ④$$

Solve ③ & ④

$4x - z = 12 \longrightarrow$ (x'y by 2)

$x - 2z = -144$

$$\Rightarrow \quad \begin{array}{rcl} 8x - \cancel{2z} &=& 24 \\ x - \cancel{2z} &=& -144 \\ (-) \quad (+) & & (+) \\ \hline 7x &=& 168 \\ \hline \end{array}$$

$x = \dfrac{\cancel{168}^{24}}{\cancel{7}}$

$\boxed{x = 24}$

Put $x = 24$ in ③

$4(24) - z = 12$

$96 - z = 12$

$-z = 12 - 96$

$+z = +84$

$\boxed{z = 84}$

Put $x = 24$, $z = 84$ in ①

$x + y + z = 159$

$\overline{24 + y + 84} = 159$

$y + 108 = 159$

$$y = 159 - 108$$

$$\boxed{y = 51}$$

∴ Age of Vani = x = 24 yrs
Age of father = y. = 51 yrs
Age of grand father = Z = 84 yrs.

④ The Sum of the digits of a three-digit number is 11. If the digits are reversed, the new number is 46 more than five times the former number. If the hundreds digit plus twice the tens digit is equal to the Units digit, then find the Original three digit number?

Sol:-

Let Hundred's digit $= x$
Tens digit $= y$
Units digit $= Z$

Let Original 3 digit Number $= 100x + 10y + Z$ $\longrightarrow \bigstar$

Sum of digits $= 11$

ie, $\boxed{x + y + Z = 11} \longrightarrow$ ①

Reversed Number = $100z + 10y + x$

Reversed number is 46 more than 5 times the former No

$$\Rightarrow 100z + 10y + x = 46 + 5(100x + 10y + z)$$

$$100z + 10y \overrightarrow{+x} = 46 + 500x + 50y + 5z$$

$$\overleftarrow{0 = 46} + 500x + 50y + 5z - 100z - 10y - x$$

$$= 499x + 40y - 95z$$

$$\Rightarrow \boxed{499x + 40y - 95z = -46} \rightarrow ②$$

Hundreds digit + twice the ten's digit = Unit digits.

$$x + 2y = z$$

$$\boxed{x + 2y - z = 0} \longrightarrow ③$$

Solve ① & ③

$$\begin{array}{l} x + y + \cancel{z} = 11 \\ x + 2y \cancel{-z} = 0 \\ \hline 2x + 3y = 11 \\ \hline \end{array} \longrightarrow ④$$

Solve ② & ③

$$499x + 40y - 95z = -46$$

$$x + 2y - z = 0 \longrightarrow \left[x^{ly} \text{ by } 95\right]$$

$$\Rightarrow \begin{array}{rl} 499x + 40y - 95\cancel{z} & = -46 \\ 95x + 190y - 95z & = 0 \\ \overset{(-)}{} \quad \overset{(-)}{} \quad \overset{(+)}{} & \\ \hline 404x - 150y & = -46 \longrightarrow ⑤ \\ \hline \end{array}$$

Solve ④ & ⑤

$$2x + 3y = 11 \longrightarrow \left(x^{ly} \text{ by } 50\right)$$

$$404x - 150y = -46$$

$$\Rightarrow \begin{array}{rl} 100x + 150\cancel{y} & = 550 \\ 404x - \cancel{150}y & = -46 \\ \hline 504x & = 504 \\ \hline \end{array}$$

$$x = \frac{5\cancel{04}}{5\cancel{04}}$$

$$\boxed{x = 1}$$

Put $x=1$ in ④

$$2(1) + 3y = 11$$

$$2 + 3y = 11$$

$$3y = 11 - 2$$
$$3y = 9$$
$$y = \frac{\cancel{9}^{3}}{\cancel{3}}$$
$$\boxed{y = 3}$$

Put $x = 1$, $y = 3$ in ①

$$x + y + z = 11$$
$$1 + 3 + z = 11$$
$$4 + z = 11$$
$$z = 11 - 4$$
$$\boxed{z = 7}$$

∴ Put $x = 1$, $y = 3$, $z = 7$ in ★

∴ Original 3 digit number $= 100x + 10y + z$
$$= 100(1) + 10(3) + 7$$
$$= 100 + 30 + 7$$
$$= 137$$

---

⑤ There are 12 pieces of five, ten, and twenty rupee currencies whose total value is ₹105. When first 2 sorts are interchanged in their numbers its value will be increased by ₹20. Find the number of currencies in each sort.

Sol :-

Let five rupee Currencies $= x$

Ten rupee Currencies $= y$

Twenty rupee Currencies $= z$

Number of Currencies $= 12$

$$\boxed{x+y+z=12} \longrightarrow ①$$

Total Value of Currencies $= 105$

$$\boxed{5x+10y+20z=105} \longrightarrow ②$$

First 2 Sorts are interchanged then Value is increased by 20

$$\Rightarrow \quad 10x+5y+20z = 105+20$$

$$\boxed{10x+5y+20z=125} \longrightarrow ③$$

Solve ① & ②

$$x+y+z=12 \longrightarrow (\times^{ly} \text{ by } 5)$$

$$5x+10y+20z=105$$

$$\Rightarrow \quad \cancel{5x}+5y+5z=60$$

$$\underset{(-)}{}\cancel{5x} \underset{(-)}{+} 10y \underset{(-)}{+} 20z = \underset{(-)}{105}$$

$$-5y-15z=-45 \longrightarrow (\div \text{ by } -5)$$

$$\boxed{y + 3z = 9} \longrightarrow ④$$

Solve ① & ③

$$x + y + z = 12 \longrightarrow (x'y \text{ by } 10)$$
$$10x + 5y + 20z = 125$$

$$\Rightarrow \quad \cancel{10x} + 10y + 10z = 120$$
$$\underset{(-)}{} \cancel{10x} \underset{(-)}{+} 5y \underset{(-)}{+} 20z \underset{(-)}{=} 125$$
$$5y - 10z = -5 \rightarrow (\div \text{ by } 5)$$

$$\boxed{y - 2z = -1} \longrightarrow ⑤$$

Solve ④ & ⑤

$$\cancel{y} + 3z = 9$$
$$\underset{(-)}{} \cancel{y} \underset{(+)}{-} 2z = \underset{(+)}{-} 1$$
$$5z = 10$$

$$z = \cancel{\frac{10}{5}}^{\,2}$$

$$\boxed{z = 2}$$

Put $z = 2$ in ④

$$y + 3(2) = 9$$
$$y + 6 = 9$$

$y = 9 - 6$

$\boxed{y = 3}$

Put $y = 3$, $Z = 2$ in ①

$x + y + Z = 12$

$x + 3 + 2 = 12$

$x + 5 = 12$

$x = 12 - 5$

$\boxed{x = 7}$

∴ Number of 5 rupee Currency = 7

Number of 10 rupee Currency = 3

Number of 20 rupee Currency = 2

Exercise - 3.2

1. Find the G.CD of the given polynomial

(i) x^4+3x^3-x-3 ; x^3+x^2-5x+3

Sol

x^4+3x^3-x-3 ; x^3+x^2-5x+3

$$\begin{array}{r|l} & x+2 \\ \hline x^3+x^2-5x+3 & \cancel{x^4}+3x^3-0x^2-x-3 \\ & (-)\cancel{x^4}\underset{(-)}{+}x^3\underset{(+)}{-}5x^2\underset{(-)}{+}3x \quad \downarrow \\ \hline & 2\cancel{x^3}+5x^2-4x-3 \\ & (-)2\cancel{x^3}\underset{(-)}{+}2x^2\underset{(+)}{-}10x\underset{(-)}{+}6 \\ \hline & 3x^2+6x-9 \end{array}$$

$3x^2+6x-9 \Rightarrow \underline{x^2+2x-3} \neq 0$

(÷ by 3)

∴ Dividig x^3+x^2-5x+3 by x^2+2x-3

$$\begin{array}{r|l} & x-1 \\ \hline x^2+2x-3 & \cancel{x^3}+x^2-5x+3 \\ & \underset{(-)}{\cancel{x^3}}\underset{(-)}{+}2x^2\underset{(+)}{-}3x \\ \hline & -x^2-2x+3 \\ & \underset{(+)}{-}x^2\underset{(+)}{-}2x\underset{(-)}{+}3 \\ \hline & 0 \end{array}$$

∴ $\boxed{GCD = x^2+2x-3}$

(ii) $x^4 - 1$; $x^3 - 11x^2 + x - 11$

Sol:

$$
\begin{array}{r|l}
 & x + 11 \\
\hline
x^3 - 11x^2 + x - 11 & \cancel{x^4} + 0x^3 + 0x^2 + 0x - 1 \\
 & \cancel{x^4} - 11x^3 + x^2 - 11x \\
 & (-) \quad (+) \quad (-) \quad (+) \\
\hline
 & 11\cancel{x^3} - x^2 + 11\cancel{x} - 1 \\
 & \cancel{11x^3} - 121x^2 + \cancel{11x} - 121 \\
 & (-) \quad (+) \quad (-) \quad (+) \\
\hline
 & 120x^2 + 120 \quad (\div \text{ by } 120)
\end{array}
$$

$\Rightarrow \quad x^2 + 1 \neq 0$

$\therefore$ Dividing $x^3 - 11x^2 + x - 11$ by $x^2 + 1$

$$
\begin{array}{r|l}
 & x - 11 \\
\hline
x^2 + 1 & \cancel{x^3} - 11x^2 + \cancel{x} - 11 \\
 & \cancel{x^3} \qquad + \cancel{x} \\
 & (-) \qquad (-) \\
\hline
 & -11\cancel{x^2} - \cancel{11} \\
 & -\cancel{11}x^2 - \cancel{11} \\
 & (+) \qquad (+) \\
\hline
 & 0
\end{array}
$$

$\therefore \boxed{GCD = x^2 + 1}$

(iii) $3x^4+6x^3-12x^2-24x$; $4x^4+14x^3+8x^2-8x$

Sol:-

$3x^4+6x^3-12x^2-24x \Rightarrow 3(x^4+2x^3-4x^2-8x)$

$4x^4+14x^3+8x^2-8x \Rightarrow 2(2x^4+7x^3+4x^2-4x)$

$$\begin{array}{r|l} & \quad 2 \\ \hline x^4+2x^3-4x^2-8x & \cancel{2x^4}+7x^3+4x^2-4x \\ & \cancel{2x^4}+4x^3-8x^2-16x \\ & (-)\quad (-) \quad\quad (+) \quad\quad (+) \\ \hline & \quad 3x^3+12x^2+12x \\ & \quad\quad\quad \longrightarrow (\div \text{ by } 3) \end{array}$$

$\Rightarrow x^3+4x^2+4x \neq 0$

Now dividing $x^4+2x^3-4x^2-8x$ by x^3+4x^2+4x.

$$\begin{array}{r|l} & \quad x-2 \\ \hline x^3+4x^2+4x & \cancel{x^4}+2x^3-4x^2-8x \\ & \cancel{x^4}+4x^3+4x^2 \\ & (-)\ (-) \quad\quad (-) \\ \hline & \quad -2x^3-8x^2-8x \\ & \quad -2x^3-8x^2-8x \\ & \quad (+) \quad\quad (+) \quad\quad (+) \\ \hline & \quad\quad\quad 0 \end{array}$$

$$\boxed{GCD = x^3+4x^2+4x}$$

(iv) $3x^3+3x^2+3x+3$; $6x^3+12x^2+6x+12$

Sol:-

$3x^3+3x^2+3x+3 = 3(x^3+x^2+x+1)$

$6x^3+12x^2+6x+12 = 6(x^3+2x^2+x+2)$

$$\begin{array}{r|l} & \quad 1 \\ \hline x^3+x^2+x+1 & \cancel{x^3}+2x^2+\cancel{x}+2 \\ & \cancel{x^3}+x^2+\cancel{x}+1 \\ & (-)\quad(-)\quad(-)\quad(-) \\ \hline & \quad x^2+1 \end{array}$$

$x^2+1 \neq 0$

Now dividing x^3+x^2+x+1 by x^2+1

$$\begin{array}{r|l} & \quad x+1 \\ \hline x^2+1 & \cancel{x^3}+x^2+\cancel{x}+1 \\ & \cancel{x^3}\qquad +\cancel{x} \\ & (-)\qquad (-) \\ \hline & \quad x^2+1 \\ & \quad x^2+1 \\ & (-)\quad (-) \\ \hline & \quad 0 \end{array}$$

$\therefore \boxed{GCD = 3(x^2+1)}$

② Find LCM of the given expressions

(i) $4x^2y, 8x^3y^2$

Sol

$$4x^2y = 2\times 2x^2y$$

$$8x^3y^2 = 2\times 2\times 2\cdot x^3y^2$$

$$LCM = 2\times 2\times 2x^3y^2$$

$$\boxed{LCM = 8x^3y^2}$$

(ii) $9a^3b^2 ; 12a^2b^2c$

Sol:-

$$9a^3b^2 = 3\times 3a^3b^2$$

$$12a^2b^2c = 2\times 2\times 3a^2b^2c$$

$$LCM = 3\times 3\times 2\times 2a^3b^2c$$

$$\boxed{LCM = 36a^3b^2c}$$

(iii) $16m ; 12m^2n^2 ; 8n^2$

Sol:-

$$16m = 2\times2\times2\times2\cdot m$$

$$12m^2n^2 = 2\times2\times3\cdot m^2\cdot n^2$$

$$8n^2 = 2\times2\times2\cdot n^2$$

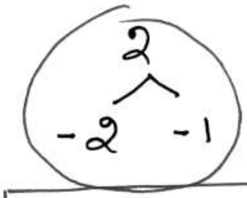
$$LCM = 2\times2\times2\times2\times3\cdot m^2\cdot n^2$$

$$\boxed{LCM = 48m^2n^2}$$

(iv) p^2-3p+2 ; p^2-4

Sol:-

$$p^2-3p+2 = (p-2)(p-1)$$



$$p^2-4 = p^2-2^2$$

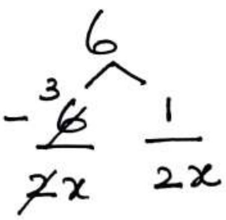
$$= (p-2)(p+2)$$

$\boxed{a^2-b^2=(a-b)(a+b)}$

$$\boxed{LCM = (p-2)(p-1)(p+2)}$$

(v) $2x^2-5x-3$; $4x^2-36$

Sol:-



$$2x^2-5x-3 = (x-3)(2x+1)$$

$$4x^2-36 = (2x)^2-6^2$$

$$= (2x-6)(2x+6)$$

$$= 2(x-3)\,2\cdot(x+3)$$

$$= 4(x-3)(x+3)$$

$$\boxed{LCM = 4(x-3)(2x+1)(x+3)}$$

(vi) $(2x^2-3xy)^2; (4x-6y)^3; (8x^3-27y^3)$

Sol :-

$$(2x^2-3xy)^2 = [x(2x-3y)]^2$$

$$= x^2(2x-3y)^2$$

$$(4x-6y)^3 = [2(2x-3y)]^3$$

$$= 2^3(2x-3y)^3$$

$$= 8(2x-3y)^3$$

$$8x^3-27y^3 = (2x)^3-(3y)^3$$

$$\boxed{a^3-b^3 = (a-b)^3+3ab(a-b)}$$

$$= (2x-3y)^3+3(2x)(3y)(2x-3y)$$

$$= (2x-3y)[(2x-3y)^2+18yx]$$

$$= (2x-3y)\left[(2x)^2+(3y)^2-2(2x)(3y) \\ +18xy\right]$$

$$= (2x-3y)[4x^2+9y^2-12xy+18xy$$

$$= (2x-3y)[4x^2+9y^2+6xy]$$

$$\boxed{LCM = 8x^2(2x-3y)^3(4x^2+9y^2+6xy)}$$

Exercise-3.3

① Find the LCM and GCD for the following and Verify that

$$f(x) \times g(x) = \text{LCM} \times \text{GCD}.$$

(i) $21x^2y$; $35xy^2$

Sol :-

$f(x) = 21x^2y = 3 \times 7 \times x^2y$

$g(x) = 35xy^2 = 5 \times 7 \times x \cdot y^2$

$\text{LCM} = 3 \times 7 \times 5\, x^2y^2$

$$\boxed{\text{LCM} = 105x^2y^2}$$

$$\boxed{\text{GCD} = 7xy}$$

$$\begin{array}{r} 21 \\ \underline{35} \\ \underline{105} \\ \underline{63} \\ 735 \end{array}$$

To Verify :- $f(x) \times g(x) = \text{LCM} \times \text{GCD}$

$$(21x^2y)(35xy^2) = (105x^2y^2)(7xy)$$

$$735x^3y^3 = 735x^3y^3$$

Hence Verified.

(ii) $(x^3-1)(x+1)$; (x^3+1)

Sol :-

$$f(x) = (x^3-1)(x+1) \qquad \boxed{a^3-b^3=(a-b)(a^2+b^2+ab)}$$

$$= (x-1)(x^2+1+x)(x+1)$$

$$g(x) = (x^3+1) \qquad \boxed{a^3+b^3=(a+b)(a^2+b^2-ab)}$$

$$= (x+1)(x^2+1-x)$$

$$LCM = (x-1)(x^2+1+x)(x+1)(x^2+1-x)$$

$$GCD = (x+1)$$

To Verify $f(x) \times g(x) = LCM \times GCD$

$$(x^3-1)(x+1) \times (x^3+1) = \underline{(x-1)(x^2+1+x)}\,\underline{(x+1)(x^2+1-x)}\,(x+1)$$

$$(x^3-1)(x^3+1)(x+1) = (x^3-1)(x^3+1)(x+1)$$

Hence Verified.

(iii) (x^2y+xy^2) ; x^2+xy

Sol :-

$$f(x) = x^2y+xy^2 = xy(x+y)$$

$$g(x) = x^2+xy = x(x+y)$$

$LCM = xy(x+y)$

$GCD = x(x+y)$

To Verify $f(x) \times g(x) = LCM \times GCD$

$$(x^2y + xy^2)(x^2 + xy) = xy(x+y) \cdot x(x+y)$$

$$(x^2y + xy^2)(x^2 + xy) = (x^2y + xy^2)(x^2 + xy)$$

Hence Verified.

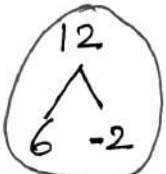
---

② Find the LCM of each pair of the following polynomials.

(i) $a^2 + 4a - 12$; $a^2 - 5a + 6$ whose $GCD = a-2$

Sol :-

$f(x) = a^2 + 4a - 12 = (a+6)(a-2)$



$g(x) = a^2 - 5a + 6 = (a-2)(a-3)$

$GCD = a-2$

$LCM \times GCD = f(x) \times g(x)$

$$LCM = \frac{f(x) \times g(x)}{GCD}$$

$$= \frac{(a+6)(a-2)\cancel{(a-2)}(a-3)}{\cancel{a-2}}$$

$$\boxed{LCM = (a+6)(a-2)(a-3)}$$

(ii) $x^4 - 27a^3x$; $(x-3a)^2$ whose GCD is $x-3a$

Sol:-

$$f(x) = x^4 - 27a^3x = x(x^3 - 27a^3)$$

$$= x(x^3 - (3a)^3)$$ $\boxed{a^3 - b^3 = (a-b)(a^2+b^2+ab)}$

$$= x(x-3a)(x^2 + (3a)^2 + (x)(3a))$$

$$= x(x-3a)(x^2 + 9a^2 + 3ax)$$

$$g(x) = (x-3a)^2$$

$$GCD = x - 3a$$

$$LCM \times GCD = f(x) \times g(x)$$

$$LCM = \frac{f(x) \times g(x)}{GCD}$$

$$= \frac{x(\cancel{x-3a})(x^2+9a^2+3ax)(x-3a)^2}{\cancel{(x-3a)}}$$

$$\boxed{LCM = x(x-3a)^2(x^2+9a^2+3ax)}$$

---

③ Find GCD of each pair of the following polynomials:-

(i) $12(x^4 - x^3)$; $8(x^4 - 3x^3 + 2x^2)$ whose LCM is $24x^3(x-1)(x-2)$

Sol

$f(x) = 12(x^4 - x^3) = 12x^3(x-1)$

$g(x) = 8(x^4 - 3x^3 + 2x^2) = 8x^2(x^2 - 3x + 2)$

$= 8x^2(x-2)(x-1)$

$\text{LCM} = 24x^3(x-1)(x-2)$

$\text{LCM} \times \text{GCD} = f(x) \times g(x)$

$$\text{GCD} = \frac{f(x) \times g(x)}{\text{LCM}}$$

$$= \frac{\overset{4}{\cancel{12}}\cancel{x^3}(x-1) \times \overset{1}{\cancel{8}}x^2\cancel{(x-1)}\cancel{(x-2)}}{\underset{\cancel{3}\,1}{\cancel{24}}\cancel{x^3}\cancel{(x-1)}\cancel{(x-2)}}$$

$$\boxed{\text{GCD} = 4x^2}$$

(ii) $(x^3 + y^3)$; $(x^4 + x^2y^2 + y^4)$ whose $\text{LCM} = (x^3+y^3)(x^2+xy+y^2)$

Sol:- $f(x) = (x^3 + y^3)$

$g(x) = (x^4 + x^2y^2 + y^4)$

$\text{LCM} = (x^3 + y^3)(x^2 + xy + y^2)$

$$\text{GCD} = \frac{f(x) \times g(x)}{\text{LCM}}$$

$$= \frac{\cancel{(x^3 + y^3)} \times (x^4 + x^2y^2 + y^4)}{\cancel{(x^3+y^3)}(x^2+xy+y^2)}$$

$$= \frac{\left((x^2)^2 + (y^2)^2 + 2x^2y^2\right) - x^2y^2}{(x^2+y^2+xy)}$$

$$= \frac{(x^2+y^2)^2 - (xy)^2}{x^2+y^2+xy}$$

$$\boxed{\begin{array}{l} a^2+b^2+2ab=(a+b)^2 \\ a^2-b^2=(a+b)(a-b) \end{array}}$$

$$= \frac{(x^2+\cancel{y^2+xy})(x^2+y^2-xy)}{x^2+\cancel{y^2+xy}}$$

$$\boxed{GCD = x^2+y^2-xy}$$

---

④ Given the LCM and GCD of the two polynomials p(x) and q(x) find the unknown polynomial in the following table:-

S.NO	(i)	(ii)
LCM	$a^3-10a^2+11a+70$	$(x^4-y^4)(x^4+x^2y^2+y^4)$
GCD	$a-7$	(x^2-y^2)
P(x)	$a^2-12a+35$	
q(x)		$(x^4-y^4)(x^2+y^2-xy)$

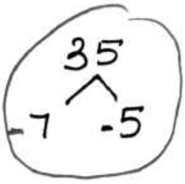
(i) $LCM = a^3-10a^2+11a+70$

$GCD = a-7$

$p(x) = a^2-12a+35$

$$p(x) \times q(x) = LCM \times GCD$$

$$q(x) = \frac{LCM \times GCD}{p(x)}$$



$$= \frac{(a^3-10a^2+11a+70)(a-7)}{a^2-12a+35}$$

$$= \frac{(a^3-10a^2+11a+70)\cancel{(a-7)}}{\cancel{(a-7)}(a-5)}$$

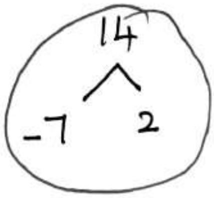
$$= \frac{a^3-10a^2+11a+70}{a-5}$$

$$\begin{array}{r|l} & a^2-5a-14 \\ \hline a-5 & \cancel{a^3}-10a^2+11a+70 \\ & -\cancel{a^3} \underset{+}{-} 5a^2 \downarrow \\ \hline & -5a^2+11a \\ & -5a^2+25a \\ & \underset{}{+} \quad\quad - \\ \hline & -14a+70 \\ & -14a+70 \\ & + \quad\quad - \\ \hline & 0 \end{array}$$

$$\boxed{q(x) = a^2-5a-14}$$

(or)

$$\boxed{q(x) = (a-7)(a+2)}$$



(ii) $$\text{LCM} = (x^4-y^4)(x^4+x^2y^2+y^4)$$

$$(x^4-y^4)\left[\left[(x^2)^2+(y^2)^2+2x^2y^2\right]-x^2y^2\right]$$

$$(x^4-y^4)\left[(x^2+y^2)^2-(xy)^2\right]$$

$$(x^4-y^4)(x^2+y^2+xy)(x^2+y^2-xy)$$

$$\text{GCD} = (x^2-y^2) = (x+y)(x-y)$$

$$q(x) = (x^4-y^4)(x^2+y^2-xy)$$

$$P(x) = \frac{\text{LCM} \times \text{GCD}}{q(x)}$$

$$= \frac{\cancel{(x^4-y^4)}(x^2+y^2+xy)\cancel{(x^2+y^2-xy)}(x+y)(x-y)}{\cancel{(x^4-y^4)}\cancel{(x^2+y^2-xy)}}$$

$$= (x^2+y^2+xy)(x+y)(x-y)$$

$$\boxed{P(x) = (x^2-y^2)(x^2+y^2+xy)}$$

Exercise - 3.4

① Reduce each of the following rational expressions to its lowest form:

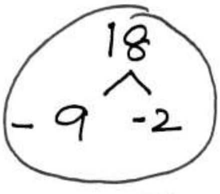
(i) $\frac{x^2-1}{x^2+x}$

Sol :- $\frac{x^2-1}{x^2+x} = \frac{x^2-1^2}{x(x+1)}$

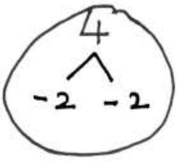
$$= \frac{(x-1)\cancel{(x+1)}}{x\cancel{(x+1)}}$$

$$= \frac{x-1}{x}$$

(ii) $\frac{x^2-11x+18}{x^2-4x+4}$



Sol $\frac{x^2-11x+18}{x^2-4x+4} = \frac{(x-9)\cancel{(x-2)}}{(x-2)\cancel{(x-2)}}$



$$= \frac{x-9}{x-2}$$

(iii) $\dfrac{9x^2+81x}{x^3+8x^2-9x}$

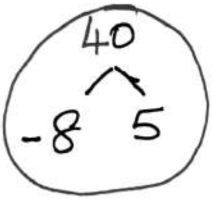
Sol:

$$\frac{9x^2+81x}{x^3+8x^2-9x} = \frac{9\cancel{x}(x+9)}{\cancel{x}(x^2+8x-9)}$$

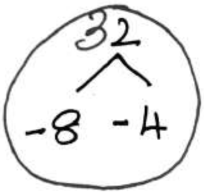
$$= \frac{9\cancel{(x+9)}}{\cancel{(x+9)}(x-1)}$$

$$= \frac{9}{x-1}$$

(iv) $\dfrac{P^2-3P-40}{2P^3-24P^2+64P}$



Sol:-



$$\frac{P^2-3P-40}{2P^3-24P^2+64P} = \frac{P^2-3P-40}{2P(P^2-12P+32)}$$

$$= \frac{\cancel{(P-8)}(P+5)}{2P\cancel{(P-8)}(P-4)}$$

$$= \frac{P+5}{2P(P-4)}$$

② Find the excluded Values, if any of the following expressions:-

(i) $\frac{y}{y^2-25}$

$\boxed{a^2-b^2=(a-b)(a+b)}$

Sol:- $\frac{y}{y^2-25} = \frac{y}{y^2-5^2}$

$$= \frac{y}{(y-5)(y+5)}$$

$$(y-5)(y+5) = 0$$

$y-5=0$ | $y+5=0$

$\boxed{y=5}$ | $\boxed{y=-5}$

$\therefore$ Excluded Values = 5, −5

(ii) $\frac{t}{t^2-5t+6}$

6

^

−2 −3

Sol

$$\frac{t}{t^2-5t+6} = \frac{t}{(t-2)(t-3)}$$

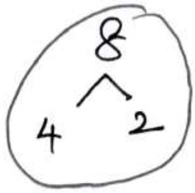
$$(t-2)(t-3) = 0$$

$t-2=0$ | $t-3=0$

$\boxed{t=2}$ | $\boxed{t=3}$

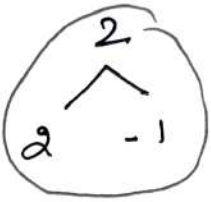
$\therefore$ Excluded Values = 2, 3

(iii) $\dfrac{x^2+6x+8}{x^2+x-2}$



Sol:-

$$\frac{x^2+6x+8}{x^2+x-2} = \frac{(x+4)\cancel{(x+2)}}{\cancel{(x+2)}(x-1)}$$



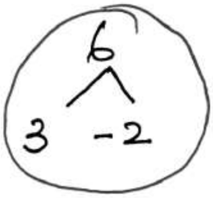
$$x-1=0$$

$$x=1$$

$\therefore$ Excluded Value $=1$

(iv) $\dfrac{x^3-27}{x^3+x^2-6x}$

$$\boxed{a^3-b^3=(a-b)(a^2+b^2+ab)}$$



Sol:-

$$\frac{x^3-27}{x^3+x^2-6x} = \frac{x^3-3^3}{x(x^2+x-6)}$$

$$= \frac{(x-3)(x^2+3^2+3x)}{x(x+3)(x-2)}$$

$$x(x+3)(x-2)=0$$

$\boxed{x=0}$ $\quad x+3=0$ $\quad x-2=0$

$\quad\boxed{x=-3}$ $\quad\boxed{x=2}$

$\therefore$ Excluded Value $=0, -3, 2$

Exercise-3.5

① Simplify :-

(i) $\frac{4x^2y}{2z^2} \times \frac{6xz^2}{20y^4}$

Sol:- $$\frac{\overset{1}{\cancel{\overset{2}{\cancel{4}}}}x^2y}{\underset{1}{\cancel{2}}z^2} \times \frac{\overset{3}{\cancel{6}}xz^2}{\underset{\cancel{10}\,5}{\cancel{20}}y^4}$$

$$\frac{x^2\cancel{y}}{\cancel{z^{\cancel{2}}}} \times \frac{3x\cancel{z^{\cancel{2}}}}{5y^{\cancel{4}\,3}}$$

$$\Rightarrow x^2 \times \frac{3x}{5y^3} \Rightarrow \frac{3x^3}{5y^3}$$

(ii) $\frac{P^2-10P+21}{P-7} \times \frac{P^2+P-12}{(P-3)^2}$

21
-7 -3

Sol $$\frac{P^2-10P+21}{P-7} \times \frac{P^2+P-12}{(P-3)^2}$$

12
4 -3

$$\frac{\cancel{(P-7)}\cancel{(P-3)}}{\cancel{(P-7)}} \times \frac{(P+4)\cancel{(P-3)}}{\cancel{(P-3)^2}} \Rightarrow P+4$$

(iii) $\frac{5t^3}{4t-8} \times \frac{6t-12}{10t}$

Sol :-

$$\frac{5t^3}{4t-8} \times \frac{6t-12}{10t} \Rightarrow \frac{\overset{1}{\cancel{5}}t^{\overset{②}{\cancel{3}}}}{\underset{2}{\cancel{4}}\cancel{(t-2)}} \times \frac{\overset{3}{\cancel{6}}\cancel{(t-2)}}{\underset{2}{\cancel{10}}\cancel{t}}$$

$$\Rightarrow \frac{t^2}{2} \times \frac{3}{2}$$

$$\Rightarrow \frac{3t^2}{4}$$

---

② Simplify:-

(i) $\frac{x+4}{3x+4y} \times \frac{9x^2-16y^2}{2x^2+3x-20}$

Sol :-

$$\frac{x+4}{3x+4y} \times \frac{9x^2-16y^2}{2x^2+3x-20}$$

40

$\frac{\overset{4}{\cancel{8}}}{\underset{1}{\cancel{2}}x} \quad \frac{-5}{2x}$

$$\frac{x\cancel{+4}}{3x+4y} \times \frac{(3x)^2-(4y)^2}{\cancel{(x+4)}(2x-5)}$$

$$\frac{1}{3x\cancel{+4y}} \times \frac{(3x-4y)\cancel{(3x+4y)}}{(2x-5)}$$

$$\Rightarrow \boxed{\frac{3x-4y}{2x-5}}$$

(ii) $\dfrac{x^3-y^3}{3x^2+9xy+6y^2} \times \dfrac{x^2+2xy+y^2}{x^2-y^2}$

Sol :-

$$\frac{x^3-y^3}{3x^2+9xy+6y^2} \times \frac{x^2+2xy+y^2}{x^2-y^2}$$

$$\frac{\cancel{(x-y)}(x^2+y^2+xy)}{3(x^2+3xy+2y^2)} \times \frac{(x+y)^{\cancel{2}}}{\cancel{(x-y)}\cancel{(x+y)}}$$

$$\frac{(x^2+y^2+xy)(x+y)}{3(x^2+3xy+2y^2)}$$

$$\frac{(x+y)(x^2+y^2+xy)}{3\left[x^2+xy+2xy+2y^2\right]}$$

$$\frac{(x+y)(x^2+y^2+xy)}{3\left[x(x+y)+2y(x+y)\right]}$$

$$\frac{\cancel{(x+y)}(x^2+y^2+xy)}{3\cancel{(x+y)}(x+2y)} \Rightarrow \boxed{\frac{x^2+y^2+xy}{3(x+2y)}}$$

③ Simplify :-

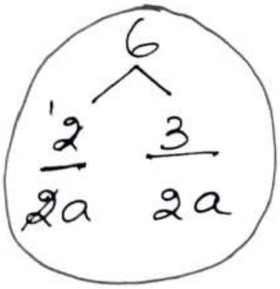
(i) $\frac{2a^2+5a+3}{2a^2+7a+6} \div \frac{a^2+6a+5}{-5a^2-35a-50}$

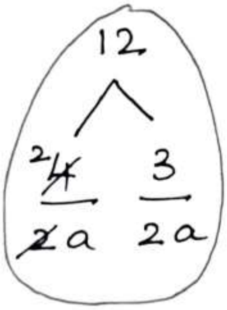
Sol :-

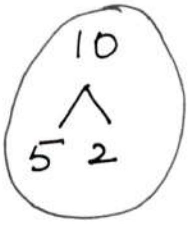
$$\frac{2a^2+5a+3}{2a^2+7a+6} \div \frac{a^2+6a+5}{-5a^2-35a-50}$$

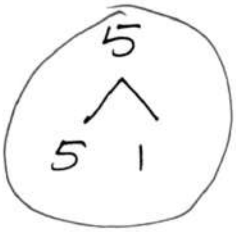
$$\frac{2a^2+5a+3}{2a^2+7a+6} \times \frac{-(5a^2+35a+50)}{a^2+6a+5}$$

$$\frac{2a^2+5a+3}{2a^2+7a+6} \times -5\frac{(a^2+7a+10)}{a^2+6a+5}$$









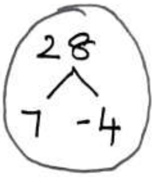
$$\frac{\cancel{(a+1)}\,\cancel{(2a+3)}}{\cancel{(a+2)}\,\cancel{(2a+3)}} \times \frac{-5\,\cancel{(a+5)}\,\cancel{(a+2)}}{\cancel{(a+5)}\,\cancel{(a+1)}}$$

$$\Rightarrow \boxed{-5}$$

(ii) $\dfrac{b^2+3b-28}{b^2+4b+4} \div \dfrac{b^2-49}{b^2-5b-14}$

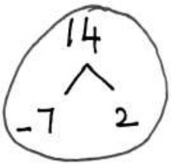
Sol:- $\dfrac{b^2+3b-28}{b^2+4b+4} \div \dfrac{b^2-49}{b^2-5b-14}$

$\dfrac{b^2+3b-28}{b^2+4b+4} \times \dfrac{b^2-5b-14}{b^2-49}$



$\dfrac{b^2+3b-28}{b^2+4b+4} \times \dfrac{b^2-5b-14}{b^2-7^2}$

$\dfrac{(\cancel{b+7})(b-4)}{(b+2)(\cancel{b+2})} \times \dfrac{(\cancel{b-7})(\cancel{b+2})}{(\cancel{b-7})(\cancel{b+7})}$

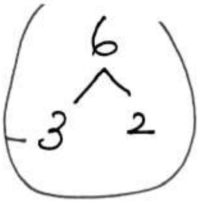


$\Rightarrow \boxed{\dfrac{b-4}{b+2}}$

(iii) $\dfrac{x+2}{4y} \div \dfrac{x^2-x-6}{12y^2}$

Sol:- $\dfrac{x+2}{4y} \div \dfrac{x^2-}{12y^2}$

$\dfrac{x+2}{4y} \times \dfrac{12y^2}{x^2-x-6}$



$\dfrac{\cancel{x+2}}{\underset{1}{\cancel{4}}\cancel{y}} \times \dfrac{\overset{3}{\cancel{12}}y^{\cancel{2}}}{(\cancel{x+2})(x-3)} \Rightarrow \boxed{\dfrac{3y}{x-3}}$

(iv) $\frac{12t^2-22t+8}{3t} \div \frac{3t^2+2t-8}{2t^2+4t}$

Sol:- $\frac{12t^2-22t+8}{3t} \div \frac{3t^2+2t-8}{2t^2+4t}$

$$\frac{12t^2-22t+8}{3t} \times \frac{2t^2+4t}{3t^2+2t-8}$$

$$\frac{2(6t^2-11t+4)}{3t} \times \frac{2t(t+2)}{3t^2+2t-8}$$

$$\frac{2\cancel{(3t-4)}(2t-1)}{3\cancel{t}} \times \frac{2\cancel{t}\cancel{(t+2)}}{\cancel{(3t-4)}\cancel{(t+2)}}$$

$$\Rightarrow \boxed{\frac{4(2t-1)}{3}}$$

24

$\frac{-\cancel{8}^{4}}{\cancel{6}_{3}t}$ $\frac{-\cancel{3}^{1}}{\cancel{4}_{2}t}$

24

$\frac{-4}{3t}$ $\frac{\cancel{6}^{2}}{\cancel{3}t}$

---

④ If $x = \frac{a^2+3a-4}{3a^2-3}$ and $y = \frac{a^2+2a-8}{2a^2-2a-4}$

find the Value of $x^2 \cdot y^{-2}$

Sol:- $x = \frac{a^2+3a-4}{3a^2-3}$

$= \frac{(a+4)(a-1)}{3(a^2-1)}$

$y = \frac{a^2+2a-8}{2(a^2-a-2)}$

$= \frac{(a+4)\cancel{(a-2)}}{2\cancel{(a-2)}(a+1)}$

4: 4, -1

8: 4, -2

2: -2, 1

$$= \frac{(a+4)\cancel{(a-1)}}{3\cancel{(a-1)}(a+1)}$$

$$x = \frac{a+4}{3(a+1)}$$

$$y = \frac{a+4}{2(a+1)}$$

To find :- $x^2 y^{-2}$

$$x^2 y^{-2} = \left[\frac{a+4}{3(a+1)}\right]^2 \cdot \left[\frac{a+4}{2(a+1)}\right]^{-2}$$

$$= \left[\frac{(a+4)}{3(a+1)}\right]^2 \cdot \left[\frac{2(a+1)}{(a+4)}\right]^2$$

$$= \frac{\cancel{(a+4)^2}}{3^2\cancel{(a+1)^2}} \times \frac{2^2\cancel{(a+1)^2}}{\cancel{(a+4)^2}}$$

$$= \frac{4}{9}$$

---

⑤ If a polynomial $p(x) = x^2 - 5x - 14$ is divided by another polynomial $q(x)$ we get $\frac{x-7}{x+2}$, find $q(x)$

Sol :- $$\frac{p(x)}{q(x)} = \frac{x-7}{x+2}$$

$$\frac{x^2-5x-14}{q(x)} = \frac{x-7}{x+2}$$

$$\frac{(x-\cancel{7})(x+2)(x+2)}{(x-\cancel{7})} = q(x)$$

14
-7 2

$$\boxed{q(x) = (x+2)^2}$$

Exercise 3.6

① Simplify :-

(i) $\frac{x(x+1)}{x-2} + \frac{x(1-x)}{x-2}$

Sol :- $\frac{x(x+1)}{x-2} + \frac{x(1-x)}{x-2}$

$$\frac{x(x+1) + x(1-x)}{x-2}$$

$$\frac{\cancel{x^2} + x + x - \cancel{x^2}}{x-2}$$

$$\boxed{\frac{2x}{x-2}}$$

(ii) $\dfrac{x+2}{x+3} + \dfrac{x-1}{x-2}$

Sol:- $\dfrac{x+2}{x+3} + \dfrac{x-1}{x-2}$

$$\frac{(x+2)(x-2) + (x-1)(x+3)}{(x+3)(x-2)}$$

$$\frac{x^2 - \cancel{2x} + \cancel{2x} - 4 + x^2 + 3x - x - 3}{(x+3)(x-2)}$$

$$\boxed{\frac{2x^2 + 2x - 7}{(x+3)(x-2)}}$$

(iii) $\dfrac{x^3}{x-y} + \dfrac{y^3}{y-x}$

Sol:- $\dfrac{x^3}{x-y} + \dfrac{y^3}{y-x}$

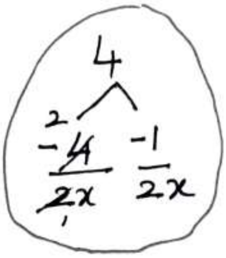
$$\frac{x^3}{x-y} - \frac{y^3}{x-y}$$

$$\frac{x^3 - y^3}{x-y} = \frac{\cancel{(x-y)}(x^2+y^2+xy)}{\cancel{(x-y)}}$$

$$\boxed{x^2+y^2+xy}$$

② Simplify :-

(i) $\dfrac{(2x+1)(x-2)}{x-4} - \dfrac{2x^2-5x+2}{x-4}$



Sol :-

$$\frac{(2x+1)(x-2)}{x-4} - \frac{2x^2-5x+2}{x-4}$$

$$\frac{(2x+1)(x-2)}{x-4} - \frac{(x-2)(2x-1)}{x-4}$$

$$\frac{(2x+1)(x-2) - (x-2)(2x-1)}{x-4}$$

$$\frac{(x-2)\left[(2x+1) - (2x-1)\right]}{x-4}$$

$$\frac{(x-2)\left[2\cancel{x}+1-2\cancel{x}+1\right]}{(x-4)}$$

$$\boxed{\frac{2(x-2)}{(x-4)}}$$

(ii) $\frac{4x}{x^2-1} - \frac{x+1}{x-1}$

Sol :-

$$\frac{4x}{x^2-1} - \frac{x+1}{x-1}$$

$$\frac{4x}{(x-1)(x+1)} - \frac{x+1}{(x-1)}$$

$$\frac{4x - \overrightarrow{(x+1)(x+1)}}{(x-1)(x+1)}$$

$$\frac{4x - [x^2 + x + x + 1]}{(x-1)(x+1)}$$

$$\frac{4x - (x^2 + 2x + 1)}{(x-1)(x+1)}$$

$$\frac{4x - x^2 - 2x - 1}{(x-1)(x+1)}$$

$$\frac{-x^2 + 2x - 1}{(x-1)(x+1)}$$

$$\frac{-[x^2 - 2x + 1]}{(x-1)(x+1)}$$

$$-\frac{(x-1)^{\cancel{2}}}{\cancel{(x-1)}(x+1)}$$

$$\frac{-(x-1)}{x+1}$$

$$\Rightarrow \frac{-x+1}{x+1} \Rightarrow \boxed{\frac{1-x}{x+1}}$$

---

③ Subtract $\frac{1}{x^2+2}$ from $\frac{2x^3+x^2+3}{(x^2+2)^2}$

Sol :-

$$\frac{2x^3+x^2+3}{(x^2+2)^2} - \frac{1}{x^2+2}$$

$$\frac{2x^3+x^2+3-(x^2+2)}{(x^2+2)^2}$$

$$\frac{2x^3+\cancel{x^2}+3-\cancel{x^2}-2}{(x^2+2)^2}$$

$$\boxed{\frac{2x^3+1}{(x^2+2)^2}}$$

---

④ Which rational expression should be subtracted from $\frac{x^2+6x+8}{x^3+8}$ to $\frac{3}{x^2-2x+4}$.

Sol:-

Let the rational expression to be subtracted $= A$

$$\frac{x^2+6x+8}{x^3+8} - A = \frac{3}{x^2-2x+4}$$

8
∧
4 2

$$\frac{(x+4)(x+2)}{x^3+2^3} - A = \frac{3}{x^2-2x+4}$$

$$\frac{(x+4)(\cancel{x+2})}{(\cancel{x+2})(x^2+2^2-2x)} - A = \frac{3}{x^2-2x+4}$$

$$\frac{x+4}{x^2+4-2x} - \frac{3}{x^2+4-2x} = A$$

$$\frac{x+4-3}{x^2+4-2x} = A$$

$$\boxed{A = \frac{x+1}{x^2+4-2x}}$$

⑤ If $A = \frac{2x+1}{2x-1}$, $B = \frac{2x-1}{2x+1}$ find

$$\frac{1}{A-B} - \frac{2B}{A^2-B^2}$$

Sol:-

To find $\frac{1}{A-B} - \frac{2B}{A^2-B^2}$

$$\frac{1}{A-B} - \frac{2B}{A^2-B^2} = \frac{1}{A-B} - \frac{2B}{(A-B)(A+B)}$$

$$= \frac{A+B-2B}{(A-B)(A+B)}$$

$$= \frac{\cancel{A-B}}{\cancel{(A-B)}(A+B)}$$

$$= \frac{1}{A+B}$$

$$= \frac{1}{\frac{2x+1}{2x-1} + \frac{2x-1}{2x+1}}$$

$$= \frac{1}{\frac{(2x+1)^2 + (2x-1)^2}{(2x-1)(2x+1)}}$$

$$= \frac{(2x-1)(2x+1)}{(2x)^2+1+\cancel{2(2x)}+(2x)^2+1-\cancel{2(2x)}}$$

$$= \frac{(2x-1)(2x+1)}{4x^2+1+4x^2+1}$$

$$= \frac{(2x)^2-1^2}{8x^2+2}$$

$$\boxed{\frac{1}{A-B}-\frac{2B}{A^2-B^2}=\frac{4x^2-1}{8x^2+2}}$$

---

⑥ If $A=\frac{x}{x+1}$, $B=\frac{1}{x+1}$ prove that

$$\frac{(A+B)^2+(A-B)^2}{A\div B}=\frac{2(x^2+1)}{x(x+1)^2}$$

Sol:- To prove

$$\frac{(A+B)^2+(A-B)^2}{A\div B}=\frac{A^2+B^2+2\cancel{BA}+A^2+B^2-2\cancel{AB}}{A\div B}$$

$$=\frac{2A^2+2B^2}{A\div B}$$

$$= \frac{2(A^2+B^2)}{A \div B}$$

$$= \frac{2\left[\left(\frac{x}{x+1}\right)^2 + \left(\frac{1}{x+1}\right)^2\right]}{\frac{x}{x+1} \div \frac{1}{x+1}}$$

$$= \frac{2\left[\frac{x^2}{(x+1)^2} + \frac{1^2}{(x+1)^2}\right]}{\frac{x}{\cancel{x+1}} \times \frac{\cancel{x+1}}{1}}$$

$$= \frac{2\left[\frac{x^2+1}{(x+1)^2}\right]}{x}$$

$$\boxed{\frac{(A+B)^2+(A-B)^2}{A \div B} = \frac{2(x^2+1)}{x(x+1)^2}}$$

---

⑦ Pari needs 4 hours to complete a work. His friend yuvan needs 6 hours to complete the Same Work. How long will it take to complete if they work together?

Sol :-

$\left.\begin{array}{l}\text{Time taken by Pari to} \\ \text{complete the Work}\end{array}\right\} = 4 \text{ hrs.}$

$\left.\begin{array}{l}\text{Time taken by yuvan to} \\ \text{complete the Work}\end{array}\right\} = 6 \text{hrs.}$

To find :-

$\left.\begin{array}{l}\text{Time taken by both to} \\ \text{complete the Work}\end{array}\right\} = ?$

Let Time taken by both $= x$

Work done by Pari in 1 hr $= \frac{1}{4}$

Work done by yuvan in 1 hr $= \frac{1}{6}$

Work done by both in 1 hr $= \frac{1}{x}$

$$\frac{1\times3}{4\times3} + \frac{1\times2}{6\times2} = \frac{1}{x}$$

$$\begin{array}{r|l} 2 & 4, 6 \\ \hline 2 & 2, 3 \\ \hline 3 & 1, 3 \\ \hline & 1, 1 \end{array}$$

$$\frac{3+2}{12} = \frac{1}{x}$$

$$\frac{5}{12} = \frac{1}{x}$$

$$x = \frac{12}{5}$$

$$\begin{array}{r} 2.4 \\ 5\overline{)12} \\ 10 \\ \hline 20 \\ 20 \\ \hline 0 \end{array}$$

$$x = 2.4 \qquad \frac{0.4\times 60}{24.0}$$

$$x = 2\text{hr } 24\text{min}$$

$\therefore$ Time taken by both to complete the Work $\Big\}$ = 2hr 24min.

---

⑧ Iniya bought 50 kg of fruits consisting of apples and bananas. She paid twice as much per kg for the apples as she did for the banana. If Iniya bought ₹1800 worth of apples and ₹600 worth bananas, then how many kgs of each fruit did she buy?

Sol

FRUIT	QUANTITY	COST
Apple	x	a
Banana	y	b

Total Quantity of fruits = 50kg

$$\Rightarrow \boxed{x + y = 50} \longrightarrow ①$$

Amount paid for Apple = Twice the amount of Banana

ie, $\boxed{a = 2b} \longrightarrow$ ②

Total Amount paid for Apple = ₹1800

$$xa = 1800 \Rightarrow \boxed{a = \frac{1800}{x}} \longrightarrow ③$$

Total Amount paid for banana = ₹600

$$yb = 600 \Rightarrow \boxed{b = \frac{600}{y}} \longrightarrow ④$$

Put ③ & ④ in ②

$$a = 2b$$

$$\frac{1800}{x} = 2\left(\frac{600}{y}\right)$$

$$\frac{1800}{x} = \frac{1200}{y}$$

$$1800y = 1200x$$

$$0 = 1200x - 1800y. \quad (\div \text{ by } 600)$$

$$\boxed{2x - 3y = 0} \longrightarrow ⑤$$

Solve ① & ⑤

$$x + y = 50 \longrightarrow (x'y \text{ by } 3)$$
$$2x - 3y = 0$$

$$\Rightarrow \quad 3x + \cancel{3y} = 150$$
$$2x - \cancel{3y} = 0$$
$$5x = 150$$

$$x = \frac{15\cancel{0}}{\cancel{5}}$$

$$\boxed{x = 30}$$

Put $x = 30$ in ①

$$30 + y = 50$$
$$y = 50 - 30$$
$$\boxed{y = 20}$$

∴ Quantity of Apple = 30kg
Quantity of banana = 20kg.

Exercise - 3.7

1. Find the square root of the following rational expressions! -

(i) $\dfrac{400x^4y^{12}z^{16}}{100x^8y^4z^4}$

Sol : -

$$\frac{400x^4y^{12}z^{16}}{100x^8y^4z^4} = \sqrt{\frac{400x^4\cdot y^{12}z^{16}}{100x^8y^4z^4}}$$

$$= \sqrt{\frac{(20)^2(x^2)^2(y^6)^2(z^8)^2}{(10)^2(x^4)^2(y^2)^2(z^2)^2}}$$

$$= \frac{\overset{2}{\cancel{20}}\,\cancel{x^2}\,y^{\cancel{6}\,\textcircled{4}}\,z^{\cancel{8}\,\textcircled{6}}}{\cancel{10}\,x^{\cancel{4}\,\textcircled{2}}\,\cancel{y^2}\,\cancel{z^2}}$$

$$= \left|\frac{2y^4z^6}{x^2}\right|$$

(ii) $$\frac{7x^2+2\sqrt{14}x+2}{x^2-\frac{1}{2}x+\frac{1}{16}}$$

Sol:-

$$\sqrt{\frac{7x^2+2\sqrt{14}x+2}{x^2-\frac{1}{2}x+\frac{1}{16}}} = \sqrt{\frac{(\sqrt{7}x)^2+2(\sqrt{7}x)(\sqrt{2})+(\sqrt{2})^2}{(x)^2-2\left(\frac{1}{4}x\right)+\left(\frac{1}{4}\right)^2}}$$

$$= \sqrt{\frac{(\sqrt{7}x+\sqrt{2})^2}{\left(x-\frac{1}{4}\right)^2}}$$

$$= \frac{\sqrt{7}x+\sqrt{2}}{x-\frac{1}{4}}$$

$$= \frac{\sqrt{7}x+\sqrt{2}}{\frac{4x-1}{4}}$$

$$= 4\left|\frac{\sqrt{7}x+\sqrt{2}}{4x-1}\right|$$

(iii) $\dfrac{121(a+b)^8(x+y)^8(b-c)^8}{81(b-c)^4(a-b)^{12}(b-c)^4}$

Sol:-

$$\frac{121(a+b)^8(x+y)^8(b-c)^8}{81\,(b-c)^4\,(a-b)^{12}\,(b-c)^4}$$

$$\sqrt{\frac{121(a+b)^8(x+y)^8\cancel{(b-c)^8}}{81\,\cancel{(b-c)^8}\,(a-b)^{12}}}$$

$$\sqrt{\frac{(11)^2\left((a+b)^4\right)^2\left((x+y)^4\right)^2}{9^2\left((a-b)^6\right)^2}}$$

$$\frac{11}{9}\left|\frac{(a+b)^4\,(x+y)^4}{(a-b)^6}\right|$$

---

② Find the Square root of the following:-

(i) $4x^2+20x+25$

Sol $\sqrt{4x^2+20x+25} = \sqrt{(2x)^2+2(2x)(5)+5^2}$

$= \sqrt{(2x+5)^2}$

$= |2x+5|$

(ii) $9x^2-24xy+30xz-40yz+25z^2+16y^2$

Sol:- $9x^2+16y^2+25z^2-24xy-40yz+30xz$

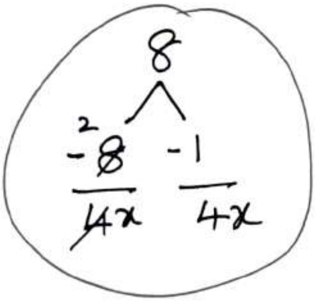
$$\sqrt{(3x)^2+(-4y)^2+(5z)^2+2(3x)(-4y)+2(-4y)(5z)+2(5z)(3x)}$$

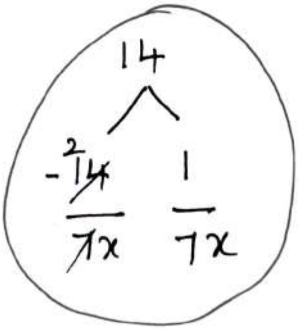
$$\sqrt{(3x-4y+5z)^2}$$

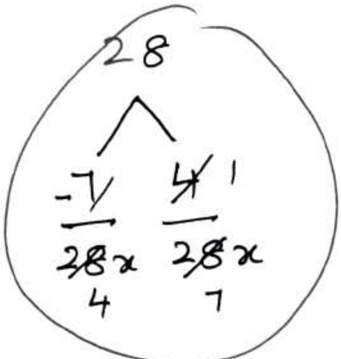
$$|3x-4y+5z|$$

(iii) $(4x^2-9x+2)(7x^2-13x-2)(28x^2-3x-1)$

Sol:- $\sqrt{(4x^2-9x+2)(7x^2-13x-2)(28x^2-3x-1)}$







$$\sqrt{(x-2)(4x-1)(x-2)(7x+1)(4x-1)(7x+1)}$$

$$\sqrt{(x-2)^2(4x-1)^2(7x+1)^2}$$

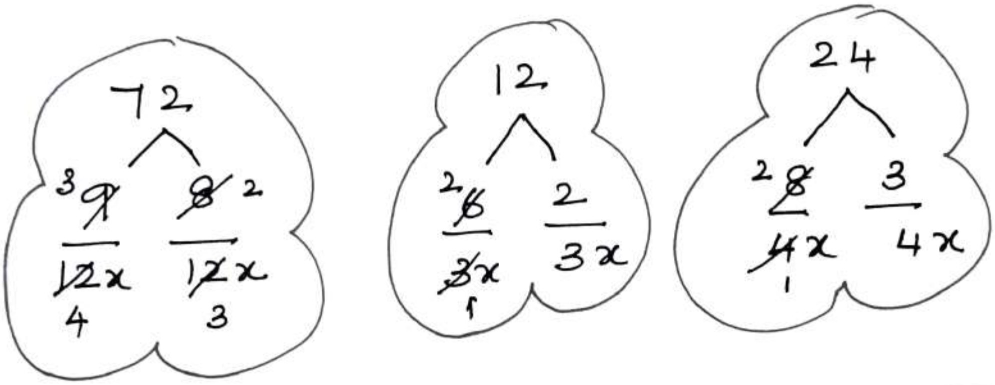
$$|(x-2)(4x-1)(7x+1)|$$

(iv) $\left(2x^2+\frac{17}{6}x+1\right)\left(\frac{3}{2}x^2+4x+2\right)\left(\frac{4}{3}x^2+\frac{11}{3}x+2\right)$

Sol

$$\sqrt{\left(2x^2+\frac{17}{6}x+1\right)\left(\frac{3}{2}x^2+4x+2\right)\left(\frac{4}{3}x^2+\frac{11}{3}x+2\right)}$$

$$\sqrt{\left(\frac{12x^2+17x+6}{6}\right)\left(\frac{3x^2+8x+4}{2}\right)\left(\frac{4x^2+11x+6}{3}\right)}$$


72
$\frac{\cancel{9}^{3}}{\cancel{12}_{4}x}$ $\frac{\cancel{8}^{2}}{\cancel{12}_{3}x}$
12
$\frac{\cancel{6}^{2}}{\cancel{3}_{1}x}$ $\frac{2}{3x}$
24
$\frac{\cancel{8}^{2}}{\cancel{4}_{1}x}$ $\frac{3}{4x}$

$$\sqrt{\frac{(4x+3)(3x+2)}{6}\times\frac{(x+2)(3x+2)}{2}\times\frac{(x+2)(4x+3)}{3}}$$

$$\sqrt{\frac{(4x+3)^2(3x+2)^2(x+2)^2}{6^2}}$$

$$\frac{1}{6}\left|(4x+3)(3x+2)(x+2)\right|$$

Exercise - 3.8

① Find the square root of the following polynomials by division method.

(i) $x^4-12x^3+42x^2-36x+9$.

Sol:- $\sqrt{x^4-12x^3+42x^2-36x+9}$

$$\begin{array}{r|l}
 & x^2-6x+3 \\ \hline
x^2 & \cancel{x^4}-12x^3+42x^2-36x+9 \\
 & \underset{(-)}{\cancel{x^4}} \quad \downarrow \quad \downarrow \qquad \downarrow \quad \downarrow \\ \hline
2x^2-6x & -\cancel{12x^3}+42x^2 \\
 & \underset{(+)}{-\cancel{12}x^3}\underset{(-)}{+}36x^2 \\ \hline
2x^2-12x+3 & \cancel{6x^2}-\cancel{36x}+\cancel{9} \\
 & \cancel{6}x^2-3\cancel{6}x+\cancel{9} \\
 & (-) \quad (+) \quad (-) \\ \hline
 & 0
\end{array}$$

$\therefore \sqrt{x^4-12x^3+42x^2-36x+9} = |x^2-6x+3|$

(ii) $37x^2 - 28x^3 + 4x^4 + 42x + 9$

Sol:-

$$\sqrt{4x^4 - 28x^3 + 37x^2 + 42x + 9}$$

$$\begin{array}{r|l}
 & 2x^2 - 7x - 3 \\
\hline
2x^2 & \cancel{4x^4} - 28x^3 + 37x^2 + 42x + 9 \\
 & \underset{(-)}{\cancel{4x^4}} \\
\hline
4x^2 - 7x & \cancel{-28x^3} + 37x^2 \\
 & \underset{(+)}{\cancel{-28x^3}} \underset{(-)}{+} 49x^2 \\
\hline
4x^2 - 14x - 3 & \cancel{-12x^2} \cancel{+42x} \cancel{+9} \\
 & \underset{(+)}{\cancel{-12x^2}} \underset{(-)}{\cancel{+42x}} \underset{(-)}{\cancel{+9}} \\
\hline
 & 0
\end{array}$$

$$\sqrt{4x^4 - 28x^3 + 37x^2 + 42x + 9} = |2x^2 - 7x - 3|$$

(iii) $16x^4 + 8x^2 + 1$

Sol:-

$$\sqrt{16x^4 + 8x^2 + 1}$$

$$\begin{array}{r|l}
 & 4x^2 + 1 \\
\hline
4x^2 & \cancel{16x^4} + 8x^2 + 1 \\
 & \underset{(-)}{\cancel{16x^4}} \\
\hline
8x^2 + 1 & 8x^2 + 1 \\
 & (-)\,8x^2 \underset{(-)}{+} 1 \\
\hline
 & 0
\end{array}
\Rightarrow |4x^2 + 1|$$

(iv) $121x^4 - 198x^3 - 183x^2 + 216x + 144$

Sol :-

$$\sqrt{121x^4 - 198x^3 - 183x^2 + 216x + 144}$$

$$\begin{array}{r|l} & 11x^2-9x-12 \\ \hline 11x^2 & \cancel{121x^4}-198x^3-183x^2+216x+144 \\ & \underset{(-)}{\cancel{121x^4}} \\ \hline 22x^2-9x & -\cancel{198x^3}-183x^2 \\ & \underset{(+)}{-\cancel{198x^3}}\underset{(-)}{+81x^2} \\ \hline 22x^2-18x-12 & -\cancel{264}x^2+\cancel{216x}+\cancel{144} \\ & \underset{(+)}{-\cancel{264}x^2}\underset{(-)}{+\cancel{216x}}\underset{(-)}{+\cancel{144}} \\ \hline & 0 \end{array}$$

$\Rightarrow \boxed{|11x^2 - 9x - 12|}$

---

② Find the Values of a and b if the following polynomials are perfect squares.

(i) $4x^4 - 12x^3 + 37x^2 + bx + a$

Sol:-

$$\begin{array}{r|l} & 2x^2-3x+7 \\ \hline 2x^2 & \cancel{4x^4}-12x^3+37x^2+bx+a \\ & \underset{(-)}{\cancel{4x^4}} \\ \hline 4x^2-3x & -12x^3+37x^2 \\ & \underset{(+)}{\mp 12x^3}\underset{(-)}{+9x^2} \\ \hline 4x^2-6x+7 & \cancel{28x^2}+bx+a \\ & \underset{(-)}{\cancel{28x^2}}-42x+49 \\ \hline \end{array}$$

$\therefore \boxed{\begin{array}{l} b=-42 \\ a=49 \end{array}}$

(ii) $ax^4+bx^3+361x^2+220x+100$

Sol :-

$$100+220x+361x^2+bx^3+ax^4$$

$$\begin{array}{r|l}
 & 10+11x+12x^2 \\
\hline
10 & \cancel{100}+220x+361x^2+bx^3+ax^4 \\
 & \cancel{100} \\
 & (-) \\
\hline
20+11x & \cancel{220x}+361x^2 \\
 & \cancel{220x}+121x^2 \\
 & (-) \qquad (-) \\
\hline
 & \cancel{240}x^2+bx^3+ax^4 \\
20+22x+12x^2 & \cancel{240}x^2+264x^3+144x^4 \\
 & (-) \\
\hline
\end{array}$$

$\therefore$ $\boxed{b=264}$ $\boxed{a=144}$

---

③ Find the Value of m and n if the following polynomials are perfect squares.

(i) $36x^4-60x^3+61x^2-mx+n$

Sol:-

$$\begin{array}{r|l}
 & 6x^2-5x+3 \\
\hline
6x^2 & \cancel{36x^4}-60x^3+61x^2-mx+n \\
 & \cancel{36x^4} \\
 & (-) \\
\hline
 & \cancel{-60x^3}+61x^2 \\
12x^2-5x & \cancel{-60x^3}+25x^2 \\
 & (+) \qquad (-) \\
\hline
 & \cancel{36x^2}-mx+n \\
12x^2-10x+3 & \cancel{36x^2}-30x+9 \\
 & (-) \\
\hline
\end{array}$$

$-m=-30$; $\boxed{n=9}$

$\boxed{m=30}$

(ii) $x^4-8x^3+mx^2+nx+16$

Sol :-

$$
\begin{array}{r|l}
 & x^2-4x+4 \\
\hline
x^2 & \cancel{x^4}-8x^3+mx^2+nx+16 \\
 & \cancel{x^4} \\
 & (-) \\
\hline
2x^2-4x & -\cancel{8x^3}+mx^2 \\
 & -\cancel{8}x^3+16x^2 \\
 & (+)\quad - \\
\hline
2x^2-8x+4 & (m-16)x^2+nx+\cancel{16} \\
 & 8x^2-32x+\cancel{16} \\
 & (-) \\
\hline
\end{array}
$$

$m-16=8$

$m=8+16$

$\boxed{m=24}$

$\boxed{n=-32}$

Exercise - 3.9

① Determine the quadratic equations, whose Sum and product of roots are

(i) $-9, 20$

Sol :- The general form of quadratic eqn

$$x^2 - (\text{Sum of roots})x + \text{product of roots} = 0$$

$$\boxed{\begin{matrix}\text{Sum} = -9 \\ \text{Product} = 20\end{matrix}}$$

$$x^2 - (-9)x + 20 = 0$$

$$\boxed{x^2 + 9x + 20 = 0}$$

(ii) $\frac{5}{3}, 4 \Rightarrow \boxed{\begin{matrix}\text{Sum} = \frac{5}{3} \\ \text{Product} = 4\end{matrix}}$

Sol:- Q.E $\Rightarrow x^2 - (\text{Sum of roots})x + \text{product of roots} = 0$

$$x^2 - \frac{5}{3}x + 4 = 0$$

$$\boxed{3x^2 - 5x + 12 = 0}$$

(iii) $\frac{-3}{2}, -1 \Rightarrow \boxed{\text{Sum} = \frac{-3}{2}}, \boxed{\text{Product} = -1}$

Sol:- Q.E $\Rightarrow x^2 - (\text{Sum of roots})x + \text{product of roots} = 0$

$$x^2 - \left(-\frac{3}{2}\right)x - 1 = 0$$

$$\boxed{2x^2 + 3x - 2 = 0}$$

(iv) $-(2-a)^2$, $(a+5)^2$

Sol:- Sum = $-(2-a)^2$

Product = $(a+5)^2$

Q.E => $x^2 -$ (Sum of roots) x + Product of roots = 0

$$x^2 - [-(2-a)^2]x + (a+5)^2 = 0$$

$$\boxed{x^2 + (2-a)^2 x + (a+5)^2 = 0}$$

---

② Find the Sum and product of the roots for each of the following quadratic equations.

(i) $x^2 + 3x - 28 = 0$

Sol $x^2 + 3x - 28 = 0$

$a = 1$ (coefficient of x^2)

$b = 3$ (coefficient of x)

$c = -28$ (constant)

Sum of roots $= -\frac{b}{a} = \frac{-3}{1}$

Product of roots $= \frac{c}{a} = \frac{-28}{1}$

(ii) $x^2 + 3x = 0$

Sol:- $a = 1$
$b = 3$
$c = 0$

Sum of roots $= -\frac{b}{a} = \frac{-3}{1}$

Product of roots $= \frac{c}{a} = \frac{0}{1} = 0$

(iii) $3 + \frac{1}{a} = \frac{10}{a^2}$

Sol $3 + \frac{1}{a} - \frac{10}{a^2} = 0$

$$\frac{3a^2 + a - 10}{a^2} = 0$$

$$3a^2 + a - 10 = 0$$

$a = 3$
$b = 1$
$c = -10$

Sum of roots $= -\frac{b}{a} = \frac{-1}{3}$

Product of roots $= \frac{c}{a} = \frac{-10}{3}$

(iv) $3y^2 - y - 4 = 0$

Sol $a = 3$
$b = -1$
$c = -4$

Sum of roots $= -\frac{b}{a} = \frac{-(-1)}{3} = \frac{1}{3}$

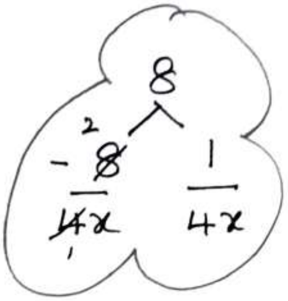
Product of roots $= \frac{c}{a} = -\frac{4}{3}$

Exercise - 3.10

① Solve the following quadratic equations by factorization method.

(i) $4x^2 - 7x - 2 = 0$

Sol



$$4x^2 - 7x - 2 = 0$$

$$(x-2)(4x+1) = 0$$

$x - 2 = 0$

$\boxed{x = 2}$

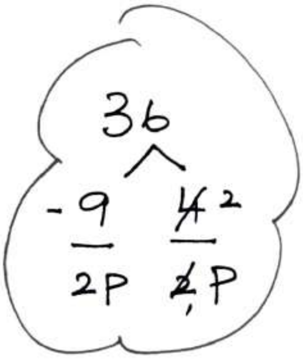
$4x + 1 = 0$

$4x = -1$

$\boxed{x = -\frac{1}{4}}$

(ii) $3(p^2 - 6) = p(p+5)$

Sol



$$3(p^2 - 6) = p(p+5)$$

$$3p^2 - 18 = p^2 + 5p$$

$$3p^2 - 18 - p^2 - 5p = 0$$

$$2p^2 - 5p - 18 = 0$$

$$(2p - 9)(p + 2) = 0$$

$2p - 9 = 0$

$2p = 9$

$\boxed{p = \frac{9}{2}}$

$p + 2 = 0$

$\boxed{p = -2}$

(iii) $\sqrt{a(a-7)} = 3\sqrt{2}$

Sol :-

$$\sqrt{a(a-7)} = 3\sqrt{2}$$

$$\sqrt{a^2-7a} = 3\sqrt{2}$$

[squaring on both sides]

$$\left(\sqrt{a^2-7a}\right)^2 = (3\sqrt{2})^2$$

$$a^2-7a = 9 \times 2$$

$$a^2-7a = 18$$

$$a^2-7a-18=0$$

$$(a-9)(a+2)=0$$

-18 ; -9 2

$a-9=0$ $\quad\|\quad$ $a+2=0$

$\boxed{a=9}$ $\quad\|\quad$ $\boxed{a=-2}$

(iv) $\sqrt{2}x^2+7x+5\sqrt{2}=0$

Sol :-

$5\times\sqrt{2}\times\sqrt{2}$ ⇓ 10 ; $\frac{5}{\sqrt{2}x}$ $\frac{2}{\sqrt{2}x}$

$$\sqrt{2}x^2+7x+5\sqrt{2}=0$$

$$(\sqrt{2}x+5)(\sqrt{2}x+2)=0$$

$\sqrt{2}x+5=0$ $\quad\|\quad$ $\sqrt{2}x+2=0$

$\sqrt{2}x=-5$ $\quad\|\quad$ $\sqrt{2}x=-2$

$\boxed{x=\frac{-5}{\sqrt{2}}}$ $\quad\|\quad$ $\boxed{x=-\frac{2}{\sqrt{2}}}$

(v) $2x^2 - x + \frac{1}{8} = 0$

Sol:-

$$2x^2 - x + \frac{1}{8} = 0$$

$$16x^2 - 8x + 1 = 0$$

$$(4x-1)(4x-1) = 0$$

$$16$$

$$\frac{-\overset{1}{\cancel{4}}}{\underset{4}{\cancel{16}}x} \qquad \frac{-\overset{1}{\cancel{4}}}{\underset{4}{\cancel{16}}x}$$

$4x - 1 = 0$	$4x - 1 = 0$
$4x = 1$	$4x = 1$
$\boxed{x = \frac{1}{4}}$	$\boxed{x = \frac{1}{4}}$

---

② The number of Volley ball games that must be Scheduled in a league with n teams is given by $G(n) = \frac{n^2 - n}{2}$ where each team plays with every other team exactly Once. A League Schedules 15 games. How many teams are in the league?

Sol:-

$$G(n) = \frac{n^2 - n}{2}$$

Number of teams = n = ?

Number of Games Scheduled by League $\}$ = 15 = G(n)

$$\therefore \frac{n^2-n}{2} = G(n)$$

$$\frac{n^2-n}{2} = 15$$

$$n^2-n = 30$$

$$\begin{array}{c} 30 \\ \wedge \\ -6 \quad 5 \end{array}$$

$$n^2-n-30=0$$

$$(n-6)(n+5)=0$$

$n-6=0$ $\qquad$ $n+5=0$

$\boxed{n=6}$ $\qquad$ $n=-5$

[Omitting -ve Value. "NO of teams cannot be -ve."]

$\therefore$ Number of teams $= 6$

QUADRATIC FORMULA

Q.E $\Rightarrow ax^2+bx+c=0$

$$\boxed{x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}}$$

Exercise - 3.11

① Solve the following quadratic equations by completing the Square method.

(i) $9x^2 - 12x + 4 = 0$

Sol:-

$$9x^2 - 12x + 4 = 0 \qquad \left[\div \text{ by } 9 \text{ Coeff of } x^2\right]$$

$$x^2 - \frac{\overset{4}{\cancel{12}}x}{\underset{3}{\cancel{9}}} + \frac{4}{9} = 0$$

$$x^2 - \frac{4x}{3} = -\frac{4}{9} \qquad \left[\text{Add } \left(\frac{2}{3}\right)^2 \text{ on both side}\right]$$

$$x^2 - 2\left(\frac{2}{3}\right)x + \left(\frac{2}{3}\right)^2 = -\frac{4}{9} + \left(\frac{2}{3}\right)^2$$

$$\left(x - \frac{2}{3}\right)^2 = -\frac{4}{9} + \frac{4}{9}$$

$$\left(x - \frac{2}{3}\right)^2 = 0$$

$$\left(x - \frac{2}{3}\right)\left(x - \frac{2}{3}\right) = 0$$

$$x - \frac{2}{3} = 0 \quad \Big\| \quad x - \frac{2}{3} = 0$$

$$\boxed{x = \frac{2}{3}} \quad \Big\| \quad \boxed{x = \frac{2}{3}}$$

(ii) $\dfrac{5x+7}{x-1} = 3x+2$

Sol

$$\frac{5x+7}{x-1} = 3x+2$$

$$5x+7 = (3x+2)(x-1)$$

$$5x+7 = 3x^2-3x+2x-2$$

$$5x+7 = 3x^2-x-2$$

$$0 = 3x^2-x-2-5x-7$$

$$0 = 3x^2-6x-9$$

$$3x^2-6x-9=0 \qquad \div \text{ by } 3$$

$$x^2-2x-3=0$$

$$x^2-2x = 3 \qquad \left[\text{Add } 1^2 \text{ on both side}\right]$$

$$x^2-2x+1^2 = 3+1^2$$

$$(x-1)^2 = 4$$

$$x-1 = \sqrt{4}$$

$$x-1 = \pm 2$$

$x-1=2$	$x-1=-2$
$x=2+1$	$x=-2+1$
$\boxed{x=3}$	$\boxed{x=-1}$

② Solve the following quadratic equations by formula method.

(i) $2x^2 - 5x + 2 = 0$

Sol:

$$2x^2 - 5x + 2 = 0$$

$a = 2$; $b = -5$; $c = 2$

$$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(2)}}{2(2)}$$

$$= \frac{5 \pm \sqrt{25 - 16}}{4}$$

$$= \frac{5 \pm \sqrt{9}}{4}$$

$$= \frac{5 \pm 3}{4}$$

$$x = \frac{5+3}{4} = \frac{8}{4} \qquad \boxed{x = 2}$$

$$x = \frac{5-3}{4} = \frac{2}{4} \qquad \boxed{x = \frac{1}{2}}$$

(ii) $\sqrt{2}f^2 - 6f + 3\sqrt{2} = 0$

Sol:-

$$\sqrt{2}f^2 - 6f + 3\sqrt{2} = 0$$

$$\boxed{a = \sqrt{2}} \; ; \; \boxed{b = -6} \; ; \; \boxed{c = 3\sqrt{2}}$$

$$f = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-6) \pm \sqrt{(-6)^2 - 4(\sqrt{2})(3\sqrt{2})}}{2\sqrt{2}}$$

$\sqrt{2} \times \sqrt{2} = 2$

$$= \frac{+6 \pm \sqrt{36 - 24}}{2\sqrt{2}}$$

$$\boxed{\begin{aligned}\sqrt{12} &= \sqrt{4 \times 3} \\ &= 2\sqrt{3}\end{aligned}}$$

$$= \frac{6 \pm \sqrt{12}}{2\sqrt{2}}$$

$$= \frac{6 \pm 2\sqrt{3}}{2\sqrt{2}}$$

$$= \frac{\cancel{2}(3 \pm \sqrt{3})}{\cancel{2}\sqrt{2}}$$

$$\boxed{f = \frac{3 + \sqrt{3}}{\sqrt{2}}} \quad \Bigg\| \quad \boxed{f = \frac{3 - \sqrt{3}}{\sqrt{2}}}$$

(iii) $3y^2 - 20y - 23 = 0$

Sol :- $3y^2 - 20y - 23 = 0$

$a = 3$ $\quad b = -20$ $\quad c = -23$

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-20) \pm \sqrt{(-20)^2 - 4(3)(-23)}}{2 \times 3}$$

$$= \frac{20 \pm \sqrt{400 + 276}}{6}$$

$$\begin{array}{r} 23 \\ \times 12 \\ \hline 46 \\ 23 \\ \hline 276 \end{array}$$

$$= \frac{20 \pm \sqrt{676}}{6}$$

$$\begin{array}{r|l} & \;\;26 \\ \hline 2 & 676 \\ & \underline{-4 \downarrow \downarrow} \\ 46 & \;276 \\ & \underline{-276} \\ & \quad 0 \end{array}$$

$$= \frac{20 \pm 26}{6}$$

$$y = \frac{20 + 26}{6} \qquad \Bigg\| \qquad y = \frac{20 - 26}{6}$$

$$= \frac{\cancel{46}^{23}}{\cancel{6}_{3}} \qquad \Bigg\| \qquad = \frac{-\cancel{6}^{1}}{\cancel{6}}$$

$$\boxed{y = \frac{23}{3}} \qquad \Bigg\| \qquad \boxed{y = -1}$$

(iv) $36y^2 - 12ay + (a^2-b^2) = 0$

Sol: $36y^2 - 12ay + a^2 - b^2 = 0$

$a = 36$ $b = -12a$ $c = a^2 - b^2$

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-12a) \pm \sqrt{(-12a)^2 - 4(36)(a^2-b^2)}}{2(36)}$$

$$\begin{array}{r}36\\ \times\ 4\\ \hline 144\end{array}$$

$$= \frac{12a \pm \sqrt{144a^2 - 144(a^2-b^2)}}{72}$$

$$\begin{array}{r}36\\ \times\ 2\\ \hline 72\end{array}$$

$$= \frac{12a \pm \sqrt{\cancel{144a^2} - \cancel{144a^2} + 144b^2}}{72}$$

$$= \frac{12a \pm \sqrt{144b^2}}{72}$$

$$= \frac{12a \pm 12b}{72}$$

$$= \frac{\overset{1}{\cancel{12}}(a \pm b)}{\cancel{72}\,6}$$

$$y = \frac{a \pm b}{6}$$

$$\boxed{y = \frac{a+b}{6}} \quad \Bigg| \quad \boxed{y = \frac{a-b}{6}}$$

---

③ A ball rolls down a slope and travels a distance $d = t^2 - 0.75t$ feet in t seconds. Find the time when the distance travelled by the ball is 11.25 feet.

Sol:-

$$d = t^2 - 0.75t$$

$$d = 11.25$$

$$t = ?$$

$$t^2 - 0.75t = 11.25$$

$$t^2 - 0.75t - 11.25 = 0 \qquad \left[\times^{ly} \text{ by } 100 \text{ on b.s}\right]$$

$$100t^2 - 75t - 1125 = 0 \qquad [\div \text{ by } 25]$$

$$4t^2 - 3t - 45 = 0$$

$$\textcircled{a = 4} \quad \textcircled{b = -3} \quad \textcircled{c = -45}$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-3) \pm \sqrt{(-3)^2 - 4(4)(-45)}}{2 \times 4}$$

$$= \frac{3 \pm \sqrt{9 + 720}}{8}$$

$$\begin{array}{r} \overset{3}{4}5 \\ \underline{16} \\ 27\,0 \\ \underline{45} \\ \underline{720} \end{array}$$

$$= \frac{3 \pm \sqrt{729}}{8}$$

$$\begin{array}{r|l} & 27 \\ 2 & \overline{729} \\ & \underline{-4} \\ 47 & 329 \\ & \underline{-329} \\ & 0 \end{array}$$

$$= \frac{3 \pm 27}{8}$$

$$\begin{array}{r} 3.75 \\ 8\overline{)30} \\ \underline{-24} \\ 60 \\ \underline{-56} \\ 40 \\ \underline{-40} \\ 0 \end{array}$$

$$t = \frac{3+27}{8} = \frac{30}{8}$$

$$\boxed{t = 3.75 \text{ Sec}}$$

$$t = \frac{3-27}{8} = \frac{-24}{8}$$

$t = -3$ (omitting -ve Value time cannot be -ve)

$$\therefore \boxed{t = 3.75 \text{ Sec}}$$

Exercise 3.12

① If the difference between a number and its reciprocal is $\frac{24}{5}$, find the number.

Sol

Let the number $= x$

Its reciprocal $= \frac{1}{x}$

Difference of number and reciprocal $\} = \frac{24}{5}$

$$\text{i'e,} \quad x - \frac{1}{\leftarrow x} \underset{\longrightarrow}{=} \frac{24}{5}$$

$$x^2 - 1 = \frac{24x}{\leftarrow 5}$$

$$5(x^2-1) = 24x$$

$$5x^2 - 5 = 24x$$

$$5x^2 - 24x - 5 = 0$$

$$(x-5)(5x+1) = 0$$

25

$$\frac{-\cancel{25}^{5}}{\cancel{5}x} \quad \frac{1}{5x}$$

$x - 5 = 0$

$\boxed{x = 5}$

$5x + 1 = 0$

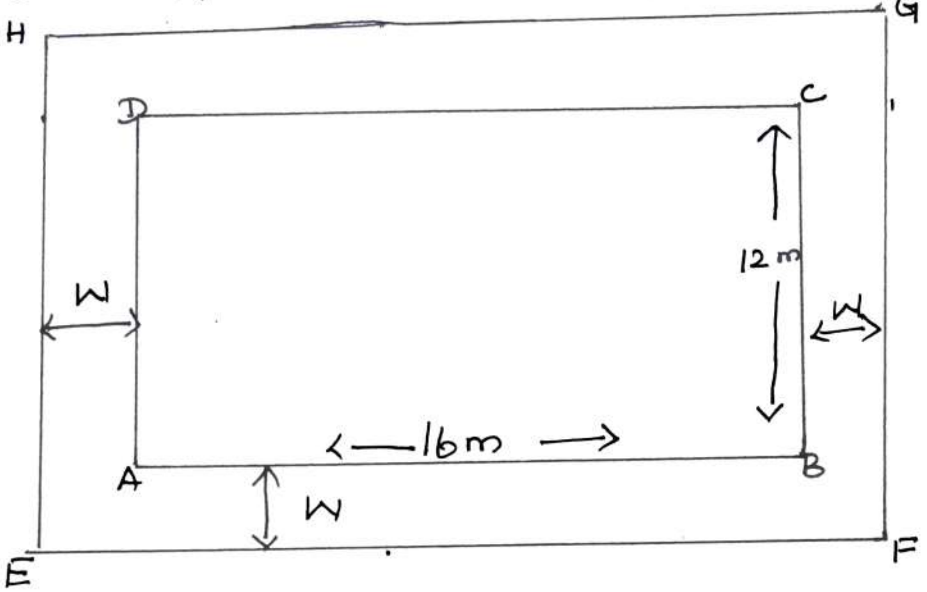
$5x = -1$

$\boxed{x = \frac{-1}{5}}$

$\therefore$ The number $= 5, \frac{-1}{5}$

② A garden measuring 12m by 16m is to have a pedestrain pathway that is 'w' meters wide installed all the way around so that it increases the total area to 285 m². what is the width of the pathway ?

Sol: -



Outer Rectangle (EFGH)

$l = 16 + w + w$

$\boxed{l = 16 + 2w}$

$b = 12 + w + w$

$\boxed{b = 12 + 2w}$

Area of the Outer Rectangle = 285

$l \times b = 285$

ie, $(16 + 2w)(12 + 2w) = 285$

$$192 + 32w + 24w + 4w^2 = 285$$

$$192 + 56w + 4w^2 - 285 = 0$$

$$4w^2 + 56w - 93 = 0$$

$$(2w-3)(2w+31) = 0$$

$$\begin{array}{r}16\\ \times 12\\ \hline 32\\ 16\\ \hline 192\end{array}$$

$$\begin{array}{r}285\\ (-)192\\ \hline 93\end{array}$$

$2w - 3 = 0$

$2w = 3$

$w = \frac{3}{2}$

$\boxed{w = 1.5}$

$2w + 31 = 0$

$2w = -31$

$w = -\frac{31}{2}$

[Width cannot be -ve
∴ Omitting -ve Value]

372

$\frac{-\overset{3}{\cancel{6}}}{\underset{2}{\cancel{4}}w} \quad \frac{\overset{31}{\cancel{62}}}{\underset{2}{\cancel{4}}w}$

∴ Width of pathway = 1.5m

---

③ A bus covers a distance of 90km at a uniform speed. Had the speed been 15km/hr more it would have taken 30 minutes less for the journey. Find the Original Speed of the bus.

Sol:-

Distande ⇒ $D = 90$km

Uniform Speed ⇒ $S = x$

Time ⇒ $T = ?$

$$\therefore T_1 = \frac{D}{S}$$

$$T_1 = \frac{90}{x} \longrightarrow ①$$

If Speed is $\underbrace{15\,\text{km more}}$

$$D = 90\,\text{km}$$

$$S = x + 15$$

$$T_2 = \frac{D}{S}$$

$$T_2 = \frac{90}{x+15} \longrightarrow ②$$

When Speed is increased, Time taken is 30 minutes less for Journey

$$\left[30\,\text{min} = \frac{1}{2}\,\text{hr}\right]$$

$$\therefore T_1 - T_2 = \frac{1}{2}$$

$$\frac{90}{x} - \frac{90}{x+15} = \frac{1}{2}$$

$$\frac{90(x+15) - 90x}{x(x+15)} = \frac{1}{2}$$

$$\frac{\cancel{90x} + 1350 - \cancel{90x}}{x^2 + 15x} = \frac{1}{2}$$

$$\frac{1350}{x^2+15x} = \frac{1}{2}$$

$$2(1350) = x^2+15x$$

$$2700 = x^2+15x$$

$$0 = x^2+15x-2700$$

$$x^2+15x-2700=0$$

2700

−45 60

$$(x-45)(x+60)=0$$

$x-45=0$

$\boxed{x=45}$

$x+60=0$

$\boxed{x=-60}$

Speed Cannot be −ve

Omitting −ve Value.

∴ The Original Speed = 45 km/hr.

---

④ A girl is twice as old as her Sister. Five years hence, the product of their ages (in years) will be 375. Find their present ages.

Sol :-

Let the girls age = x

Sisters age = y.

Girl = Twice as old as her Sister

$\boxed{x = 2y} \longrightarrow ①$

Five years hence [after ⇒ 5+]

Product of their ages = 375

$(x+\overrightarrow{5)\,(y}+5) = 375$

$xy + 5x + 5y + 25 - 375 = 0$

Put $\boxed{x = 2y}$

$$\begin{array}{r} 375 \\ -\ 25 \\ \hline 350 \end{array}$$

$2y(y) + 5(2y) + 5y - 350 = 0$

$2y^2 + 10y + 5y - 350 = 0$

$2y^2 + 15y - 350 = 0$

$(y-10)(2y+35) = 0$

$$700: \quad \frac{\overset{10}{\cancel{-20}}}{2y} \quad \frac{35}{2y}$$

$y - 10 = 0$

$\boxed{y = 10}$

$2y + 35 = 0$

$2y = -35$

$y = -\frac{35}{2}$

Age cannot be −ve
Omitting −ve value.

$\therefore$ Age of her Sister = $\boxed{y = 10\text{yrs.}}$

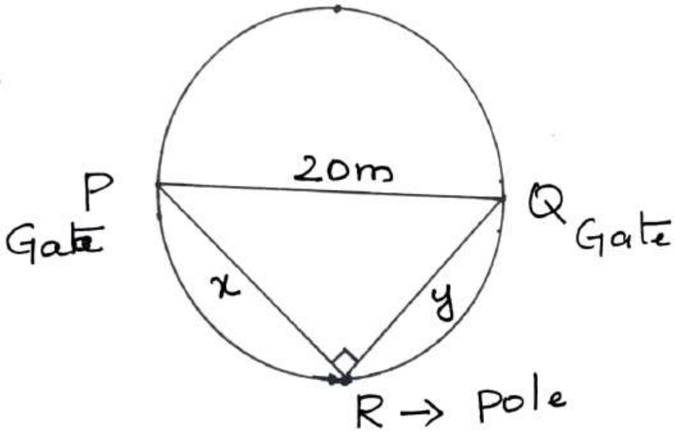
Age of the girl = $x = 2y$

$= 2(10)$

$\boxed{x = 20\text{yrs}}$

⑤ A pole has to be erected at a point on the boundary of a circular ground of diameter 20m in such away that the difference of its distances from two diametrically opposite fixed gates P and Q on the boundary is 4m. Is it possible to do so? If answer is yes at what distance from the two gates should the pole be erected?

Sol: –



By Pythagoras theorem

$$PR^2 + RQ^2 = PQ^2$$

$$x^2 + y^2 = 20^2$$

$$x^2 + y^2 = 400 \longrightarrow ①$$

Difference of its distance from two opposite fixed gates = 4m

$$\boxed{x-y=4} \rightarrow ②$$

$$\therefore x = 4+y \rightarrow ③$$

Put ③ in ①

$$(4+y)^2 + y^2 = 400 \qquad \boxed{(a+b)^2 = a^2+b^2+2ab}$$

$$4^2+y^2+2(4)(y)+y^2-400=0$$

$$16+2y^2+8y-400=0$$

$$2y^2+8y-384=0 \quad (\div \text{ by } 2)$$

$$y^2+4y-192=0$$

$$\begin{array}{cc} \multicolumn{2}{c}{192} \\ \multicolumn{2}{c}{\wedge} \\ -12 & 16 \end{array}$$

$$(y-12)(y+16)=0$$

$$y-12=0 \qquad y+16=0$$

$$\boxed{y=12} \qquad y=-16$$

[Omitting -ve Value]

Put $y=12$ in ③

$$x = 4+12$$

$$\boxed{x=16}$$

$\therefore$ Distance of two gates from pole.

$PR = x = 16m$

$QR = y = 12m$

It is possible.

⑥ From a group of $2x^2$ black bees, square root of half of the group went to a tree. Again eight-ninth of the bees went to the same tree. The remaining two got caught up in a fragrant lotus. How many bees were there in total?

Sol:-

Total number of black bees $= 2x^2$

Number of bees went to tree $= \sqrt{\frac{1}{\cancel{2}}(\cancel{2}x^2)}$

$$= \sqrt{x^2}$$

$$= x$$

Again, Number of bees went to tree $\Big\} = \frac{8}{9}(2x^2)$

$$= \frac{16x^2}{9}$$

Number of bees caught in lotus $\Big\} = 2$

Total bees $= ?$

$$\Rightarrow \quad 2x^2 = x + \frac{16x^2}{9} + 2$$

$$18x^2 = 9x + 16x^2 + 18$$

$$18x^2 - 16x^2 - 9x - 18 = 0$$

$2x^2 - 9x - 18 = 0$

$(x-6)\ (2x+3) = 0$

36

$\overset{6}{\cancel{-12}} \quad 3$

$\frac{\cancel{-12}}{\underset{1}{\cancel{2}}x} \quad \frac{3}{2x}$

$x - 6 = 0$

$\boxed{x = 6}$

$2x + 3 = 0$

$2x = -3$

$x = -\frac{3}{2}$

[Bees cannot be -ve
Omitting -ve value.]

$\therefore$ No of bees $= 2x^2$

$= 2(6)^2$

$= 2(36)$

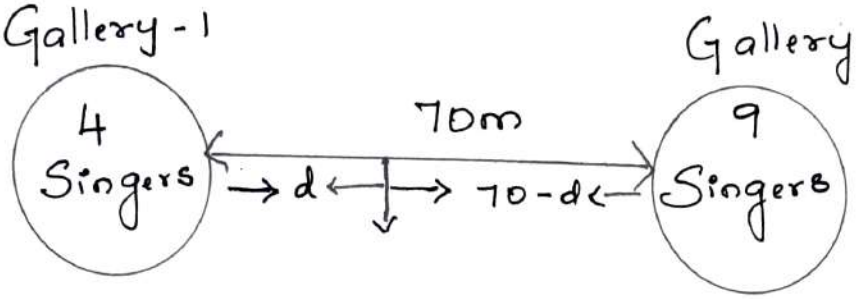
$= 72$

---

(7) Music is been played in two opposite galleries with certain group of people. In the first gallery a group of 4 singers were singing and in the second gallery 9 singers were singing. The two galleries are separated by the distance of 70m. Where should a person stand for hearing the same intensity of the singers voice? [Hint: The ratio of the

Sound intensity is equal to the square of the ratio of their corresponding distances).

Sol



Ratio of sound intensity } = Square of the ratio of their distance

$$\frac{4}{9} = \frac{d^2}{(70-d)^2}$$ [Taking square root]

$$\sqrt{\frac{4}{9}} = \sqrt{\frac{d^2}{(70-d)^2}}$$

$$\frac{2}{3} = \frac{d}{70-d}$$

$$2(70-d) = 3d$$

$$140 - 2d = 3d$$

$$140 = 3d + 2d$$

$$140 = 5d$$

$$5d = 140$$

$$d = \frac{\cancel{140}^{28}}{\cancel{5}}$$

$$\boxed{d = 28\,m}$$

$\therefore$ From Gallery 1 the person stands at $\Big\} = 28\,m$

From Gallery 2, the Person stands at $\Big\} = 70 - d$

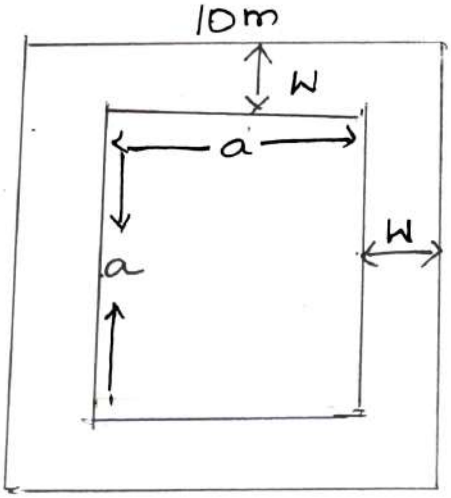
$= 70 - 28$

$= 42\,m$

---

⑧ There is a square field whose side is 10m. A square flower bed is prepared in its centre leaving a gravel path all round the flower bed. The total cost of laying the flower bed and gravelling the path at ₹3 and ₹4 per square meter respectively is ₹364. Find the width of the gravel path.

Sol:-



$$\boxed{a = 10 - 2w} \longrightarrow ①$$

Area of square field $= 10 \times 10$
$= 100\,m^2$

Area of flower bed $= a \times a$
$= a^2\,m^2$

Area of gravel path = (Area of square field) − (Area of flower bed)
$= (100 - a^2)\,m^2$

Cost of laying flower bed = ₹ 3
Cost of laying gravel path = ₹ 4
∴ Total cost = ₹ 364

$$\Rightarrow 3a^2 + 4(100 - a^2) = 364$$
$$3a^2 + 400 - 4a^2 = 364$$
$$-a^2 = 364 - 400$$
$$\cancel{-}a^2 = \cancel{-}36$$
$$a^2 = 6^2$$
$$a = 6$$

$$a = 10 - 2w \qquad [\text{from } ①]$$

$$6 \overset{\leftarrow}{=} 10 - 2w$$

$$6 - 10 = -2w$$

$$\cancel{-}4 = \cancel{-}2w$$

$$2w = 4$$

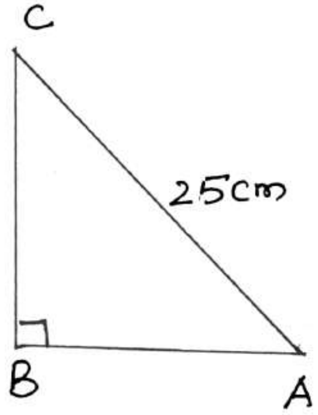
$$w = \frac{4}{2}$$

$$\boxed{w = 2\,m}$$

∴ Width of gravel path = 2m

---

⑨ The hypotenuse of a right angled triangle is 25cm. and its perimeter 56cm. Find the length of the smallest side.



Sol:-

$$AC = 25\,cm \longrightarrow ①$$

$$\text{Hypotenuse} = 25\,cm$$

$$\text{Perimeter} = 56\,cm$$

$$AB + BC + AC = 56$$

$$AB + BC + 25 \overset{\longrightarrow}{=} 56$$

$$AB + BC = 56 - 25$$

$$AB + BC = 31 \longrightarrow ②$$

$$AB = 31 - BC \longrightarrow ③$$

By Pythagoras theorem

$$AB^2 + BC^2 = AC^2$$

From ① & ⑤

$$(31 - BC)^2 + BC^2 = 25^2$$

$$\boxed{(a-b)^2 = a^2 + b^2 - 2ab}$$

$$(31)^2 + BC^2 - 2(31)BC + BC^2 = 625$$

$$961 + BC^2 - 62\,BC + BC^2 = 625$$

$$2BC^2 - 62\,BC + 961 - 625 = 0$$

$$2BC^2 - 62\,BC + 336 = 0$$

÷ by 2

$$BC^2 - 31\,BC + 168 = 0$$

168

$-24 \quad -7$

$$(BC - 24)(BC - 7) = 0$$

$$BC - 24 = 0$$

$$\boxed{BC = 24}$$

$$AB = 31 - BC$$

$$= 31 - 24$$

$$\boxed{AB = 7\text{cm}}$$

$$BC - 7 = 0$$

$$\boxed{BC = 7}$$

$$AB = 31 - BC$$

$$= 31 - 7$$

$$\boxed{AB = 24\text{cm}}$$

∴ Length of Smallest Side = 7cm.

Nature of roots of a Quadratic Equation

VALUES OF DISCRIMINANT $\Delta = b^2-4ac$	NATURE OF ROOTS
$\Delta > 0$ [+ve Values]	Real and unequal roots
$\Delta = 0$	Real and equal roots
$\Delta < 0$ [−ve Values]	No Real roots.

Exercise - 3.13

(1.) Determine the nature of the roots for the following quadratic equations.

(i) $15x^2+11x+2=0$

Sol: $15x^2+11x+2=0$

$\boxed{a=15}$, $\boxed{b=11}$, $\boxed{c=2}$

$$b^2-4ac = (11)^2-4(15)(2)$$

$$= 121-120$$

$$\boxed{b^2-4ac = 1}$$

$$\Delta > 0$$

∴ Roots are Real and unequal.

(ii) $x^2 - x - 1 = 0$

Sol:- $x^2 - x - 1 = 0$

$a = 1$ $\quad b = -1$ $\quad c = -1$

$$b^2 - 4ac = (-1)^2 - 4(1)(-1)$$

$$= 1 + 4$$

$$\boxed{b^2 - 4ac = 5}$$

$$\boxed{\Delta > 0}$$

∴ Roots are real and unequal.

(iii) $\sqrt{2}t^2 - 3t + 3\sqrt{2} = 0$

Sol:- $\sqrt{2}t^2 - 3t + 3\sqrt{2} = 0$

$a = \sqrt{2}$, $b = -3$, $c = 3\sqrt{2}$

$$b^2 - 4ac = (-3)^2 - 4(\sqrt{2})(3\sqrt{2})$$

$$= 9 - 12(2)$$

$$= 9 - 24$$

$$\boxed{b^2 - 4ac = -15}$$

$$\boxed{\Delta < 0}$$

∴ No Real roots.

(iv) $9y^2 - 6\sqrt{2}y + 2 = 0$

Sol:- $9y^2 - 6\sqrt{2}y + 2 = 0$

$a = 9$ $b = -6\sqrt{2}$ $c = 2$

$b^2 - 4ac = (-6\sqrt{2})^2 - 4(9)(2)$

$= 36(2) - 72$

$= 72 - 72$

$\boxed{b^2 - 4ac = 0}$

$\boxed{\Delta = 0}$

$\therefore$ Roots are real and equal.

(v) $9a^2b^2x^2 - 24abcdx + 16c^2d^2 = 0$ $a \neq 0, b \neq 0$

Sol:- $9a^2b^2x^2 - 24abcdx + 16c^2d^2 = 0$

$a = 9a^2b^2$, $b = -24abcd$, $c = 16c^2d^2$

$b^2 - 4ac = (-24abcd)^2 - 4(9a^2b^2)(16c^2d^2)$

$= 576a^2b^2c^2d^2 - 576a^2b^2c^2d^2$

$\boxed{b^2 - 4ac = 0}$

$\boxed{\Delta = 0}$

$\therefore$ Roots are real and equal.

② Find the Value(s) of K for which the roots of the following equations are real and equal.

(i) $(5k-6)x^2 + 2kx + 1 = 0$

Sol:- $(5k-6)x^2 + 2kx + 1 = 0$

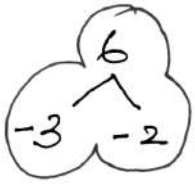
$\boxed{a = 5k-6}$ $\boxed{b = 2k}$ $\boxed{c = 1}$

$b^2 - 4ac = 0$ [Given:- roots are real and equal]

$(2k)^2 - 4(5k-6)(1) = 0$

$4k^2 - 20k + 24 = 0$ $\boxed{\div \text{ by } 4}$

$k^2 - 5k + 6 = 0$



$(k-3)(k-2) = 0$

$k - 3 = 0$ | $k - 2 = 0$

$\boxed{k = 3}$ | $\boxed{k = 2}$

(ii) $kx^2 + (6k+2)x + 16 = 0$

Sol $kx^2 + (6k+2)x + 16 = 0$

$a = k$; $b = 6k+2$; $c = 16$

$b^2 - 4ac = 0$ (Given)

$(6k+2)^2 - 4(k)(16) = 0$

$(6k)^2 + 2^2 + 2(6k)(2) - 64k = 0$

$36k^2 + 4 + 24k - 64k = 0$

$36k^2 - 40k + 4 = 0$ ($\div$ by 4)

$9k^2 - 10k + 1 = 0$

$(k-1)(9k-1) = 0$

$$9 \qquad -\frac{\cancel{9}^{1}}{\cancel{9}k} \quad \frac{-1}{9k}$$

$k - 1 = 0$

$\boxed{k = 1}$

$9k - 1 = 0$

$9k = 1$

$\boxed{k = \frac{1}{9}}$

---

③ If the roots of $(a-b)x^2 + (b-c)x + (c-a) = 0$ are real and equal, then prove that b, a, c are in arithmetic progression.

Sol :- $(a-b)x^2 + (b-c)x + (c-a) = 0$

$a = a-b$ $\quad b = b-c$ $\quad c = c-a$

To Prove :- b, a, c are in A.P

i.e, $a - b = c - a$

Given :- Roots are real and equal

ie, $b^2 - 4ac = 0$

$$(b-c)^2 - 4(a-b)(c-a) = 0$$

$$b^2+c^2-2bc-4(ac-a^2-bc+ba)=0$$

$$b^2+c^2-2bc-4ac+4a^2+4bc-4ab=0$$

$$4a^2+b^2+c^2-4ab+2bc\ \ -4ac=0$$

[writing in the form of $a^2+b^2+c^2+2ab+2bc+2ca = (a+b+c)^2$]

$$(-2a)^2+b^2+c^2+2(-2a)(b)+2(b)(c)+2(-2a)(c)=0$$

$$(-2a+b+c)^2=0$$ [Taking Square root]

$$-2a+b+c=0$$

$$\boxed{b+c=2a}$$ (or)

$$b+c=a+a$$

$$\boxed{c-a=a-b}$$

$\therefore b, a, c$ are in A.P

---

④ If a, b are real then Show that the roots of the equation $(a-b)x^2-6(a+b)x-9(a-b)=0$ are real and unequal.

Sol:-

$(a-b)x^2 - 6(a+b)x - 9(a-b) = 0$

To Show:- Roots are real and Unequal

ie, $\boxed{\Delta > 0}$

$a = a-b$. $b = -6(a+b)$ $c = -9(a-b)$

$$b^2 - 4ac = [-6(a+b)]^2 - 4(a-b)(-9(a-b))$$

$$= 36(a+b)^2 + 36(a-b)^2$$

$$= 36[(a+b)^2 + (a-b)^2]$$

$$= 36[a^2+b^2+\cancel{2ab} + a^2+b^2 - \cancel{2ab}]$$

$$= 36[2a^2 + 2b^2]$$

$$= 36 \times 2(a^2+b^2)$$

$$b^2-4ac = 72(a^2+b^2)$$

$\therefore \Delta > 0$ [$72(a^2+b^2)$ is +ve Value]

Hence Roots are real and Unequal

⑤ If the roots of the equation $(c^2-ab)x^2-2(a^2-bc)x+b^2-ac=0$ are real and equal prove that either $a=0$ (or) $a^3+b^3+c^3=3abc$.

Sol :-

$$(c^2-ab)x^2-2(a^2-bc)x+b^2-ac=0$$

To prove :- $a=0$

$a^3+b^3+c^3=3abc$.

$a=c^2-ab$ $b=-2(a^2-bc)$ $c=b^2-ac$

$$b^2-4ac=0 \quad \left[\text{Given: roots real and equal}\right]$$

$$\left[-2(a^2-bc)\right]^2-4(c^2-ab)(b^2-ac)=0$$

$$4(a^2-bc)^2-4(c^2-ab)(b^2-ac)=0$$

$$4\left[(a^2)^2+(bc)^2-2(a^2)(bc)\right]-4\left[c^2b^2-ac^3-ab^3+a^2bc\right]=0$$

$$4\left[a^4+\cancel{b^2c^2}-2a^2bc-\cancel{c^2b^2}+ac^3+ab^3-a^2bc\right]=0$$

$$4\left[a^4-3a^2bc+ac^3+ab^3\right]=0$$

$$4a\left[a^3 - 3abc + c^3 + b^3\right] = 0$$

$4a = 0$ | $a^3 + b^3 + c^3 - 3abc = 0$

$\boxed{a = 0}$ | $\boxed{a^3 + b^3 + c^3 = 3abc}$

Hence proved.

Formulae

1. $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$
2. $\alpha^3 + \beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)$
3. $\alpha - \beta = \sqrt{(\alpha+\beta)^2 - 4\alpha\beta}$.

Exercise - 3.14

(1) Write each of the following expression in terms of $\alpha+\beta$ and $\alpha\beta$.

(i) $\frac{\alpha}{3\beta} + \frac{\beta}{3\alpha}$

Sol:- $\dfrac{\alpha}{3\beta}+\dfrac{\beta}{3\alpha} = \dfrac{\alpha^2+\beta^2}{3\alpha\beta}$

$= \dfrac{(\alpha+\beta)^2-2\alpha\beta}{3\alpha\beta}$

(ii) $\dfrac{1}{\alpha^2\beta}+\dfrac{1}{\beta^2\alpha}$

Sol:-

$\dfrac{1}{\alpha^2\beta}+\dfrac{1}{\beta^2\alpha} = \dfrac{\beta+\alpha}{\alpha^2\beta^2}$

$= \dfrac{\alpha+\beta}{(\alpha\beta)^2}$

(iii) $(3\alpha-1)(3\beta-1)$

Sol:-

$(3\alpha-1)(3\beta-1) = 9\alpha\beta-3\alpha-3\beta+1$

$= 9\alpha\beta-3(\alpha+\beta)+1$

(iv) $\dfrac{\alpha+3}{\beta}+\dfrac{\beta+3}{\alpha}$

Sol:- $\dfrac{\alpha+3}{\beta}+\dfrac{\beta+3}{\alpha} = \dfrac{\alpha(\alpha+3)+\beta(\beta+3)}{\alpha\beta}$

$$= \frac{\alpha^2 + 3\alpha + \beta^2 + 3\beta}{\alpha\beta}$$

$$= \frac{\alpha^2 + \beta^2 + 3(\alpha+\beta)}{\alpha\beta}$$

$$= \frac{(\alpha+\beta)^2 - 2\alpha\beta + 3(\alpha+\beta)}{\alpha\beta}$$

---

② The roots of the equation $2x^2-7x+5=0$ are α and β. without solving for the roots find.

(i) $\frac{1}{\alpha} + \frac{1}{\beta}$.

Sol:- $2x^2 - 7x + 5 = 0$

$$\boxed{\begin{array}{l} a = 2 \\ b = -7 \\ c = 5 \end{array}}$$

Sum of roots $\Rightarrow \alpha+\beta = -\frac{b}{a}$

$$= \frac{-(-7)}{2}$$

$$\boxed{\alpha+\beta = \frac{7}{2}}$$

Product of roots $\Rightarrow \alpha\beta = \frac{c}{a} = \frac{5}{2}$

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta + \alpha}{\alpha\beta}$$

$$= \frac{\alpha + \beta}{\alpha\beta}$$

$$= \frac{\frac{7}{2}}{\frac{5}{2}}$$

$$= \frac{7}{\cancel{2}} \times \frac{\cancel{2}}{5}$$

$$\boxed{\frac{1}{\alpha} + \frac{1}{\beta} = \frac{7}{5}}$$

(ii) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$

$$\boxed{\begin{aligned} \alpha+\beta &= \frac{7}{2} \\ \alpha\beta &= \frac{5}{2} \end{aligned}}$$

SOL:- $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta}$

$$= \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}$$

$$= \frac{\left(\frac{7}{2}\right)^2 - \cancel{2}\left(\frac{5}{\cancel{2}}\right)}{\frac{5}{2}}$$

$$= \left(\frac{49}{4} - 5\right) \times \frac{2}{5}$$

$$= \left(\frac{49-20}{\cancel{4}_{2}}\right) \times \frac{\cancel{2}^{1}}{5}$$

$$= \frac{29}{10}$$

(iii) $\frac{\alpha+2}{\beta+2} + \frac{\beta+2}{\alpha+2}$

$$\boxed{\begin{aligned} \alpha+\beta &= \frac{7}{2} \\ \alpha\beta &= \frac{5}{2} \end{aligned}}$$

Sol :-

$$\frac{\alpha+2}{\beta+2} + \frac{\beta+2}{\alpha+2} = \frac{(\alpha+2)^2 + (\beta+2)^2}{(\alpha+2)(\beta+2)}$$

$$= \frac{\alpha^2+2^2+2(\alpha)(2)+\beta^2+2^2+2(\beta)(2)}{\alpha\beta+2\alpha+2\beta+4}$$

$$= \frac{\alpha^2+4+4\alpha+\beta^2+4+4\beta}{\alpha\beta+2(\alpha+\beta)+4}$$

$$= \frac{\alpha^2+\beta^2+4(\alpha+\beta)+8}{\alpha\beta+2(\alpha+\beta)+4}$$

$$= \frac{(\alpha+\beta)^2-2\alpha\beta+4(\alpha+\beta)+8}{\alpha\beta+2(\alpha+\beta)+4}$$

$$= \frac{\left(\frac{7}{2}\right)^2 - \cancel{2}\left(\frac{5}{\cancel{2}}\right) + \overset{2}{\cancel{4}}\left(\frac{7}{\cancel{2}}\right) + 8}{\frac{5}{2} + \cancel{2}\left(\frac{7}{\cancel{2}}\right) + 4}$$

$$= \frac{\frac{49}{4} - 5 + 14 + 8}{\frac{5}{2} + 7 + 4}$$

$$\begin{array}{r} 14 \\ +\ 8 \\ \hline 22 \\ -\ 5 \\ \hline 17 \end{array}$$

$$= \frac{\frac{49}{4} \nearrow + 17}{\frac{5}{2} \nearrow + 11}$$

$$\begin{array}{r} 17 \\ \times\ 4 \\ \hline 68 \end{array}$$

$$= \frac{\frac{49 + 68}{4}}{\frac{5 + 22}{2}}$$

$$= \frac{\frac{117}{4}}{\frac{27}{2}}$$

$$= \frac{\overset{13}{\cancel{117}}}{\underset{2}{\cancel{4}}} \times \frac{\overset{1}{\cancel{2}}}{\underset{3}{\cancel{27}}}$$

$$\boxed{\frac{\alpha+2}{\beta+2} + \frac{\beta+2}{\alpha+2} = \frac{13}{6}}$$

③ The roots of the equation $x^2 + 6x - 4 = 0$ are α, β Find the quadratic equation whose roots are

(i) α^2 and β^2.

Sol :-

$$x^2 + 6x - 4 = 0$$

$$\boxed{\begin{aligned} a &= 1 \\ b &= 6 \\ c &= -4 \end{aligned}}$$

Sum $\Rightarrow \alpha + \beta = \frac{-b}{a} = \frac{-6}{1}$

Product $\Rightarrow \alpha\beta = \frac{c}{a} = \frac{-4}{1}$

Here the roots are α^2, β^2

$$\text{Sum} = \alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$$

$$= (-6)^2 - 2(-4)$$

$$= 36 + 8$$

$$\boxed{\text{Sum} = 44}$$

$$\text{Product} = \alpha^2\beta^2 = (\alpha\beta)^2$$

$$= (-4)^2$$

$$\boxed{\text{Product} = 16}$$

Q.E $\Rightarrow x^2 - (\text{Sum of roots})x + \text{Product of roots} = 0$

$$x^2 - 44x + 16 = 0$$

(ii) $\frac{2}{\alpha}$ and $\frac{2}{\beta}$

$$\boxed{\begin{aligned}\alpha+\beta &= -6\\ \alpha\beta &= -4\end{aligned}}$$

Sol Here the roots are $\frac{2}{\alpha}$ and $\frac{2}{\beta}$

Sum of roots $= \frac{2}{\alpha} + \frac{2}{\beta}$

$$= \frac{2\beta + 2\alpha}{\alpha\beta}$$

$$= \frac{2(\alpha+\beta)}{\alpha\beta}$$

$$= \frac{\overset{1}{\cancel{2}}(\cancel{-}\overset{3}{\cancel{6}})}{\cancel{-}\cancel{4}_{\cancel{2}_{1}}}$$

$$\boxed{\text{Sum} = 3}$$

Product $= \frac{2}{\alpha} \times \frac{2}{\beta}$

$$= \frac{4}{\alpha\beta}$$

$$= \frac{4}{-4}$$

$$\boxed{\text{Product} = -1}$$

(Q.E $\Rightarrow$ $x^2 - (\text{Sum of roots})x + \text{Product of roots} = 0$

$$x^2 - 3x - 1 = 0$$

(iii) $\alpha^2\beta$ and $\beta^2\alpha$

$$\boxed{\begin{aligned}\alpha+\beta &= -6\\ \alpha\beta &= -4\end{aligned}}$$

Sol:- Here the roots are $\alpha^2\beta$ & $\beta^2\alpha$

$$\text{Sum} = \alpha^2\beta + \beta^2\alpha$$

$$= \alpha\beta(\alpha+\beta)$$

$$= -4(-6)$$

$$\boxed{\text{Sum} = 24}$$

$$\text{Product} = (\alpha^2\beta)(\beta^2\alpha)$$

$$= \alpha^3\beta^3$$

$$= (\alpha\beta)^3$$

$$= (-4)^3$$

$$\boxed{\text{Product} = -64}$$

Q.E $\Rightarrow$ $x^2 - (\text{Sum of roots})x + \text{Product of roots} = 0$

$$x^2 - 24x - 64 = 0$$

④ If α, β are the roots of $7x^2+ax+2=0$ and if $\beta-\alpha = -\frac{13}{7}$ Find the Value of a.

Sol:- $7x^2+ax+2=0$

$$\boxed{\begin{aligned} a &= 7 \\ b &= a \\ c &= 2 \end{aligned}}$$

$$\text{Sum} = -\frac{b}{a} = -\frac{a}{7}$$

$$\therefore \boxed{\alpha+\beta = -\frac{a}{7}}$$

$$\text{Product} = \frac{c}{a} = \frac{2}{7}$$

$$\boxed{\alpha\beta = \frac{2}{7}}$$

Given:- $\beta - \alpha = -\frac{13}{7}$

We know $\alpha-\beta = \sqrt{(\alpha+\beta)^2-4\alpha\beta}$

lly $\beta-\alpha = \sqrt{(\beta+\alpha)^2-4\beta\alpha}$

$$\beta-\alpha = -\frac{13}{7}$$

$$\sqrt{(\beta+\alpha)^2-4\beta\alpha} = \frac{-13}{7}$$ [Squaring on both sides]

$$(\beta+\alpha)^2-4\beta\alpha = \left(\frac{-13}{7}\right)^2$$

$$\left(-\frac{a}{7}\right)^2 - 4\left(\frac{2}{7}\right) = \frac{169}{49}$$

$$\frac{a^2}{49} - \frac{8\times7}{7} = \frac{169}{49}$$

$$\frac{a^2-56}{\cancel{49}} = \frac{169}{\cancel{49}}$$

$$a^2 = 169+56$$

$$a^2 = 225$$

$$a = \sqrt{225}$$

$$\boxed{a = \pm 15}$$

---

⑤ If One root of the equation $2y^2-ay+64=0$ is twice the Other then find the Values of a.

Sol:- $2y^2-ay+64=0$

$$\boxed{\begin{array}{l} a=2 \\ b=-a \\ c=64 \end{array}}$$

$\therefore$ One root = twice the other

Let One root = α

then another root = 2α

Sum $\Rightarrow \alpha + 2\alpha = \frac{-b}{a}$

$$3\alpha = \frac{-(-a)}{2}$$

$$\boxed{3\alpha = \frac{a}{2}} \longrightarrow ①$$

Product $\Rightarrow (\alpha)(2\alpha) = \frac{c}{a}$

$$2\alpha^2 = \frac{\cancel{64}^{\,32}}{\cancel{2}_{\,1}}$$

$$2\alpha^2 = 32$$

$$\alpha^2 = \frac{32}{2}$$

$$\alpha^2 = 16$$

$$\alpha = \sqrt{16}$$

$$\boxed{\alpha = \pm 4}$$

Put $\alpha = 4$ in ①

$$3(4) = \frac{a}{2}$$

$$12 \times 2 = a$$

$$\boxed{a = 24}$$

Put $\alpha = -4$ in ①

$$3(-4) = \frac{a}{2}$$

$$-12 \times 2 = a$$

$$\boxed{a = -24}$$

⑥ If one root of equation $3x^2+kx+81=0$ (having real roots) is the square of the other then find k.

Sol $3x^2+kx+81=0$

$a=3$
$b=k$
$c=81$

One root = square of other

ie, Let one root $=\alpha$
another root $=\alpha^2$

$$\text{Sum} = \alpha+\alpha^2 = -\frac{b}{a}$$

$$\alpha+\alpha^2 = \frac{-k}{3} \longrightarrow ①$$

$$\text{Product} = \alpha(\alpha^2) = \frac{c}{a}$$

$$\alpha^3 = \frac{81}{3}$$

$$\alpha^3 = 27$$

$$\alpha^3 = 3^3$$

$$\boxed{\alpha = 3}$$

Put $\alpha = 3$ in ①

$$\alpha^2 + \alpha = \frac{-k}{3}$$

$$3^2 + 3 = \frac{-k}{3}$$

$$9 + 3 = \frac{-k}{3}$$

$$12 = \frac{-k}{3}$$

$$12 \times 3 = -k$$

$$-k = 36$$

$$\boxed{k = -36}$$

Exercise 3.15

① A garment Shop announces a flat 50% discount on every Purchase of items for their customers. Draw the graph for the relation between the Marked Price and the Discount. Hence Find

(i) the marked price when a Customer gets a discount of ₹3250 [from graph]

(ii) the discount when the marked price is ₹2500

Sol :-

Marked Price (x)	1000	2000	3000	4000	5000
Discount (y)	500	1000	1500	2000	2500

As x increases, y also increases.

It is direct Variation.

$$\boxed{y = kx}$$

$$k = \frac{y}{x} = \frac{\cancel{500}^{1}}{\cancel{1000}_{2}} = \frac{1000}{2000} = \cdots$$

$$\boxed{k = \frac{1}{2}}$$

$$\therefore \quad \boxed{y = \frac{1}{2}x}$$

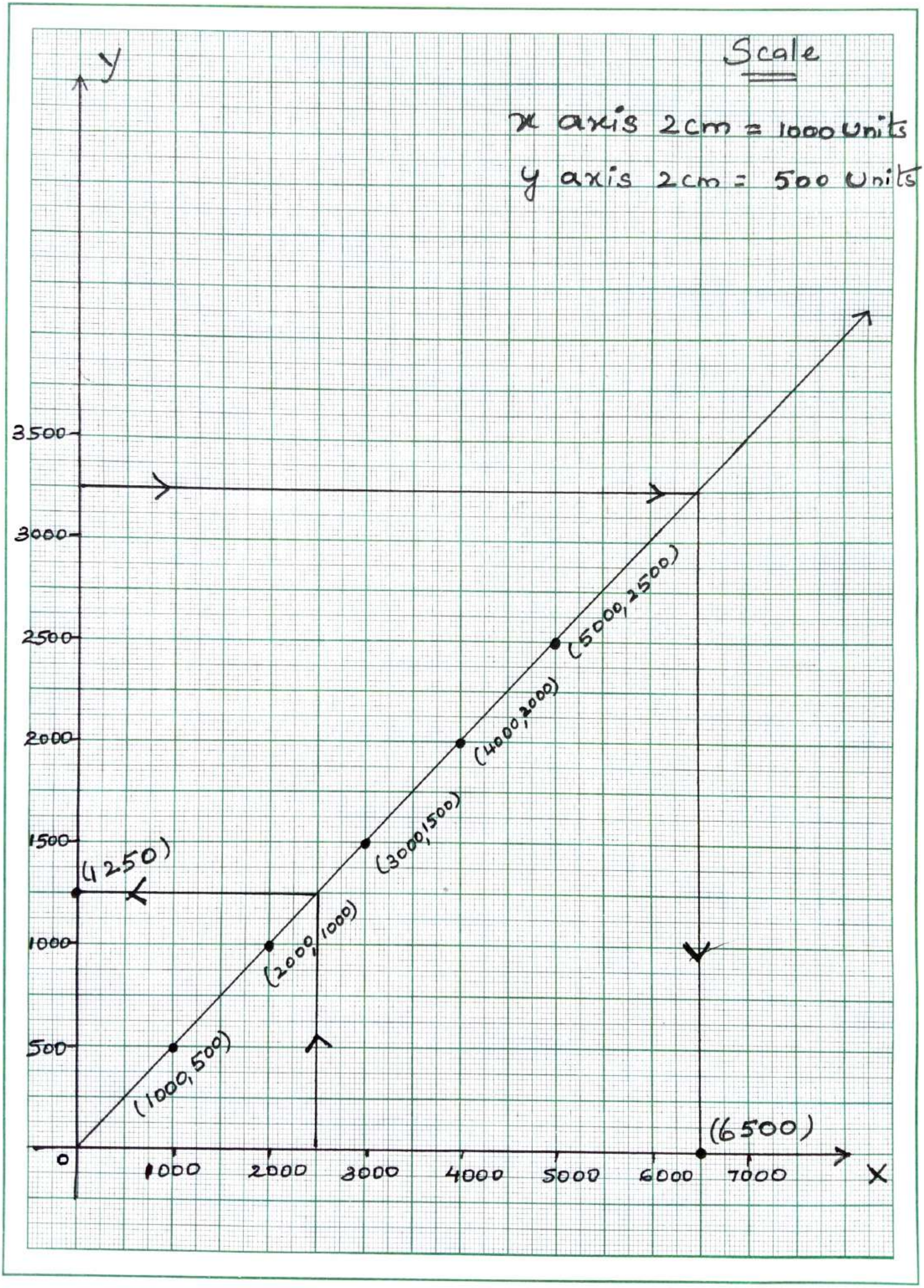
(i) Discount = ₹3250

$\therefore$ y = ₹3250

$\therefore$ Marked Price ⇒ $x = 6500$ [From graph]

(ii) Marked Price ⇒ x = ₹2500

$\therefore$ Discount ⇒ $y = 1250$ [From graph]


Scale
x axis 2cm = 1000 units
y axis 2cm = 500 units
Y
X
0
500
1000
1500
2000
2500
3000
3500
1000
2000
3000
4000
5000
6000
7000
(1000, 500)
(2000, 1000)
(3000,1500)
(4000,2000)
(5000,2500)
(1250)
(6500)

② Draw the graph of $xy = 24, x, y > 0$.
Using the graph find
(i) y when $x = 3$
(ii) x when $y = 6$.

Sol :-

$$xy = 24$$

$$y = \frac{24}{x}$$

$x = 2$; $y = \frac{\overset{12}{\cancel{24}}}{\underset{1}{\cancel{2}}} = 12$

$x = 4$; $y = \frac{\overset{6}{\cancel{24}}}{\underset{1}{\cancel{4}}} = 6$

$x = 6$; $y = \frac{\overset{4}{\cancel{24}}}{\underset{1}{\cancel{6}}} = 4$

$x = 8$; $y = \frac{\overset{3}{\cancel{24}}}{\underset{1}{\cancel{8}}} = 3$

$x = 12$; $y = \frac{\overset{2}{\cancel{24}}}{\cancel{12}} = 2$

x	2	4	6	8	12
y	12	6	4	3	2

As x increases, y decreases.

It is Indirect Variation

$$\boxed{xy = k}$$

$$k = (2\times12) = (4\times6) = \cdots$$

$$\boxed{k = 24}$$

$$\therefore \boxed{xy = 24}$$

(i) $x = 3$

$y = 8$

[From graph]

(ii) $y = 6$

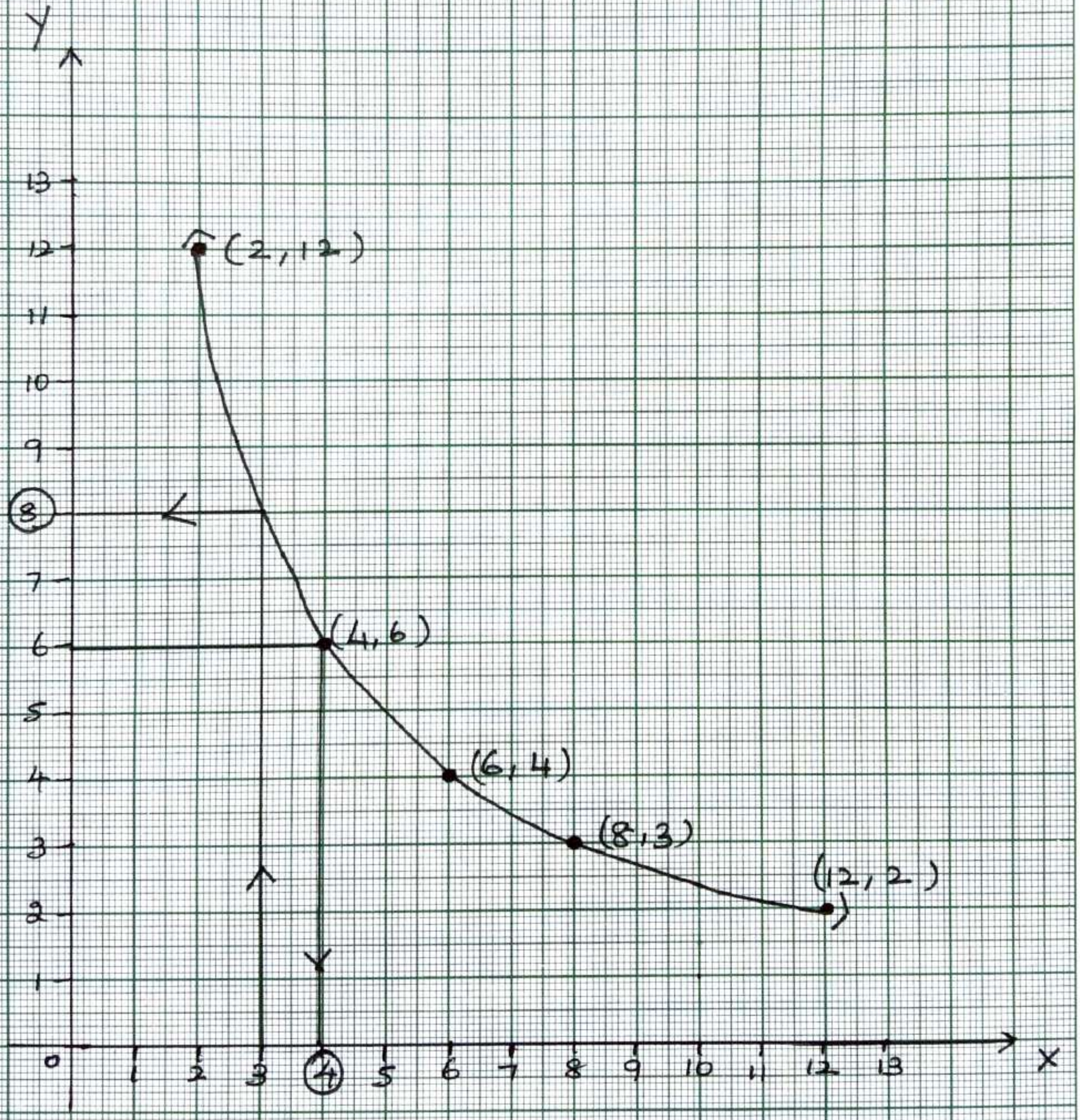
$x = 4$

[From graph]

Scale

x axis 1cm = 1 unit

y axis 1cm = 1 unit



③ Graph the following linear function $y = \frac{1}{2}x$. Identify the constant of Variation and Verify it with the graph. Also

(i) find y when $x = 9$.

(ii) find x when $y = 7.5$

Sol :- $y = \frac{1}{2}x$

$x = 2$; $y = \frac{1}{\cancel{2}}(\cancel{2}) = 1$

$x = 4$; $y = \frac{1}{\cancel{2}}(\cancel{4}^{2}) = 2$

$x = 6$; $y = \frac{1}{\cancel{2}_{1}}(\cancel{6}^{3}) = 3$

$x = 8$; $y = \frac{1}{\cancel{2}}(\cancel{8}^{4}) = 4$

$x = 10$; $y = \frac{1}{\cancel{2}_{1}}(\cancel{10}^{5}) = 5$

x	2	4	6	8	10
y	1	2	3	4	5

As x increases, y also increases.

It is direct Variation.

$$\boxed{y = kx}$$

$$K = \frac{y}{x} = \frac{1}{2} = \frac{\cancel{2}^{1}}{\cancel{4}_{2}} = \cdots\cdots$$

$$\boxed{k = \frac{1}{2}}$$

$$\therefore \boxed{y = \frac{1}{2}x}$$

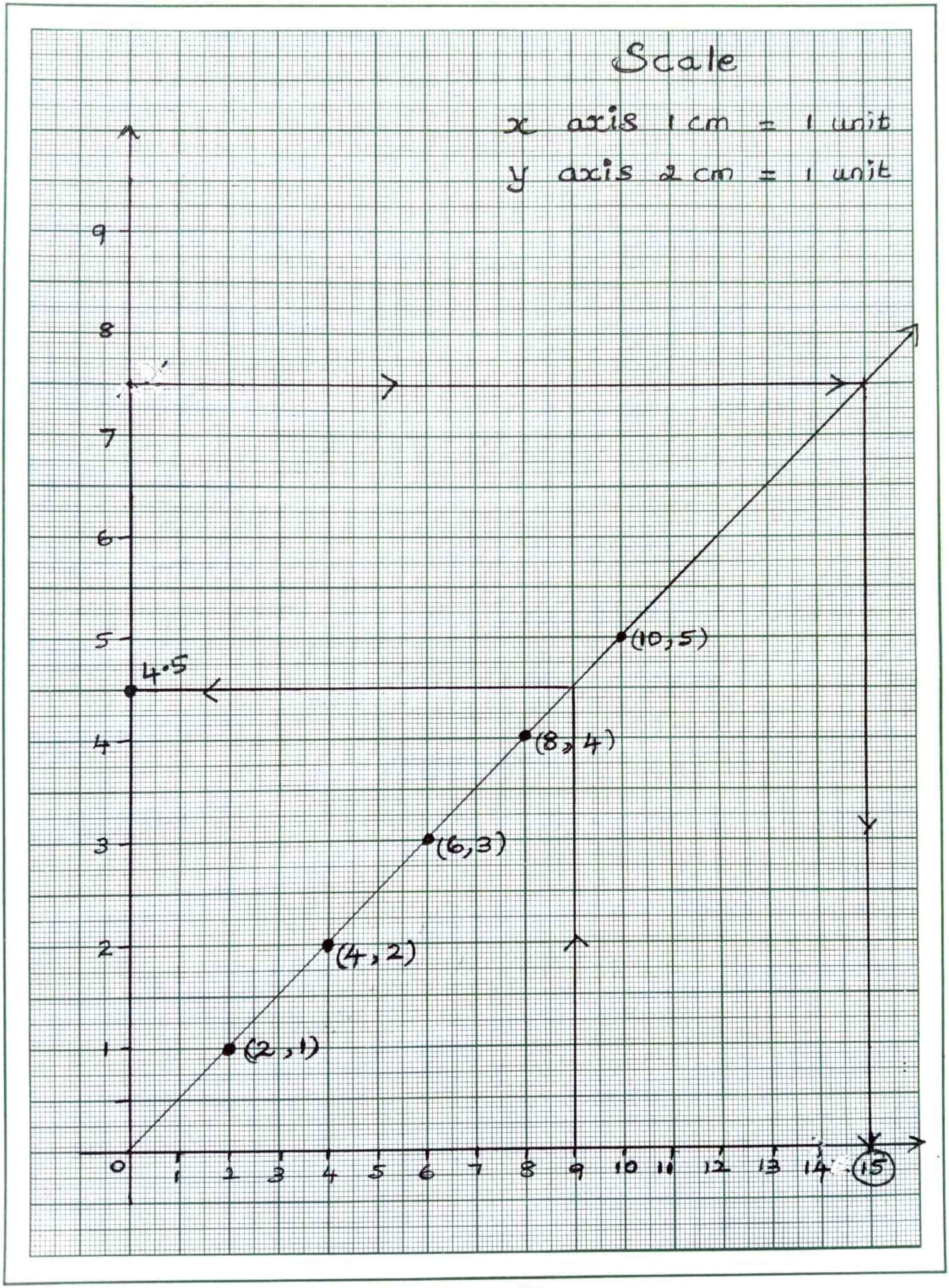
Constant of Variation $\Rightarrow \boxed{k = \frac{1}{2}}$

(i) $x = 9$

$y = 4.5$ [From graph]

(ii) $y = 7.5$

$x = 15$ [From graph]


Scale
x axis 1 cm = 1 unit
y axis 2 cm = 1 unit
9
8
7
6
5
4·5
4
3
2
1
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
(2, 1)
(4, 2)
(6, 3)
(8, 4)
(10, 5)

④ The following table shows the data about the number of pipes and the time taken to fill the same tank

No. of Pipes (x)	2	3	6	9
Time Taken (in min) (y)	45	30	15	10

Draw the graph for the above data and hence

(i) find the time taken to fill the tank when five pipes are used.

(ii) find the number of pipes when the time is 9 minutes.

Sol :-

No. of pipes (x)	2	3	6	9
Time Taken (in min) (y)	45	30	15	10

As x increases, y decreases.

It is Indirect Variation

$$\boxed{xy = k}$$

$$k = (2 \times 45) = (3 \times 30) = \cdots$$

$$\boxed{k = 90}$$

$$\boxed{xy = 90}$$

(i) Find Time Taken to fill the tank when 5 pipes are used.

$\therefore$ $x = 5$

$y = 19$

(ii) Find the number of pipes when the time is 9 minutes.

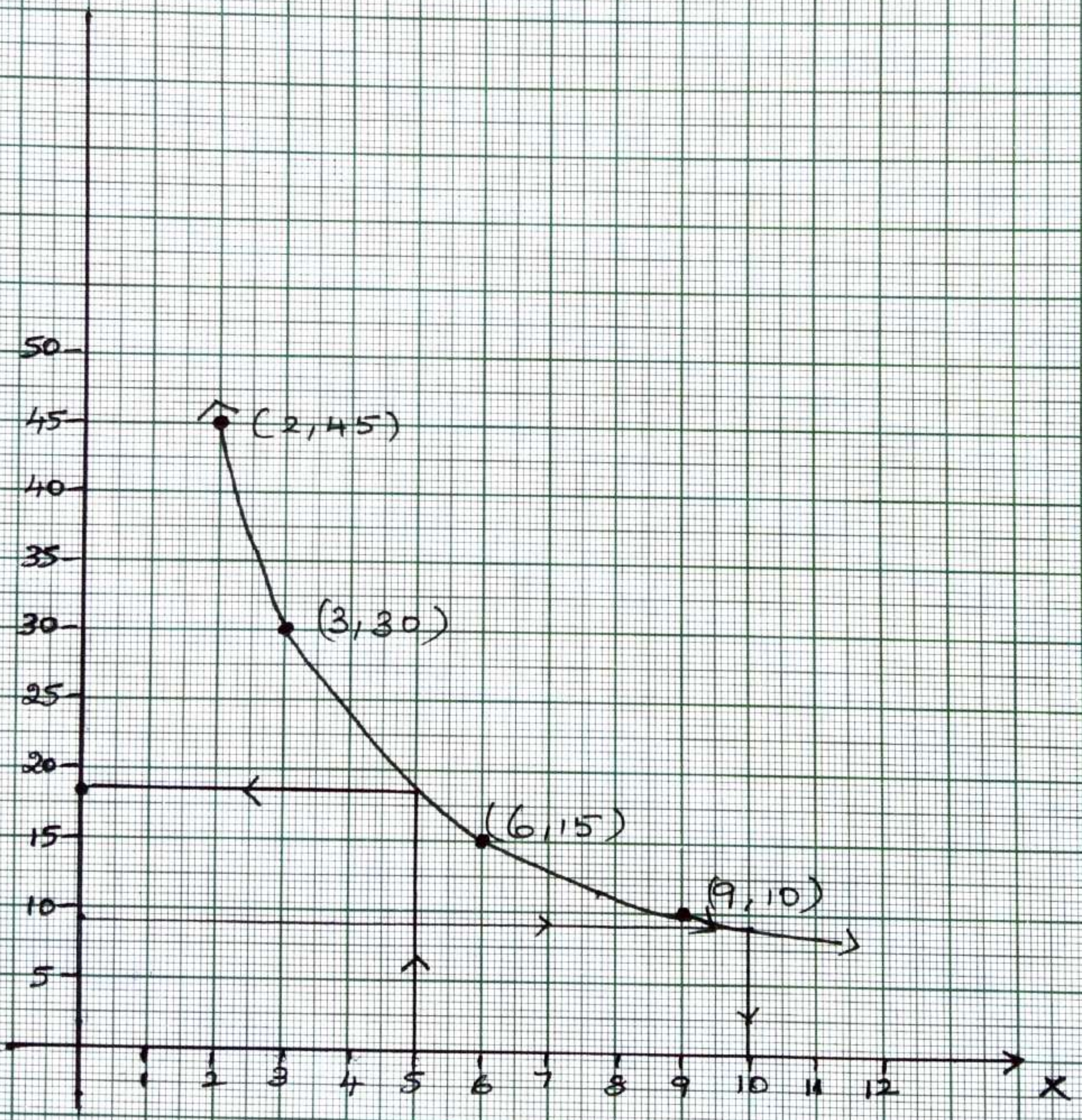
Time taken $= y = 9$

Number Pipes $x = 10$

Scale

x axis 1 cm = 1 unit

y axis 1 cm = 5 units



(5) A School announces that for a Certain competitions, the cash price will be distributed for all the participants equally as show below:-

No. of Participants (x)	2	4	6	8	10
Amount for each participant in ₹ (y)	180	90	60	45	36

(i) Find the constants of Variation

(ii) Graph the above data and hence, find how much will each participants get if the number of participants are 12.

Sol :-

No. of Participants	2	4	6	8	10
Amt for each Participant	180	90	60	45	36

As x increases, y decreases

It is indirect Variation.

$$\boxed{xy = k}$$

$$k = (2 \times 180) = (4 \times 90) = \ldots$$

$$\boxed{k = 360} \Rightarrow \boxed{xy = 360}$$

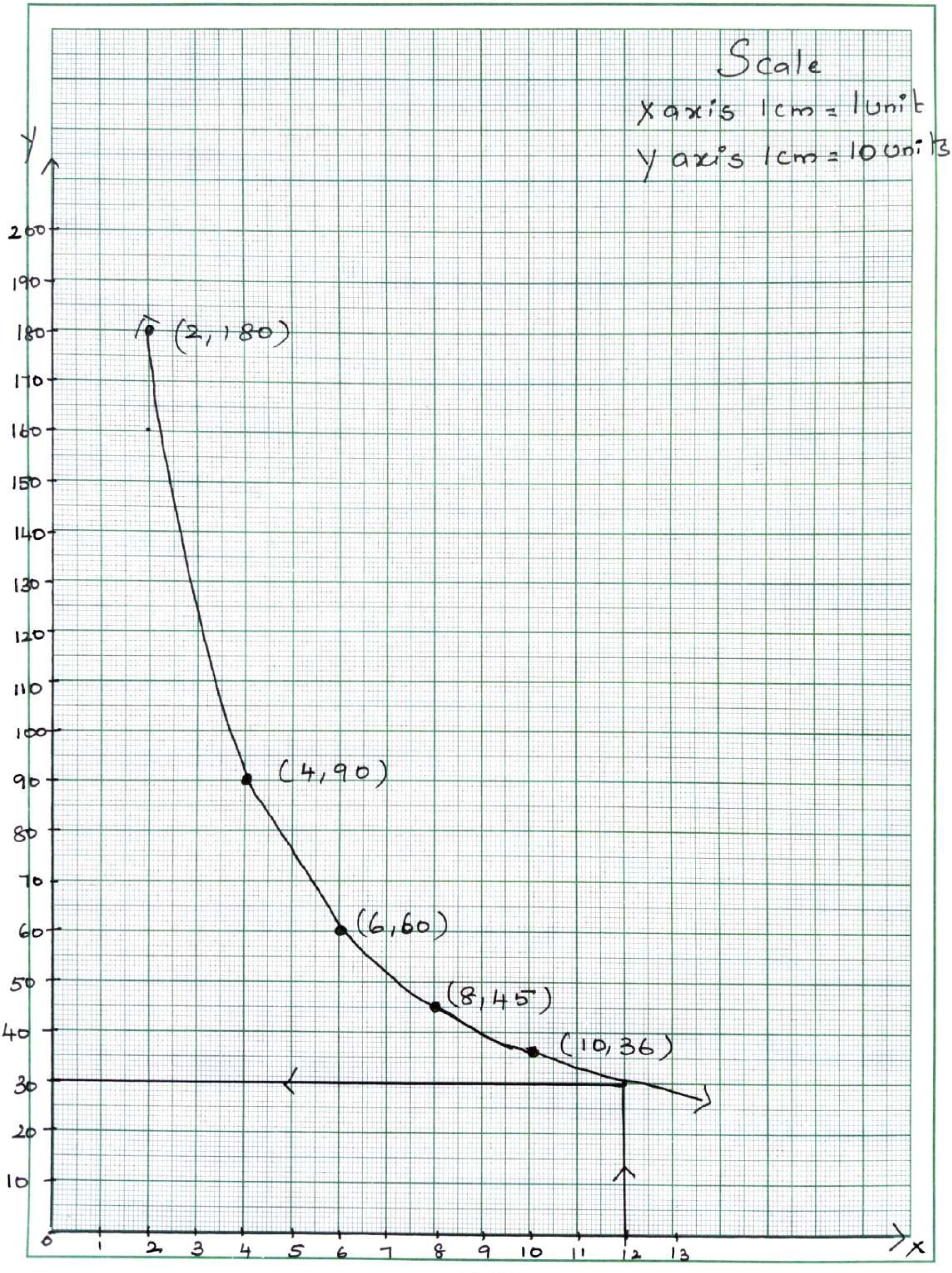
(i) Constant of Variation

$$\boxed{k = 360}$$

(ii) Graph the above data and hence, find how much will each participant get if the number of participant are 12.

Sol:- Number of participant $= 12$

$$\therefore x = 12$$
$$y = 30$$


Scale
X axis 1cm = 1unit
Y axis 1cm = 10 units
y
x
0
1
2
3
4
5
6
7
8
9
10
11
12
13
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
160
170
180
190
200
(2, 180)
(4, 90)
(6, 60)
(8, 45)
(10, 36)

⑥ A two wheeler parking Zone near bus Stand charges as below.

Time (in hours) (x)	4	8	12	24
Amount ₹ (y)	60	120	180	360

Check if the amount charged are in direct Variation or in inverse Variation to the parking time. Graph the data. Also

(i) Find the amount to be paid when parking time is 6 hr.

(ii) Find the parking duration when the amount paid is ₹ 150.

Sol :-

Time (x)	4	8	12	24
Amount (y)	60	120	180	360

As x increases, y also increases.
It is direct Variation.

$$\boxed{y = kx}$$

$$k = \frac{y}{x} = \frac{\overset{15}{\cancel{60}}}{\underset{1}{\cancel{4}}} = \frac{\overset{15}{\cancel{120}}}{\cancel{8}} = \cdots$$

$$\therefore \boxed{k = 15}$$

$$\therefore \boxed{y = 15x}$$

(i) Find the amount to be paid when parking time is 6 hr.

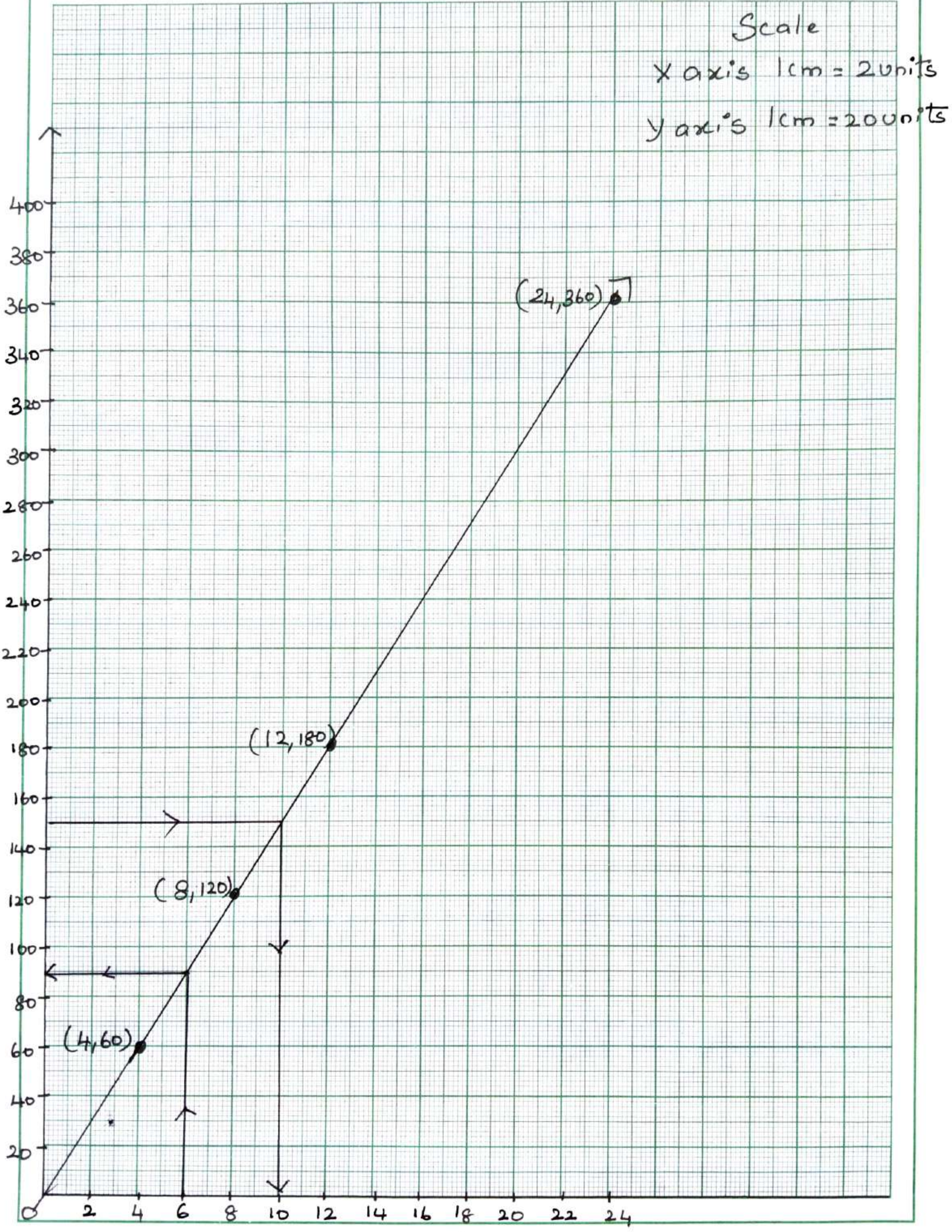
Sol:- $x = 6$ hrs.

$y = ₹90$

(ii) Find the parking duration when the amount paid ₹ 150.

Sol: $y = 150$

$x = 10$ hrs


Scale
X axis 1cm = 2units
Y axis 1cm = 20units
(24, 360)
(12, 180)
(8, 120)
(4, 60)
400
380
360
340
320
300
280
260
240
220
200
180
160
140
120
100
80
60
40
20
0
2
4
6
8
10
12
14
16
18
20
22
24

Exercise - 3.16

① Graph the following quadratic equations and state their nature of solutions.

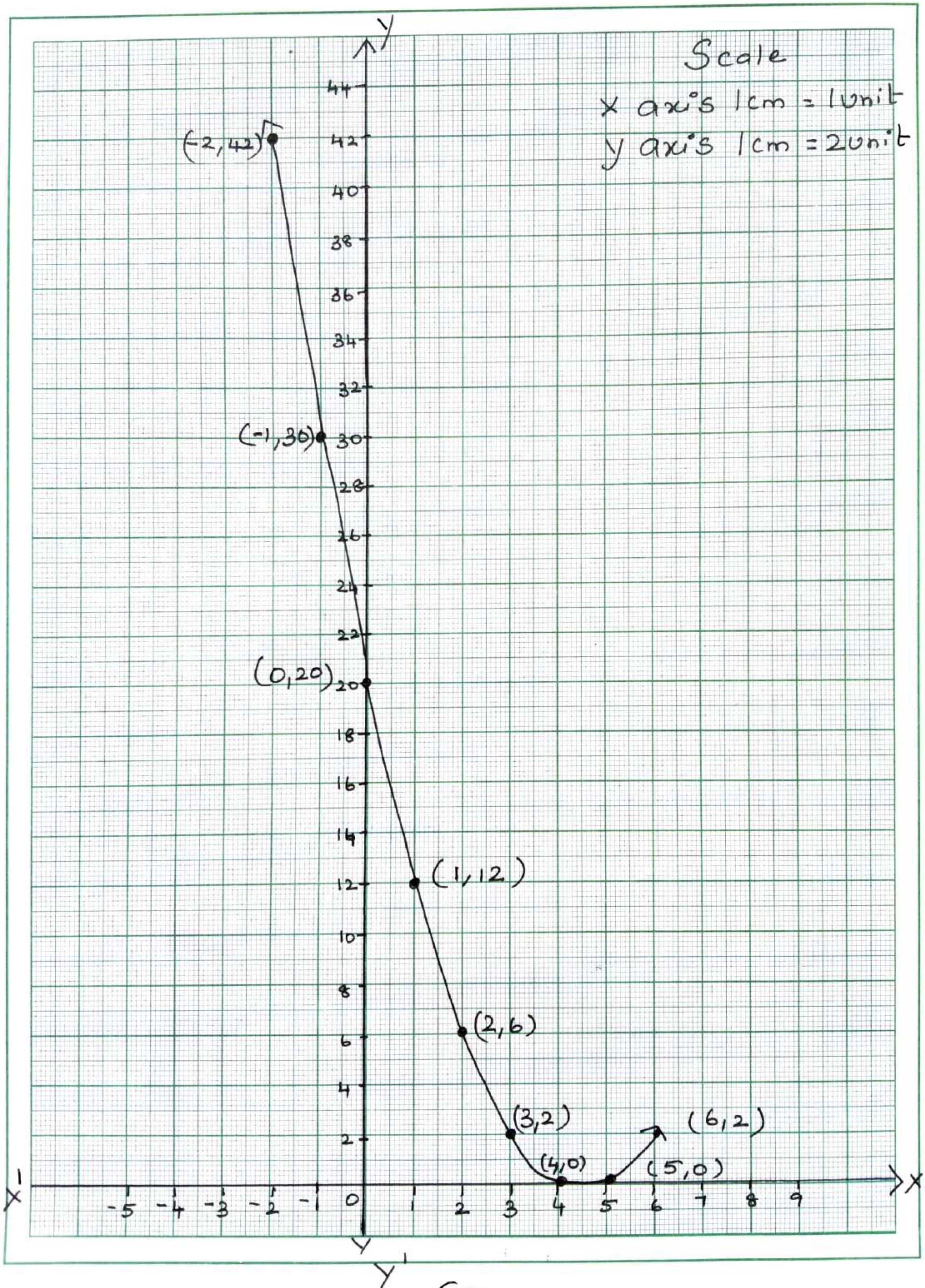
(i) $x^2 - 9x + 20 = 0$

$$y = x^2 - 9x + 20$$

x	-2	-1	0	1	2	3	4	5	6
x^2	4	1	0	1	4	9	16	25	36
$-9x$	18	9	0	-9	-18	-27	-36	-45	-54
20	20	20	20	20	20	20	20	20	20
y	42	30	20	12	6	2	0	0	2

Plot:- (-2, 42), (-1, 30) (0, 20) (1, 12) (2, 6) (3, 2)
(4, 0) (5, 0) (6, 2)

Nature :- Roots are Real and Unequal.



Scale

x axis 1cm = 1unit

y axis 1cm = 2unit

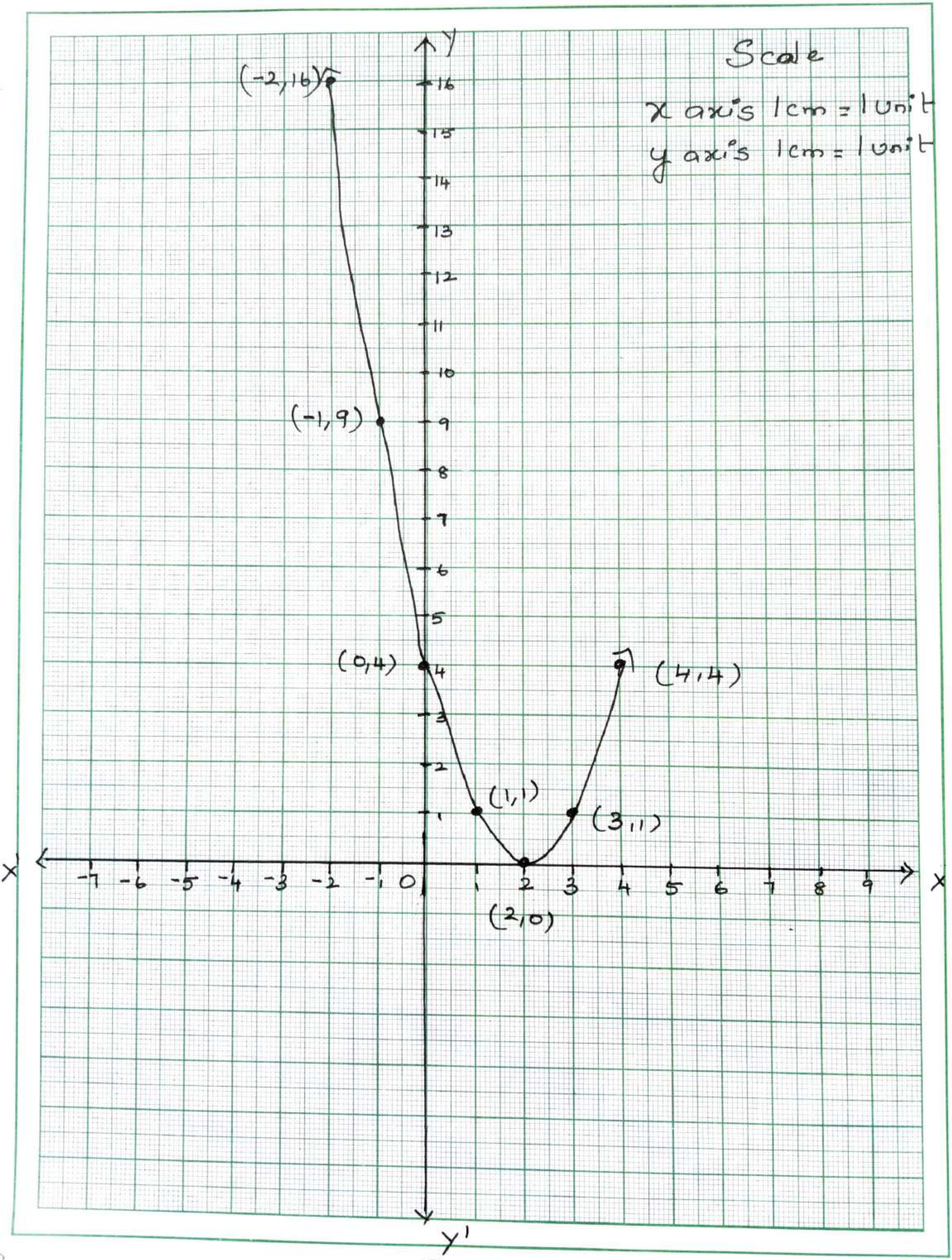
(ii) $x^2 - 4x + 4 = 0$

$y = x^2 - 4x + 4$

x	-2	-1	0	1	2	3	4
x^2	4	1	0	1	4	9	16
$-4x$	8	4	0	-4	-8	-12	-16
4	4	4	4	4	4	4	4
y	16	9	4	1	0	1	4

Plot: $(-2, 16)$ $(-1, 9)$ $(0, 4)$ $(1, 1)$ $(2, 0)$
$(3, 1)$ $(4, 4)$

Nature :- Roots are real and equal.



Scale

x axis 1cm = 1 unit

y axis 1cm = 1 unit

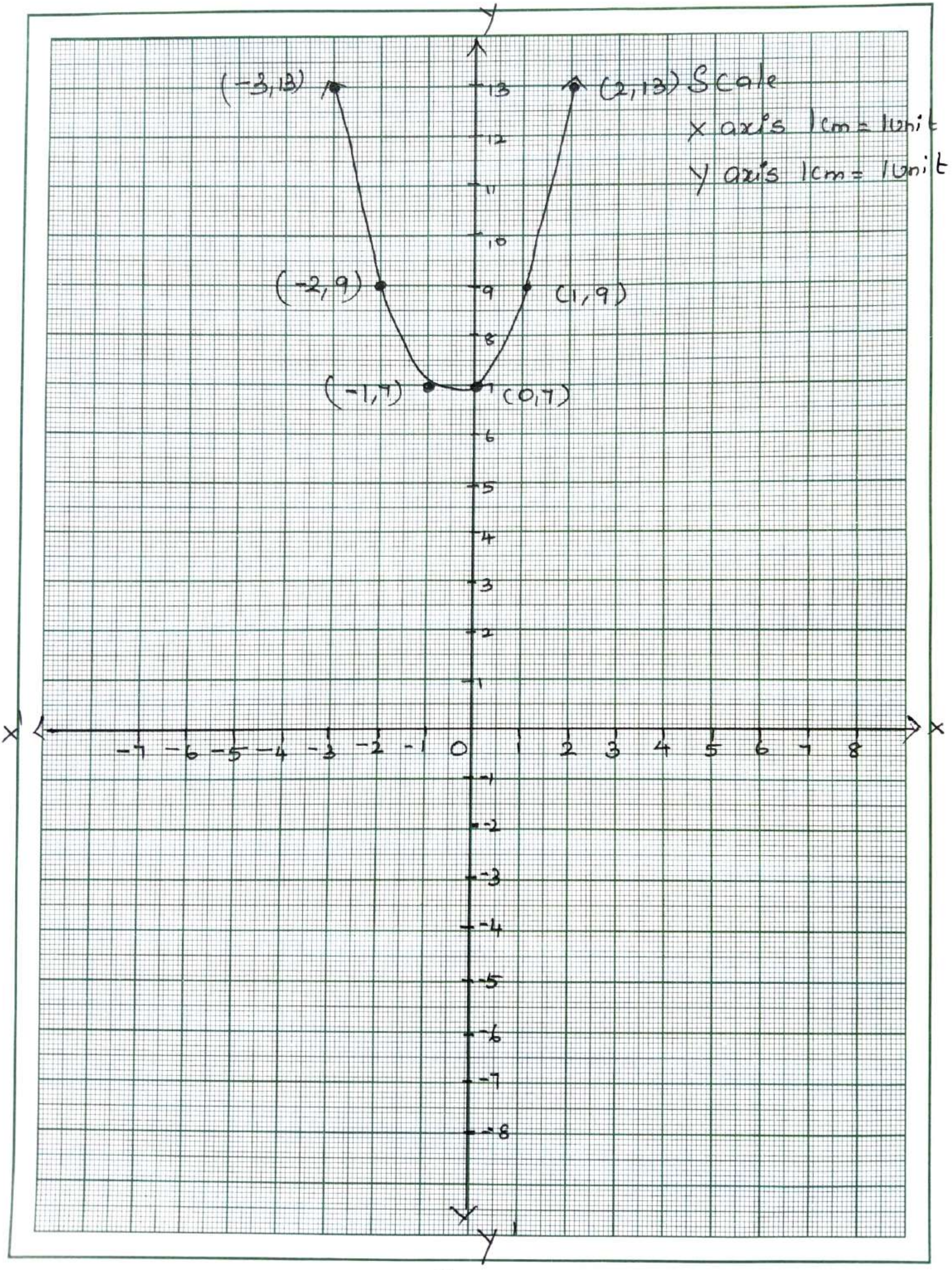
(iii) $x^2 + x + 7 = 0$

$y = x^2 + x + 7$

x	-3	-2	-1	0	1	2
x^2	9	4	1	0	1	4
x	-3	-2	-1	0	1	2
7	7	7	7	7	7	7
y	13	9	7	7	9	13

Plot:- (-3, 13) (-2, 9) (-1,7) (0,7) (1,9) (2,13)

Nature:- Roots are Imaginary.


Y
X'
X
Y'
(-3,13)
(2,13)
Scale
x axis 1cm = 1unit
y axis 1cm = 1unit
(-2,9)
(1,9)
(-1,7)
(0,7)
13
12
11
10
9
8
7
6
5
4
3
2
1
-1
-2
-3
-4
-5
-6
-7
-8
-7
-6
-5
-4
-3
-2
-1
0
2
3
4
5
6
7
8

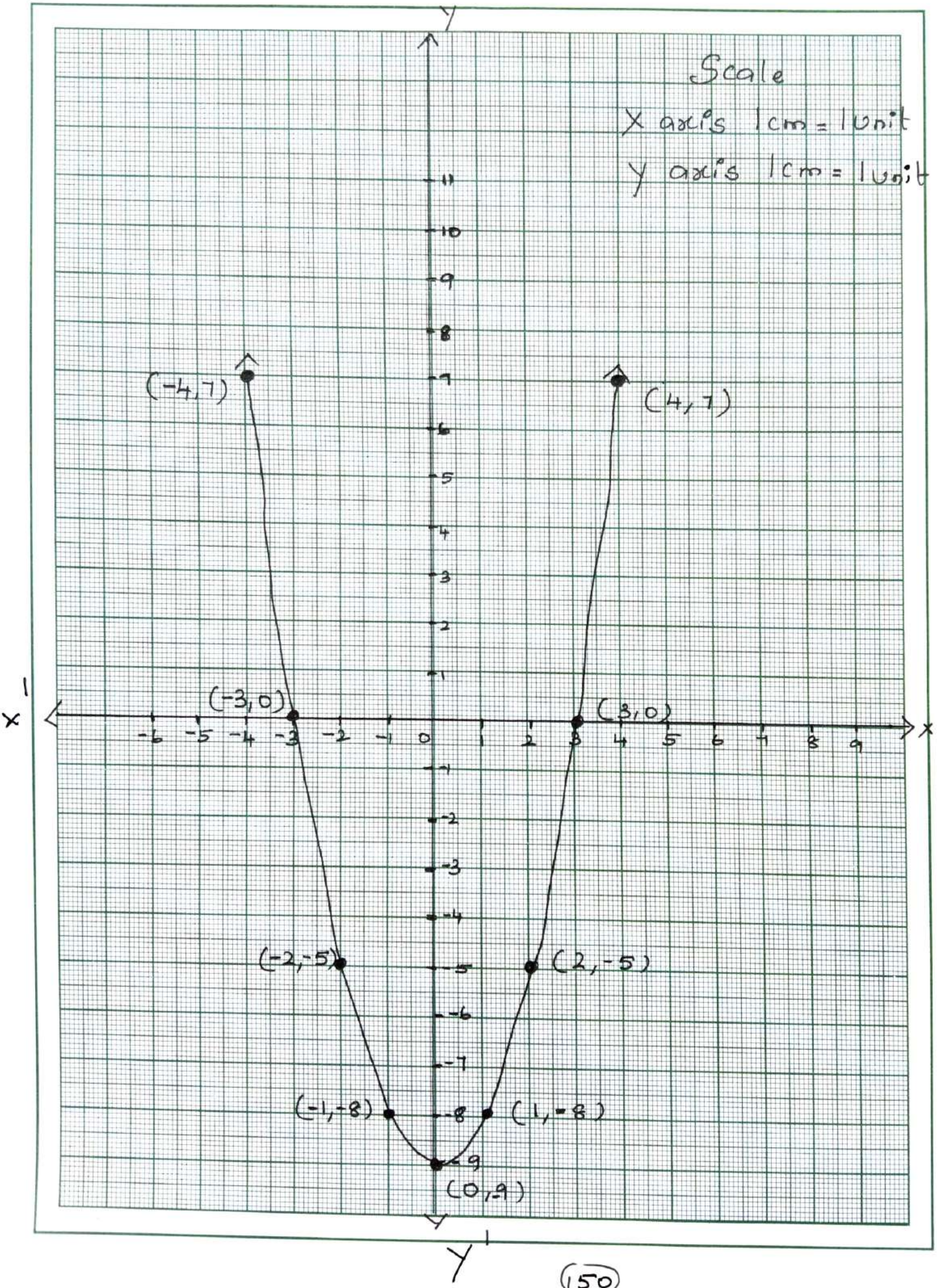
(iv) $x^2 - 9 = 0$

$$y = x^2 - 9$$

x	-4	-3	-2	-1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
-9	-9	-9	-9	-9	-9	-9	-9	-9	-9
y	7	0	-5	-8	-9	-8	-5	0	7

plot :- $(-4,7)$ $(-3,0)$ $(-2,-5)$, $(-1,-8)$, $(0,-9)$
$(1,-8)$, $(2,-5)$, $(3,0)$, $(4,7)$

Nature :- Roots are real and unequal.


Scale
X axis 1cm = 1unit
Y axis 1cm = 1unit
(-4,7)
(4,7)
(-3,0)
(3,0)
(-2,-5)
(2,-5)
(-1,-8)
(1,-8)
(0,9)
X'
X
Y
Y'
-6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9
11 10 9 8 7 6 5 4 3 2 1 -1 -2 -3 -4 -5 -6 -7 -8 -9

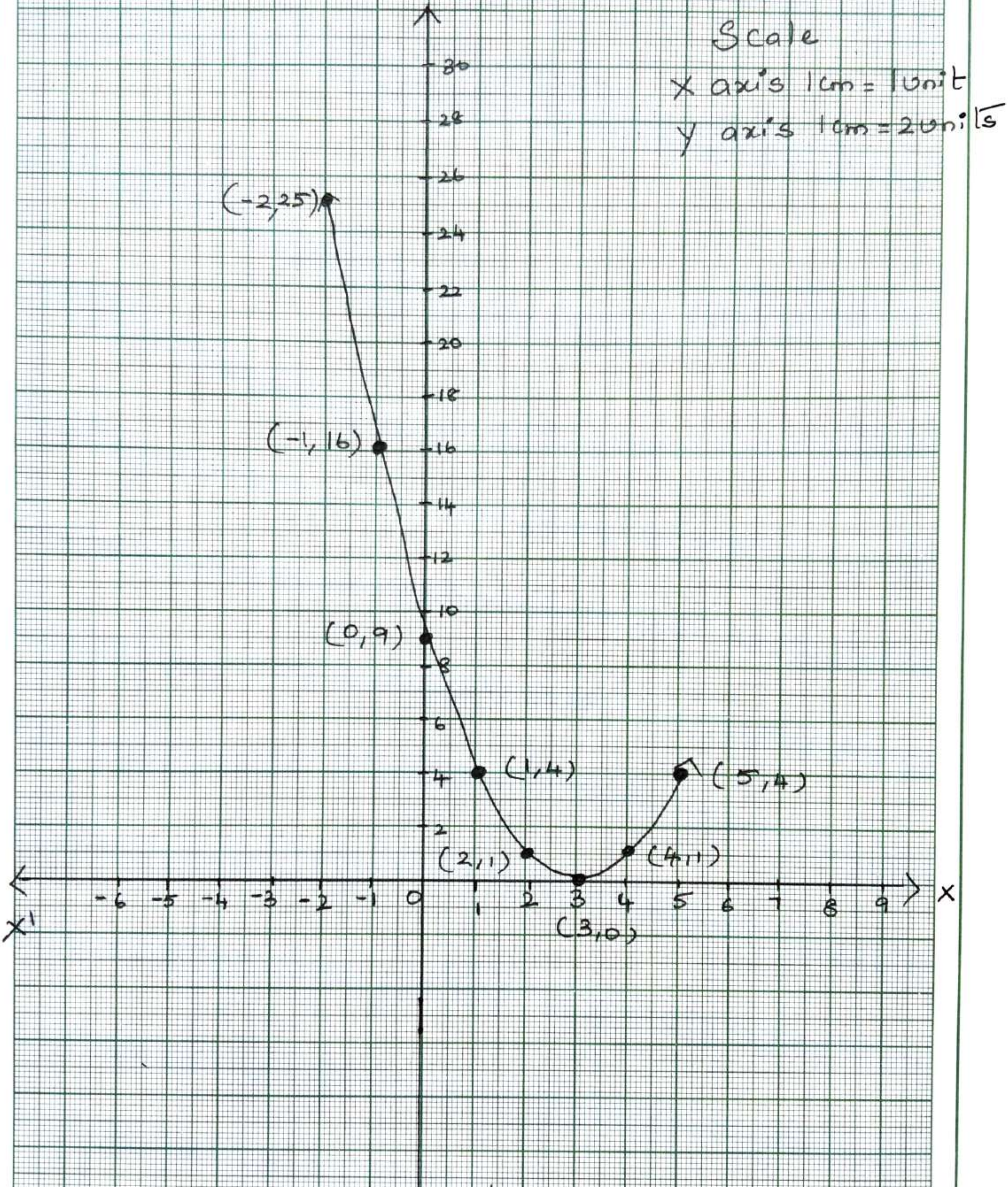
(v) $x^2 - 6x + 9 = 0$

$y = x^2 - 6x + 9$

x	-2	-1	0	1	2	3	4	5
x^2	4	1	0	1	4	9	16	25
$-6x$	12	6	0	-6	-12	-18	-24	-30
9	9	9	9	9	9	9	9	9
y	25	16	9	4	1	0	1	4

Plot:- $(-2,25)$ $(-,16)$ $(0,9)$ $(1,4)$ $(2,1)$ $(3,0)$
$(4,1)$ $(5,4)$

Nature :- Roots are real and Equal.


Scale
X axis 1cm = 1unit
Y axis 1cm = 2units
(-2,25)
(-1,16)
(0,9)
(1,4)
(2,1)
(3,0)
(4,1)
(5,4)
30
28
26
24
22
20
18
16
14
12
10
8
6
4
2
-6
-5
-4
-3
-2
-1
0
1
2
3
4
5
6
7
8
9
X
X'
Y'

(vi) $(2x-3)(x+2)=0$

$$y=(2x-3)(x+2)$$

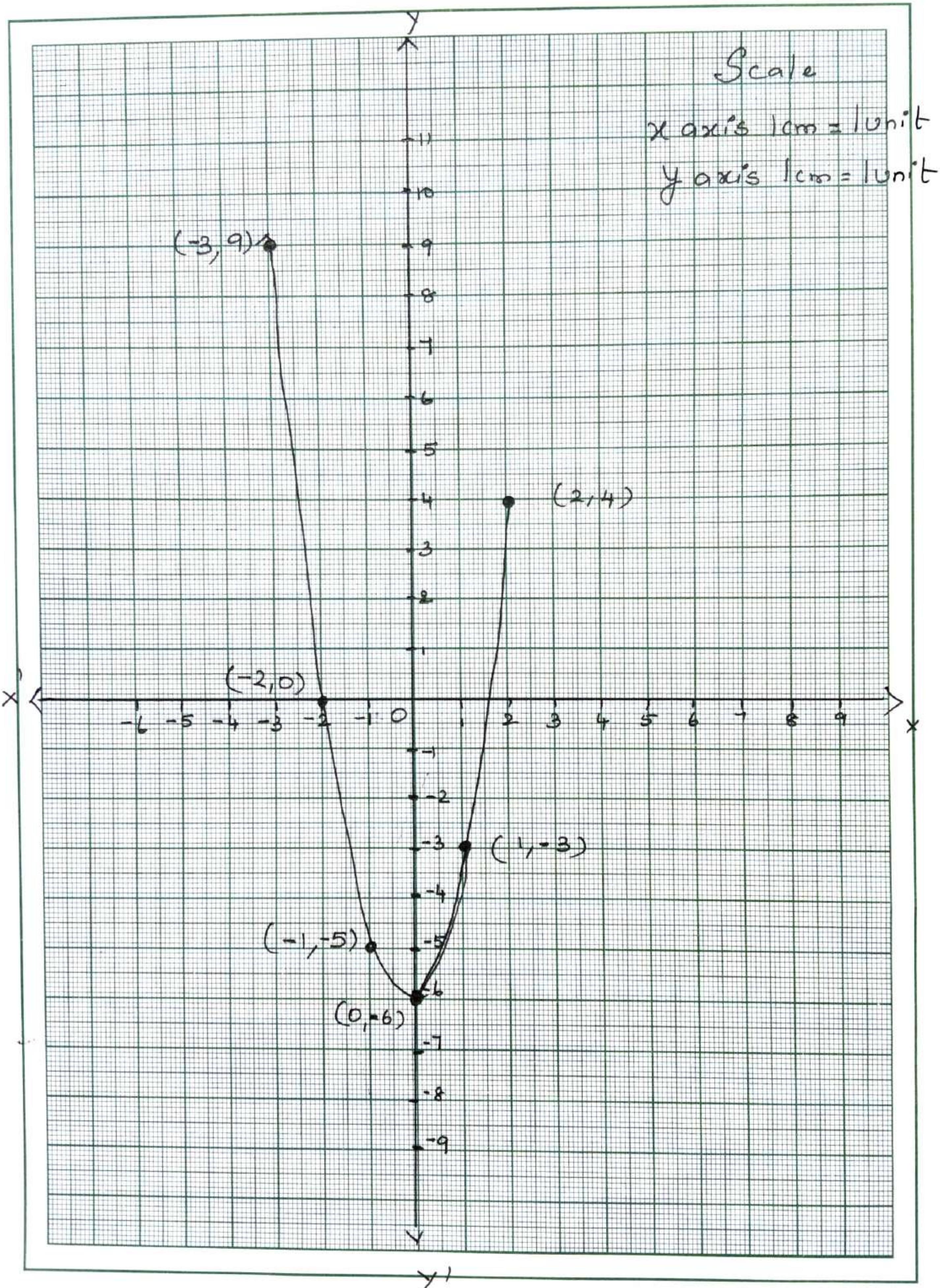
$$=2x^2+4x-3x-6$$

$$y=2x^2+x-6$$

x	-3	-2	-1	0	1	2
x^2	9	4	1	0	1	4
$2x^2$	18	8	2	0	2	8
x	-3	-2	-1	0	1	2
-6	-6	-6	-6	-6	-6	-6
y	9	0	-5	-6	-3	4

Plot : $(-3,9)$ $(-2,0)$ $(-1,-5)$ $(0,-6)$ $(1,-3)$, $(2,4)$

Nature :- Roots are Real and unequal.



Scale

x axis 1cm = 1unit

y axis 1cm = 1unit

Exercise - 3.16

② Draw the graph of $y = x^2 - 4$ and hence Solve $x^2 - x - 12 = 0$

Sol

$$y = x^2 - 4 \longrightarrow ①$$

$$x^2 - x - 12 = 0 \longrightarrow ②$$

From ① $y = x^2 - 4$

x	-3	-2	-1	0	1	2	3	4
x^2	9	4	1	0	1	4	9	16
-4	-4	-4	-4	-4	-4	-4	-4	-4
y	5	0	-3	-4	-3	0	5	12

Plot: $(-3,5)$ $(-2,0)$ $(-1,-3)$ $(0,-4)$ $(1,-3)$, $(2,0)$ $(3,5)$ $(4,12)$

Solve ① & ②

$$y = \cancel{x^2} - 4$$
$$0 = \cancel{x^2} - x - 12$$
$$\quad - \quad + \quad +$$

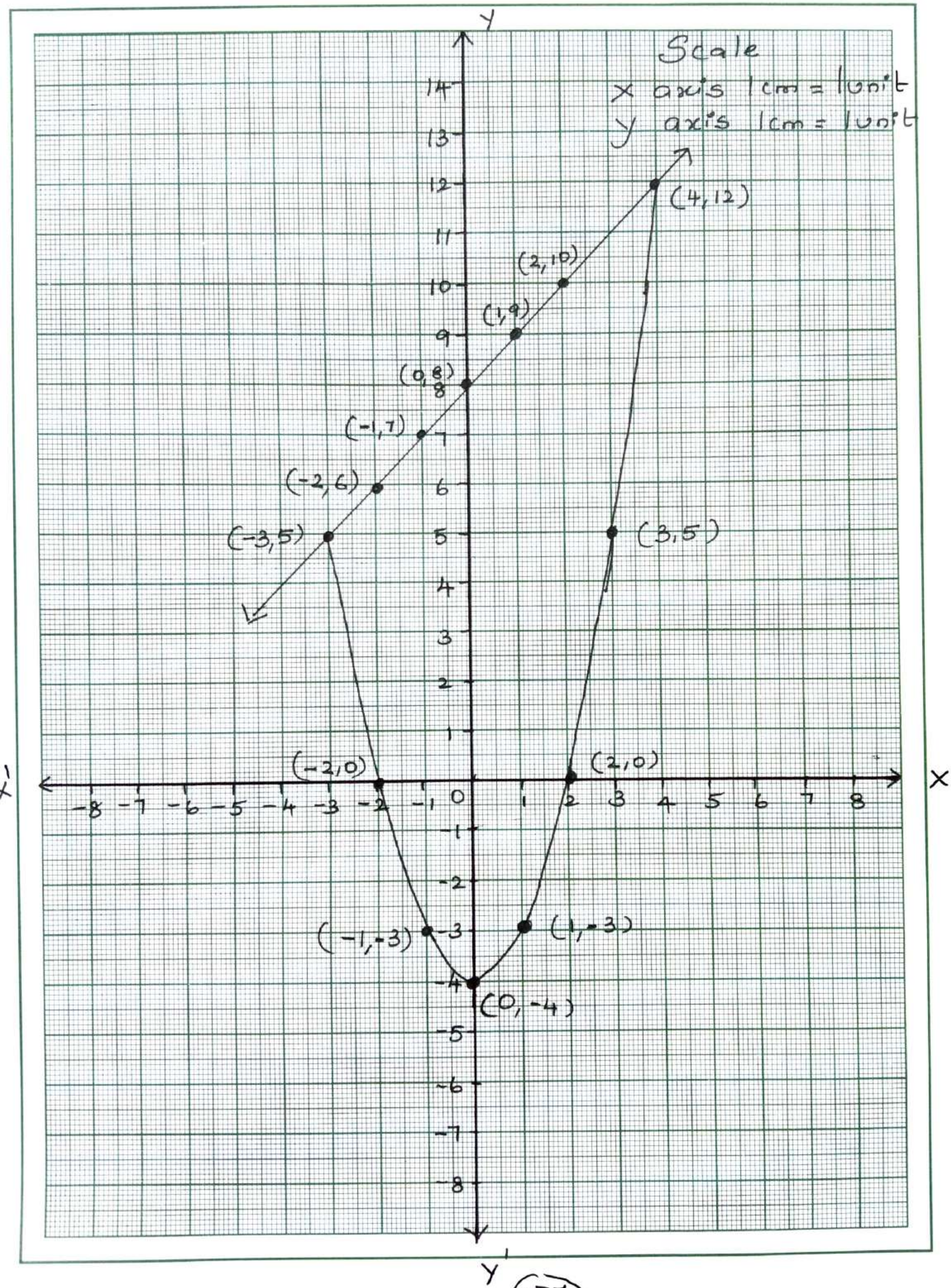
$$\underline{y = x + 8}$$

x	-2	-1	0	1	2
8	8	8	8	8	8
y	6	7	8	9	10

Plot: $(-2,6)$ $(-1,7)$ $(0,8)$ $(1,9)$ $(2,10)$

The line touches the curve at $(-3,5)$ and $(4,12)$

$\therefore$ Solution Set $= \{-3, 4\}$


Y
Scale
x axis 1cm = 1unit
y axis 1cm = 1unit
(4,12)
(2,10)
(1,9)
(0,8)
(-1,7)
(-2,6)
(-3,5)
(3,5)
(-2,0)
(2,0)
(-1,-3)
(1,-3)
(0,-4)
X
X'
Y'
14
13
12
11
10
9
8
7
6
5
4
3
2
1
0
-1
-2
-3
-4
-5
-6
-7
-8
-8 -7 -6 -5 -4 -3 -2 -1
1 2 3 4 5 6 7 8

③ Draw the graph of $y = x^2 + x$ and hence solve $x^2 + 1 = 0$

Sol $y = x^2 + x \longrightarrow$ ①

$x^2 + 1 = 0 \longrightarrow$ ②

From ① $y = x^2 + x$

x	-3	-2	-1	0	1	2
x^2	9	4	1	0	1	4
y	6	2	0	0	2	6

Plot :- $(-3,6)$ $(-2,2)$ $(-1,0)$ $(0,0)$ $(1,2)$ $(2,6)$

Solve ① & ②

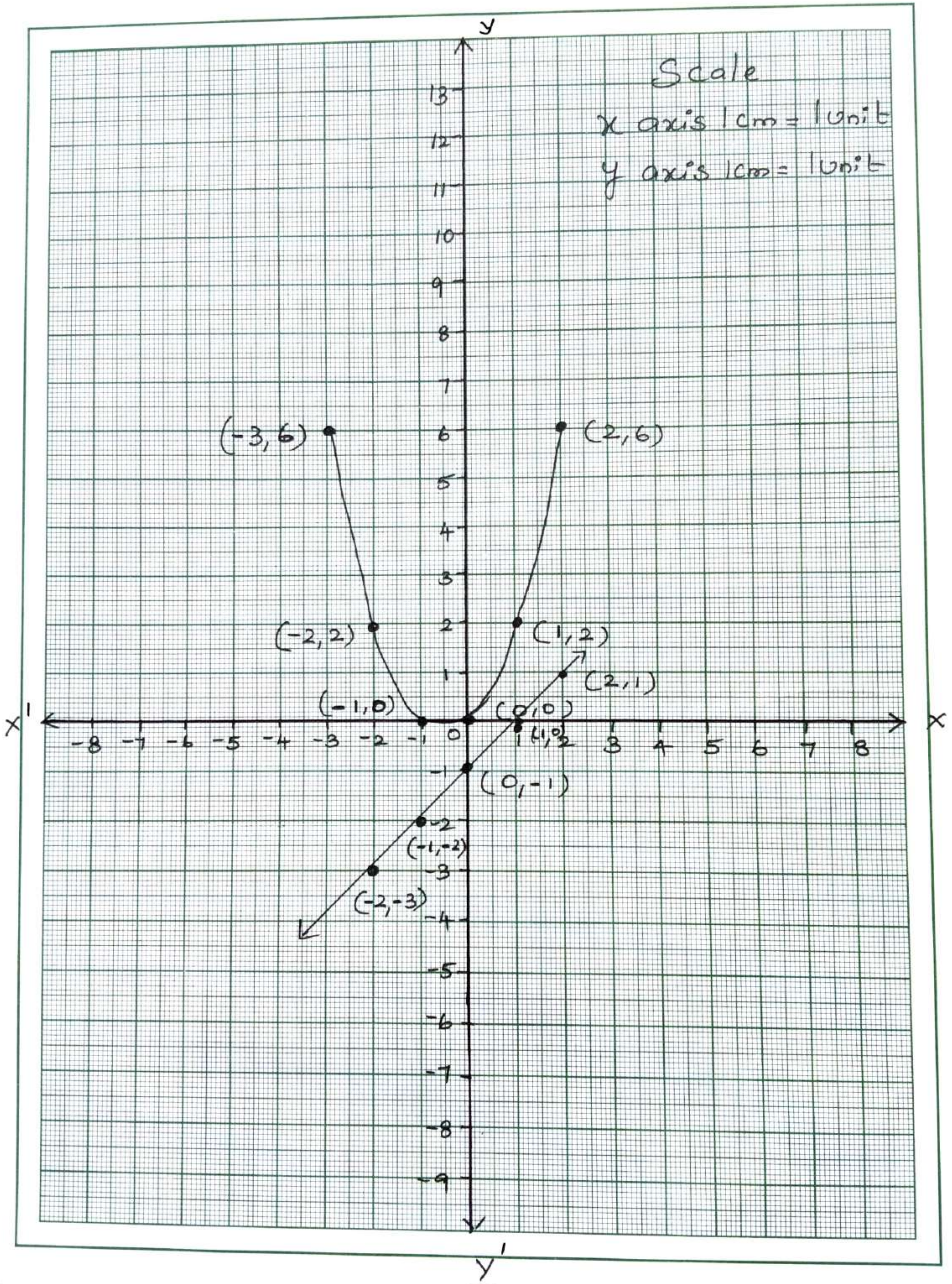
$$y = \cancel{x^2} + x$$
$$0 = \underset{(-)}{\cancel{x^2}} \quad \underset{(-)}{+} 1$$
$$\underline{y = x - 1}$$

x	-2	-1	0	1	2
-1	-1	-1	-1	-1	-1
y	-3	-2	-1	0	1

Plot: $(-2,-3)$ $(-1,-2)$ $(0,-1)$ $(1,0)$ $(2,1)$

The line doesnot touch the curve at any point.

$\therefore$ No Solution Set.


Scale
x axis 1cm = 1unit
y axis 1cm = 1unit
y
y'
x
x'
13
12
11
10
9
8
7
6
5
4
3
2
1
-1
-2
-3
-4
-5
-6
-7
-8
-9
-8
-7
-6
-5
-4
-3
-2
-1
0
1
2
3
4
5
6
7
8
(-3, 6)
(2, 6)
(-2, 2)
(1, 2)
(2, 1)
(-1, 0)
(0, 0)
(1, 0)
(0, -1)
(-1, -2)
(-2, -3)

④ Draw the graph of $y = x^2 + 3x + 2$. and use it to solve $x^2 + 2x + 1 = 0$

Sol :-

$$y = x^2 + 3x + 2 \longrightarrow ①$$

$$x^2 + 2x + 1 = 0 \longrightarrow ②$$

From ①

$$y = x^2 + 3x + 2$$

x	-4	-3	-2	-1	0	1	2
x^2	16	9	4	1	0	1	4
$3x$	-12	-9	-6	-3	0	3	6
2	2	2	2	2	2	2	2
y	6	2	0	0	2	6	12

Plot:- $(-4,6)$ $(-3,2)$ $(-2,0)$ $(-1,0)$ $(0,2)$
$(1,6)$ $(2,12)$

Solve ① & ②

$$y = \cancel{x^2} + 3x + 2$$

$$0 = \cancel{x^2} + 2x + 1$$

$$(-) \qquad (-) \qquad (-)$$

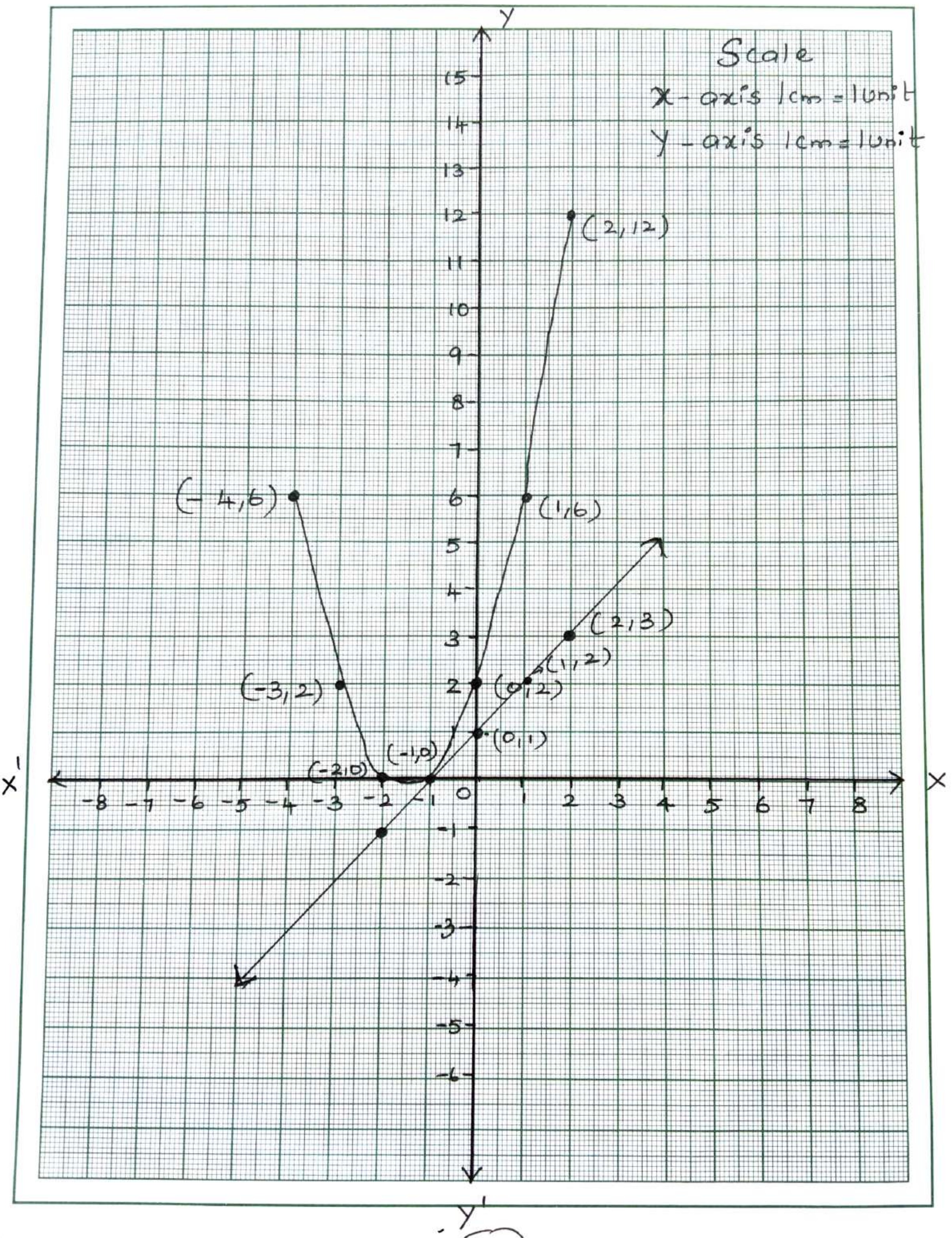
$$\underline{y = x + 1}$$

x	-2	-1	0	1	2
1	1	1	1	1	1
y	-1	0	1	2	3

Plot:- $(-2,-1)$ $(-1,0)$ $(0,1)$ $(1,2)$ $(2,3)$

The line touches the Curve at $(-1, 0)$

$\therefore$ Solution Set = $\{-1\}$


Scale
X - axis 1cm = 1unit
Y - axis 1cm = 1unit
Y
Y'
X
X'
(2,12)
(-4,6)
(1,6)
(2,3)
(1,2)
(-3,2)
(0,2)
(0,1)
(-1,0)
(-2,0)
0
-8
-7
-6
-5
-4
-3
-2
-1
1
2
3
4
5
6
7
8
15
14
13
12
11
10
9
8
7
6
5
4
3
2
-1
-2
-3
-4
-5
-6

⑤ Draw the graph of $y = x^2 + 3x - 4$ and hence use it to solve $x^2 + 3x - 4 = 0$

Sol :-

$$y = x^2 + 3x - 4 \longrightarrow ①$$

$$x^2 + 3x - 4 = 0 \longrightarrow ②$$

From ①

$$y = x^2 + 3x - 4$$

x	-5	-4	-3	-2	-1	0	1	2	3
x^2	25	16	9	4	1	0	1	4	9
$3x$	-15	-12	-9	-6	-3	0	3	6	9
-4	-4	-4	-4	-4	-4	-4	-4	-4	-4
y	6	0	-4	-6	-6	-4	0	6	14

Plot :- $(-5, 6)$ $(-4, 0)$ $(-3, -4)$ $(-2, -6)$, $(-1, -6)$
$(0, -4)$ $(1, 0)$ $(2, 6)$ $(3, 14)$

Solve ① & ②

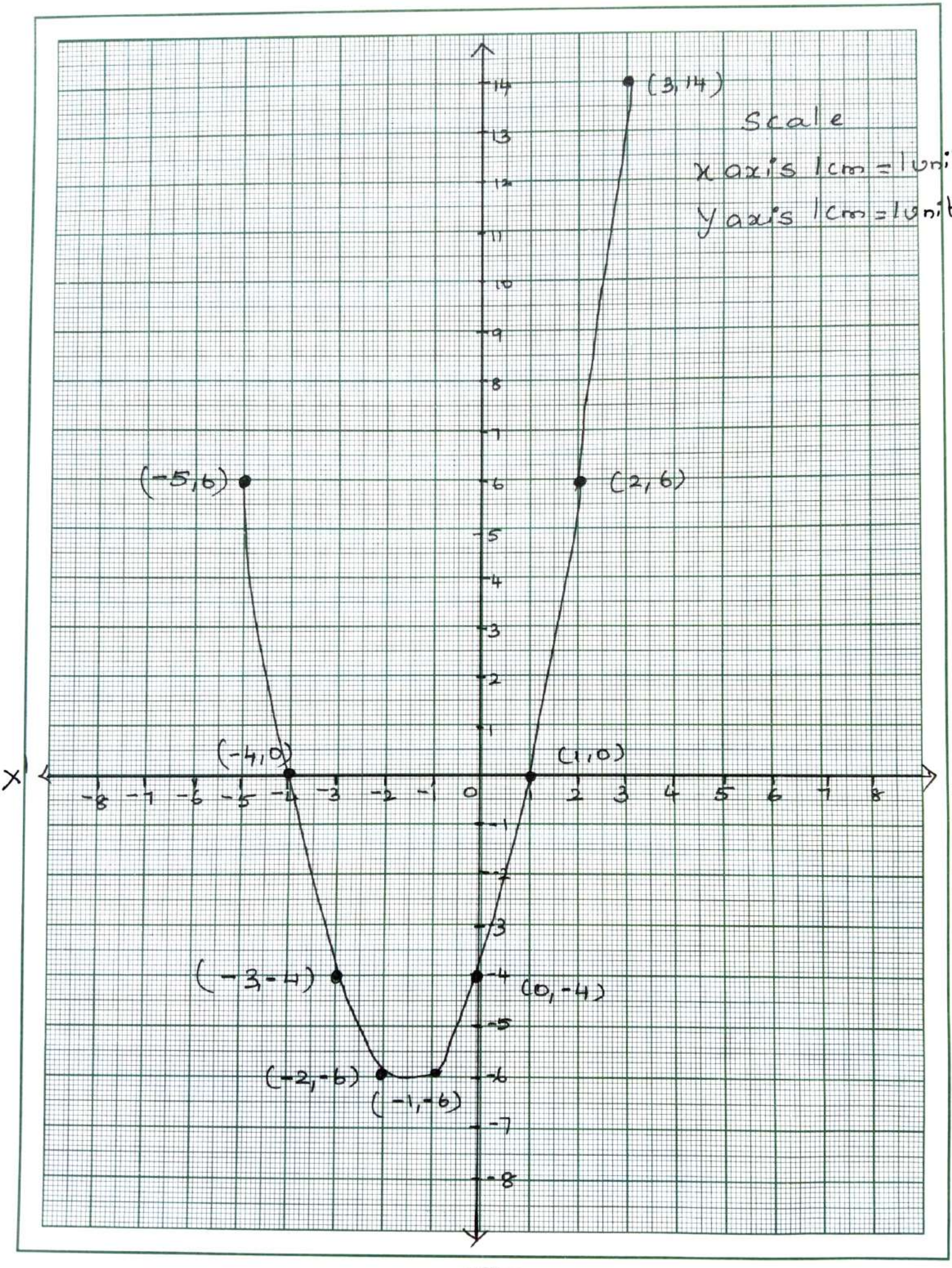
$$y = \cancel{x^2} \cancel{+ 3x} \cancel{- 4}$$

$$\underline{0 = \underset{(-)}{\cancel{x^2}} \underset{(-)}{\cancel{+ 3x}} \underset{(+)}{\cancel{- 4}}}$$

$$\underline{y = 0}$$

∴ The line touches the curve at $(-4, 0)$ & $(1, 0)$

Solution Set = $\{-4, 1\}$



Scale

x axis 1cm = 1unit

y axis 1cm = 1unit

⑥ Draw the graph of $y = x^2 - 5x - 6$ and hence Solve $x^2 - 5x - 14 = 0$

Sol:-

$$y = x^2 - 5x - 6 \rightarrow ①$$

$$x^2 - 5x - 14 = 0 \rightarrow ②$$

From ①

$$y = x^2 - 5x - 6$$

x	-3	-2	-1	0	1	2	3	4	5	6	7
x^2	9	4	1	0	1	4	9	16	25	36	49
$-5x$	15	10	5	0	-5	-10	-15	-20	-25	-30	-35
-6	-6	-6	-6	-6	-6	-6	-6	-6	-6	-6	-6
y	18	8	0	-6	-10	-12	-12	-10	-6	0	8

Plot: (-3,18) (-2, 8) (-1,0) (0,-6) (1,-10) (2,-12)

(3,-12) (4,-10) (5,-6) (6,0) (7,8)

Solve ① & ②

$$y = \cancel{x^2} \cancel{-5x} - 6$$

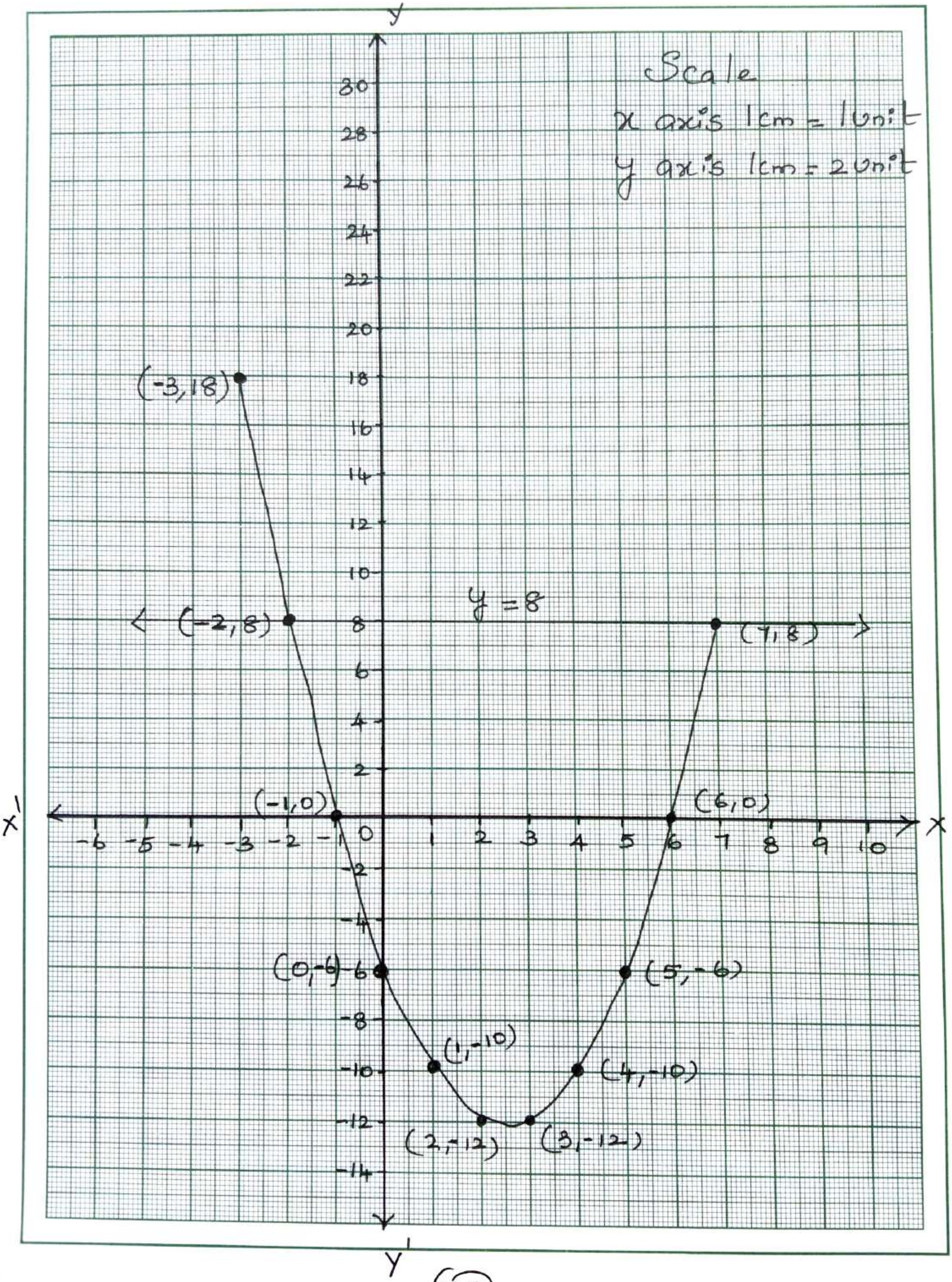
$$0 = \cancel{x^2} - \cancel{5x} - 14$$

$$(-) \quad (+) \quad (+)$$

$$\underline{y = 8}$$

∴ The line touches the curve at $(-2, 8)$ & $(7, 8)$

Solution Set = $\{-2, 7\}$

y
Scale
x axis 1cm = 1unit
y axis 1cm = 2unit
30
28
26
24
22
20
18
16
14
12
10
8
6
4
2
-2
-4
-6
-8
-10
-12
-14
(-3,18)
(-2,8)
y = 8
(7,8)
(-1,0)
(6,0)
X'
X
-6
-5
-4
-3
-2
-1
0
1
2
3
4
5
6
7
8
9
10
(0,-6)
(5,-6)
(1,-10)
(4,-10)
(2,-12)
(3,-12)
Y'

⑦ Draw the graph of $y = 2x^2 - 3x - 5$ and hence Solve $2x^2 - 4x - 6 = 0$

Sol :-

$$y = 2x^2 - 3x - 5 \rightarrow ①$$

$$2x^2 - 4x - 6 = 0 \rightarrow ②$$

From ①

$$y = 2x^2 - 3x - 5$$

x	-2	-1	0	1	2	3	4
x^2	4	1	0	1	4	9	16
$2x^2$	8	2	0	2	8	18	32
$-3x$	6	3	0	-3	-6	-9	-12
-5	-5	-5	-5	-5	-5	-5	-5
y	9	0	-5	-6	-3	4	15

Plot :- (-2, 9) (-1, 0) (0, -5) (1, -6) (2, -3)
(3, 4) (4, 15)

Solve ① & ②

$$y = \cancel{2x^2} - 3x - 5$$
$$0 = \underset{(-)}{\cancel{2x^2}} \underset{(+)}{- 4x} \underset{(+)}{\mp 6}$$

$$\underline{y = x + 1}$$

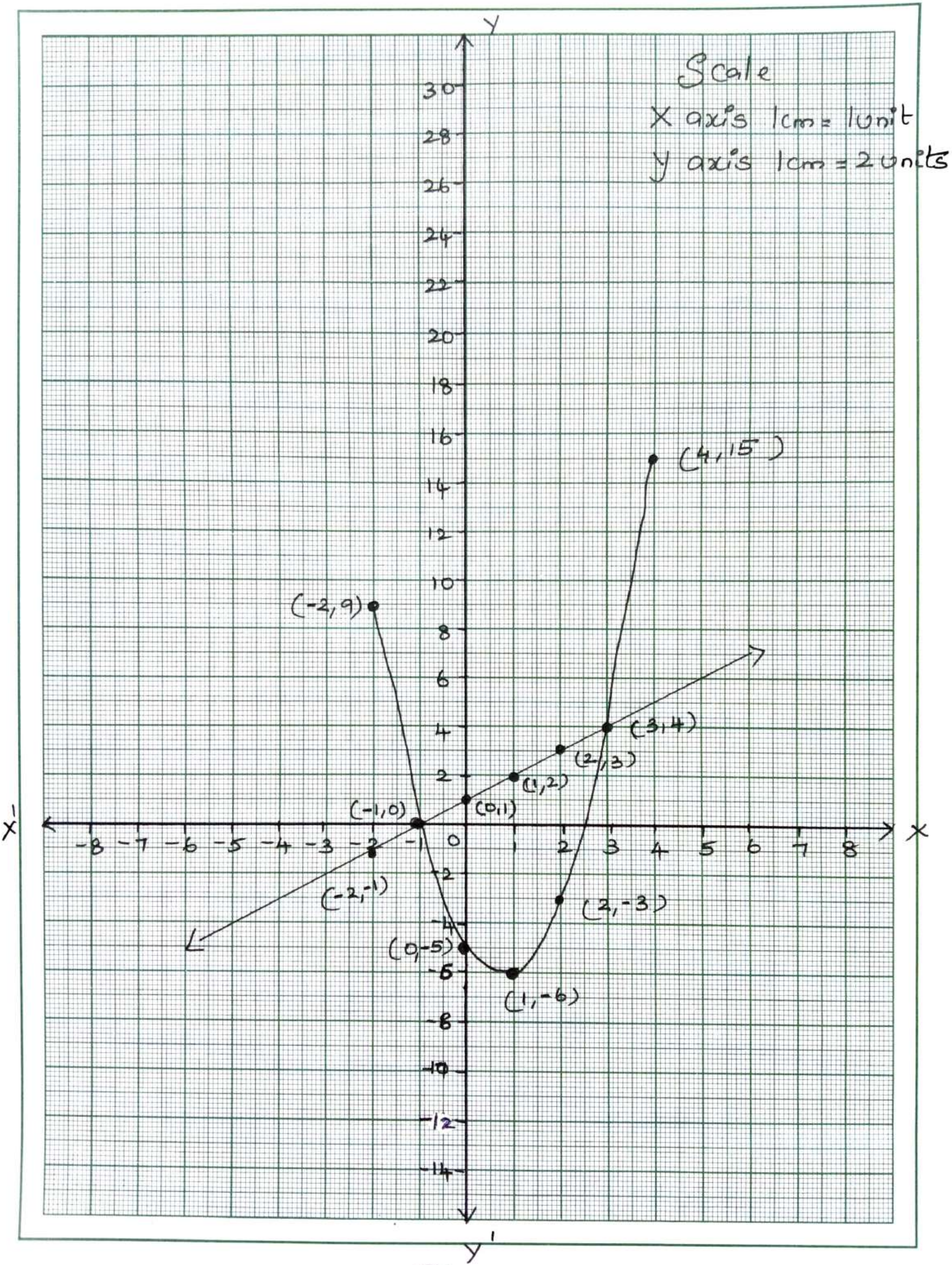
x	-2	-1	0	1	2	3
1	1	1	1	1	1	1
y	-1	0	1	2	3	4

Plot :- $(-2,-1)$ $(-1,0)$ $(0,1)$ $(1,2)$ $(2,3)$ $(3,4)$

The line touches the Curve at

$(-1,0)$ & $(3,4)$

Solution set $= \{-1, 3\}$



Scale
X axis 1cm = 1unit
y axis 1cm = 2 units

⑧ Draw the graph of $y = (x-1)(x+3)$ and hence solve $x^2 - x - 6 = 0$

Sol :-

$$y = (x-1)(x+3)$$

$$= x^2 + 3x - x - 3$$

$$y = x^2 + 2x - 3 \longrightarrow ①$$

$$x^2 - x - 6 = 0 \longrightarrow ②$$

From ①

$$y = x^2 + 2x - 3$$

x	-4	-3	-2	-1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
$2x$	-8	-6	-4	-2	0	2	4	6	8
-3	-3	-3	-3	-3	-3	-3	-3	-3	-3
y	5	0	-3	-4	-3	0	5	12	21

Plot: $(-4, 5)$ $(-3, 0)$ $(-2, -3)$ $(-1, -4)$ $(0, -3)$ $(1, 0)$

$(2, 5)$ $(3, 12)$ $(4, 21)$

Solve ① & ②

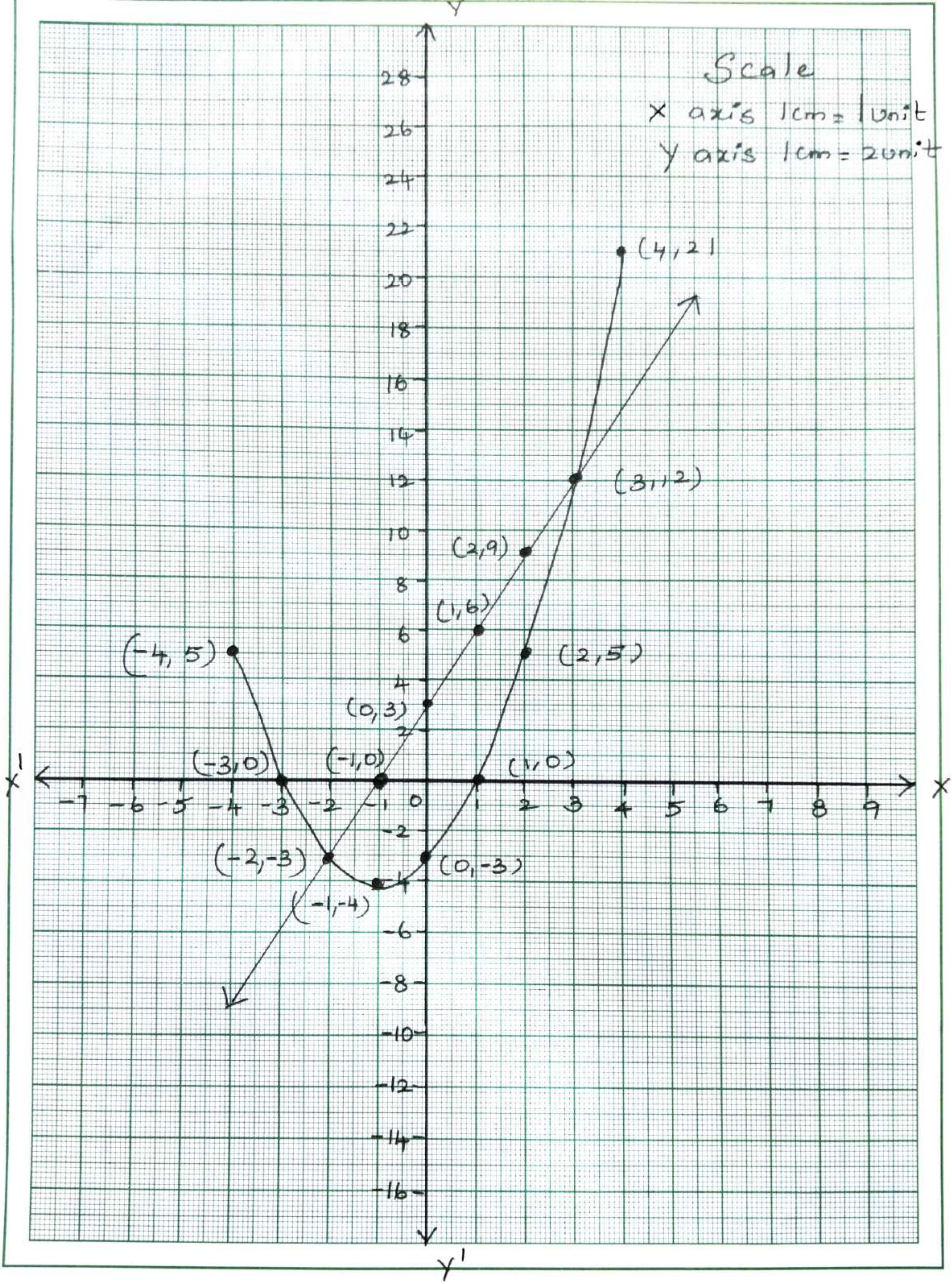
$$y = \cancel{x^2} + 2x - 3$$
$$0 = \underset{(-)}{\cancel{x^2}} \; \underset{(+)}{- x} \; \underset{(+)}{- 6}$$
$$y = 3x + 3$$

x	-2	-1	0	1	2
$3x$	-6	-3	0	3	6
3	3	3	3	3	3
y	-3	0	3	6	9

Plot:- $(-2,-3)$ $(-1,0)$ $(0,3)$ $(1,6)$ $(2,9)$

The line touches the Curve at $(-2,-3)$ &
$(3,12)$

$\therefore$ Solution Set = $\{-2, 3\}$


Scale
x axis 1cm = 1unit
y axis 1cm = 2unit
X
X'
Y'
28
26
24
22
20
18
16
14
12
10
8
6
4
2
-2
-4
-6
-8
-10
-12
-14
-16
-7
-6
-5
-4
-3
-2
-1
0
1
2
3
4
5
6
7
8
9
(4, 21)
(3, 12)
(2, 9)
(1, 6)
(-4, 5)
(2, 5)
(0, 3)
(-3, 0)
(-1, 0)
(1, 0)
(-2, -3)
(0, -3)
(-1, -4)

CHAPTER-3 EXERCISE 3.17 MATRICES

1) In the matrix $A = \begin{bmatrix} 8 & 9 & 4 & 3 \\ -1 & \sqrt{7} & \frac{\sqrt{3}}{2} & 5 \\ 1 & 4 & 3 & 0 \\ 6 & 8 & -11 & 1 \end{bmatrix}$, write

(i) The number of elements

(ii) The order of the matrix

(iii) Write the elements $a_{22}, a_{23}, a_{24}, a_{34}, a_{43}, a_{44}$.

$$A = \begin{bmatrix} 8 & 9 & 4 & 3 \\ -1 & \sqrt{7} & \frac{\sqrt{3}}{2} & 5 \\ 1 & 4 & 3 & 0 \\ 6 & 8 & -11 & 1 \end{bmatrix}$$

(i) The number of elements = 16

(ii) The Order of the matrix = 4×4

(iii) elements:

$a_{22} = \sqrt{7}$

$a_{23} = \frac{\sqrt{3}}{2}$

$a_{24} = 5$

$a_{34} = 0$

$a_{43} = -11$

$a_{44} = 1$

---

2) If a matrix has 18 elements, what are the possible orders it can have? What if it has 6 elements?

18 elements

Possible Orders ⇒

1×18	18×1
2×9	9×2
3×6	6×3

6 elements

Possible Orders $\Rightarrow$

1×6 2×3	6×1 3×2

---

3) Construct a 3×3 matrix whose elements are given by

(i) $a_{ij} = |i-2j|$

3×3 matrix

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$a_{ij} = |i-2j|$

$a_{11} = |1-2(1)| = |1-2| = |-1| = 1$

$a_{12} = |1-2(2)| = |1-4| = |-3| = 3$

$a_{13} = |1-2(3)| = |1-6| = |-5| = 5$

$a_{21} = |2-2(1)| = |2-2| = |0| = 0$

$a_{22} = |2-2(2)| = |2-4| = |-2| = 2$

$a_{23} = |2-2(3)| = |2-6| = |-4| = 4$

$a_{31} = |3-2(1)| = |3-2| = |1| = 1$

$a_{32} = |3-2(2)| = |3-4| = |-1| = 1$

$a_{33} = |3-2(3)| = |3-6| = |-3| = 3$

$$A = \begin{bmatrix} 1 & 3 & 5 \\ 0 & 2 & 4 \\ 1 & 1 & 3 \end{bmatrix}$$

(ii) $a_{ij} = \dfrac{(i+j)^3}{3}$

3×3 Matrix

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$\boxed{a_{ij} = \frac{(i+j)^3}{3}}$$

$$a_{11} = \frac{(1+1)^3}{3} = \frac{(2)^3}{3} = \frac{8}{3}$$

$$a_{12} = \frac{(1+2)^3}{3} = \frac{(3)^3}{3} = \frac{\cancel{27}^{\,9}}{\cancel{3}} = 9$$

$$a_{13} = \frac{(1+3)^3}{3} = \frac{(4)^3}{3} = \frac{64}{3}$$

$$a_{21} = \frac{(2+1)^3}{3} = \frac{(3)^3}{3} = \frac{\cancel{27}^{\,9}}{\cancel{3}} = 9$$

$$a_{22} = \frac{(2+2)^3}{3} = \frac{(4)^3}{3} = \frac{64}{3}$$

$$a_{23} = \frac{(2+3)^3}{3} = \frac{(5)^3}{3} = \frac{125}{3}$$

$$a_{31} = \frac{(3+1)^3}{3} = \frac{(4)^3}{3} = \frac{64}{3}$$

$$a_{32} = \frac{(3+2)^3}{3} = \frac{(5)^3}{3} = \frac{125}{3}$$

$$a_{33} = \frac{(3+3)^3}{3} = \frac{(6)^3}{3} = \frac{\cancel{216}^{\,72}}{\cancel{3}} = 72$$

$$\therefore\ A = \begin{bmatrix} \frac{8}{3} & 9 & \frac{64}{3} \\ 9 & \frac{64}{3} & \frac{125}{3} \\ \frac{64}{3} & \frac{125}{3} & 72 \end{bmatrix}$$

---

4) If $A = \begin{bmatrix} 5 & 4 & 3 \\ 1 & -7 & 9 \\ 3 & 8 & 2 \end{bmatrix}$ then find the transpose of A.

$$A = \begin{bmatrix} 5 & 4 & 3 \\ 1 & -7 & 9 \\ 3 & 8 & 2 \end{bmatrix}$$

Transpose of A

$$A^T = \begin{bmatrix} 5 & 4 & 3 \\ 1 & -7 & 9 \\ 3 & 8 & 2 \end{bmatrix}^T$$

$$A^T = \begin{bmatrix} 5 & 1 & 3 \\ 4 & -7 & 8 \\ 3 & 9 & 2 \end{bmatrix}$$

---

5) If $A = \begin{bmatrix} \sqrt{7} & -3 \\ -\sqrt{5} & 2 \\ \sqrt{3} & -5 \end{bmatrix}$ then find the transpose of $-A$.

$$A = \begin{bmatrix} \sqrt{7} & -3 \\ -\sqrt{5} & 2 \\ \sqrt{3} & -5 \end{bmatrix}$$

$$-A = -\begin{bmatrix} \sqrt{7} & -3 \\ -\sqrt{5} & 2 \\ \sqrt{3} & -5 \end{bmatrix} = \begin{bmatrix} -\sqrt{7} & 3 \\ \sqrt{5} & -2 \\ -\sqrt{3} & 5 \end{bmatrix}$$

$\therefore$ Transpose of $-A$

$$(-A)^T = \begin{pmatrix} -\sqrt{7} & 3 \\ \sqrt{5} & -2 \\ -\sqrt{3} & 5 \end{pmatrix}^T = \begin{pmatrix} -\sqrt{7} & \sqrt{5} & -\sqrt{3} \\ 3 & -2 & 5 \end{pmatrix}$$

6) If $A = \begin{bmatrix} 5 & 2 & 2 \\ -\sqrt{17} & 0.7 & \frac{5}{2} \\ 8 & 3 & 1 \end{bmatrix}$ then verify $(A^T)^T = A$.

$$A = \begin{bmatrix} 5 & 2 & 2 \\ -\sqrt{17} & 0.7 & \frac{5}{2} \\ 8 & 3 & 1 \end{bmatrix}$$

$$\boxed{(A^T)^T = A}$$

$$\left[\begin{bmatrix} 5 & 2 & 2 \\ -\sqrt{17} & 0.7 & \frac{5}{2} \\ 8 & 3 & 1 \end{bmatrix}^T\right]^T = \begin{bmatrix} 5 & 2 & 2 \\ -\sqrt{17} & 0.7 & \frac{5}{2} \\ 8 & 3 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 5 & -\sqrt{17} & 8 \\ 2 & 0.7 & 3 \\ 2 & \frac{5}{2} & 1 \end{bmatrix}^T = \begin{bmatrix} 5 & 2 & 2 \\ -\sqrt{17} & 0.7 & \frac{5}{2} \\ 8 & 3 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 2 & 2 \\ -\sqrt{17} & 0.7 & \frac{5}{2} \\ 8 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 2 & 2 \\ -\sqrt{17} & 0.7 & \frac{5}{2} \\ 8 & 3 & 1 \end{bmatrix}$$

L.H.S = R.H.S

---

7) Find the values of x, y, z from the following equations.

(i) $\begin{pmatrix} 12 & 3 \\ x & 5 \end{pmatrix} = \begin{pmatrix} y & z \\ 3 & 5 \end{pmatrix}$

$12 = y$

$\boxed{y = 12}$

$3 = z$

$\boxed{z = 3}$

$\boxed{x = 3}$

(ii) $\begin{pmatrix} x+y & 2 \\ 5+z & xy \end{pmatrix} = \begin{pmatrix} 6 & 2 \\ 5 & 8 \end{pmatrix}$

$x + y = 6$

$\boxed{y = 6 - x}$ $\to$ ①

when $x = 2$

① $\Rightarrow$ $y = 6 - 2$

$\boxed{y = 4}$

when $x = 4$

① $\Rightarrow$ $y = 6 - 4$

$\boxed{y = 2}$

$5 + z = 5$

$z = 5 - 5$

$\boxed{z = 0}$

$xy = 8$

$x(6 - x) = 8$ (by ①)

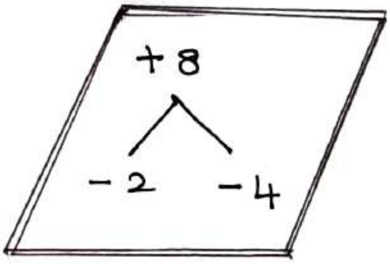
$6x - x^2 = 8$

$x^2 - 6x + 8 = 0$

$(x - 2)(x - 4) = 0$

$x - 2 = 0$ $x - 4 = 0$

$\boxed{x = 2}$ $\boxed{x = 4}$



$\Rightarrow$ $\boxed{x, y, z \Rightarrow 2, 4, 0}$

(or)

$\boxed{x, y, z \Rightarrow 4, 2, 0}$

(iii) $\begin{pmatrix} x+y+z \\ x+z \\ y+z \end{pmatrix} = \begin{pmatrix} 9 \\ 5 \\ 7 \end{pmatrix}$

$x + y + z = 9$

$5 + y = 9$

$y = 9 - 5$

$\boxed{y = 4}$

$x + z = 5$

$x + 3 = 5$

$x = 5 - 3$

$\boxed{x = 2}$

$y + z = 7$

$4 + z = 7$

$z = 7 - 4$

$\boxed{z = 3}$

EXERCISE 3.18

1) If $A = \begin{bmatrix} 1 & 9 \\ 3 & 4 \\ 8 & -3 \end{bmatrix}$, $B = \begin{bmatrix} 5 & 7 \\ 3 & 3 \\ 1 & 0 \end{bmatrix}$ then verify that

(i) $A+B = B+A$ (ii) $A+(-A) = (-A)+A = 0$

$$A = \begin{bmatrix} 1 & 9 \\ 3 & 4 \\ 8 & -3 \end{bmatrix} \quad B = \begin{bmatrix} 5 & 7 \\ 3 & 3 \\ 1 & 0 \end{bmatrix}$$

(i) $A+B = B+A$

$$\begin{pmatrix} 1 & 9 \\ 3 & 4 \\ 8 & -3 \end{pmatrix} + \begin{pmatrix} 5 & 7 \\ 3 & 3 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 5 & 7 \\ 3 & 3 \\ 1 & 0 \end{pmatrix} + \begin{pmatrix} 1 & 9 \\ 3 & 4 \\ 8 & -3 \end{pmatrix}$$

$$\begin{pmatrix} 6 & 16 \\ 6 & 7 \\ 9 & -3 \end{pmatrix} = \begin{pmatrix} 6 & 16 \\ 6 & 7 \\ 9 & -3 \end{pmatrix}$$

L.H.S = R.H.S

(ii) $\boxed{A+(-A) = (-A)+A = 0}$

$$\begin{pmatrix} 1 & 9 \\ 3 & 4 \\ 8 & -3 \end{pmatrix} + \begin{pmatrix} -1 & -9 \\ -3 & -4 \\ -8 & 3 \end{pmatrix} = \begin{pmatrix} -1 & -9 \\ -3 & -4 \\ -8 & 3 \end{pmatrix} + \begin{pmatrix} 1 & 9 \\ 3 & 4 \\ 8 & -3 \end{pmatrix} = 0$$

$$\begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} = 0$$

$$0 = 0 = 0$$

Hence Proved.

2) If $A = \begin{bmatrix} 4 & 3 & 1 \\ 2 & 3 & -8 \\ 1 & 0 & -4 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 9 & 2 \\ -7 & 1 & -1 \end{bmatrix}$ and

$C = \begin{bmatrix} 8 & 3 & 4 \\ 1 & -2 & 3 \\ 2 & 4 & -1 \end{bmatrix}$ then verify that

$A + (B + C) = (A + B) + C$.

$$A = \begin{bmatrix} 4 & 3 & 1 \\ 2 & 3 & -8 \\ 1 & 0 & -4 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 9 & 2 \\ -7 & 1 & -1 \end{bmatrix} \quad C = \begin{bmatrix} 8 & 3 & 4 \\ 1 & -2 & 3 \\ 2 & 4 & -1 \end{bmatrix}$$

$$\boxed{A + (B + C) = (A + B) + C}$$

L.H.S

$$A + (B + C)$$

$$= \begin{pmatrix} 4 & 3 & 1 \\ 2 & 3 & -8 \\ 1 & 0 & -4 \end{pmatrix} + \left[\begin{pmatrix} 2 & 3 & 4 \\ 1 & 9 & 2 \\ -7 & 1 & -1 \end{pmatrix} + \begin{pmatrix} 8 & 3 & 4 \\ 1 & -2 & 3 \\ 2 & 4 & -1 \end{pmatrix} \right]$$

$$= \begin{pmatrix} 4 & 3 & 1 \\ 2 & 3 & -8 \\ 1 & 0 & -4 \end{pmatrix} + \begin{pmatrix} 10 & 6 & 8 \\ 2 & 7 & 5 \\ -5 & 5 & -2 \end{pmatrix}$$

$$= \begin{pmatrix} 14 & 9 & 9 \\ 4 & 10 & -3 \\ -4 & 5 & -6 \end{pmatrix} \checkmark$$

R.H.S $(A + B) + C$

$$= \left[\begin{pmatrix} 4 & 3 & 1 \\ 2 & 3 & -8 \\ 1 & 0 & -4 \end{pmatrix} + \begin{pmatrix} 2 & 3 & 4 \\ 1 & 9 & 2 \\ -7 & 1 & -1 \end{pmatrix} \right] + \begin{pmatrix} 8 & 3 & 4 \\ 1 & -2 & 3 \\ 2 & 4 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 6 & 6 & 5 \\ 3 & 12 & -6 \\ -6 & 1 & -5 \end{pmatrix} + \begin{pmatrix} 8 & 3 & 4 \\ 1 & -2 & 3 \\ 2 & 4 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 14 & 9 & 9 \\ 4 & 10 & -3 \\ -4 & 5 & -6 \end{pmatrix} \checkmark$$

$$\boxed{L.H.S = R.H.S}$$

---

3) Find X and Y if $X+Y = \begin{pmatrix} 7 & 0 \\ 3 & 5 \end{pmatrix}$ and $X-Y = \begin{pmatrix} 3 & 0 \\ 0 & 4 \end{pmatrix}$

$$X+\cancel{Y} = \begin{pmatrix} 7 & 0 \\ 3 & 5 \end{pmatrix} \rightarrow ①$$

$$X\cancel{-Y} = \begin{pmatrix} 3 & 0 \\ 0 & 4 \end{pmatrix} \rightarrow ②$$

$$2X = \begin{pmatrix} 10 & 0 \\ 3 & 9 \end{pmatrix}$$

$$X = \begin{pmatrix} \cancel{\frac{10}{2}}^{5} & \frac{0}{2} \\ \frac{3}{2} & \frac{9}{2} \end{pmatrix}$$

$$\boxed{X = \begin{pmatrix} 5 & 0/2 \\ 3/2 & 9/2 \end{pmatrix}}$$

$$\cancel{X}+Y = \begin{pmatrix} 7 & 0 \\ 3 & 5 \end{pmatrix} \rightarrow ①$$

$$X-Y = \begin{pmatrix} 3 & 0 \\ 0 & 4 \end{pmatrix} \rightarrow ②$$

$$(-) \quad (+) \quad (-)$$

$$2Y = \begin{pmatrix} 4 & 0 \\ 3 & 1 \end{pmatrix}$$

$$Y = \begin{pmatrix} \cancel{\frac{4}{2}}^{2} & \frac{0}{2} \\ \frac{3}{2} & \frac{1}{2} \end{pmatrix}$$

$$\boxed{Y = \begin{pmatrix} 2 & 0 \\ 3/2 & 1/2 \end{pmatrix}}$$

---

4) If $A = \begin{pmatrix} 0 & 4 & 9 \\ 8 & 3 & 7 \end{pmatrix}$, $B = \begin{pmatrix} 7 & 3 & 8 \\ 1 & 4 & 9 \end{pmatrix}$, find the value of (i) $B-5A$ (ii) $3A-9B$

$$A = \begin{pmatrix} 0 & 4 & 9 \\ 8 & 3 & 7 \end{pmatrix} \quad B = \begin{pmatrix} 7 & 3 & 8 \\ 1 & 4 & 9 \end{pmatrix}$$

(i) $\boxed{B - 5A}$

$$= \begin{pmatrix} 7 & 3 & 8 \\ 1 & 4 & 9 \end{pmatrix} - 5\begin{pmatrix} 0 & 4 & 9 \\ 8 & 3 & 7 \end{pmatrix}$$

$$= \begin{pmatrix} 7 & 3 & 8 \\ 1 & 4 & 9 \end{pmatrix} - \begin{pmatrix} 0 & 20 & 45 \\ 40 & 15 & 35 \end{pmatrix}$$

$$= \begin{pmatrix} 7 & -17 & -37 \\ -39 & -11 & -26 \end{pmatrix}$$

$7 - 0 = 7$
$3 - 20 = -17$
$8 - 45 = -37$
$1 - 40 = -39$
$4 - 15 = -11$
$9 - 35 = -26$

(ii) $\boxed{3A - 9B}$

$$= 3\begin{pmatrix} 0 & 4 & 9 \\ 8 & 3 & 7 \end{pmatrix} - 9\begin{pmatrix} 7 & 3 & 8 \\ 1 & 4 & 9 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 12 & 27 \\ 24 & 9 & 21 \end{pmatrix} - \begin{pmatrix} 63 & 27 & 72 \\ 9 & 36 & 81 \end{pmatrix}$$

$$= \begin{pmatrix} -63 & -15 & -45 \\ 15 & -27 & -60 \end{pmatrix}$$

$0 - 63 = -63$
$12 - 27 = -15$
$27 - 72 = -45$
$24 - 9 = 15$
$9 - 36 = -27$
$21 - 81 = -60$

---

5) Find the value of x, y, z if

(i) $\begin{pmatrix} x-3 & 3x-z \\ x+y+7 & x+y+z \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 6 \end{pmatrix}$

$x - 3 = 1$
$x = 1 + 3$
$\boxed{x = 4}$

$3x - z = 0$
$3(4) - z = 0$
$12 - z = 0$
$\cancel{-}z = \cancel{-}12$
$\boxed{z = 12}$

$x + y + z = 6$
$4 + y + 12 = 6$
$y + 16 = 6$
$y = 6 - 16$
$\boxed{y = -10}$

(ii) $\begin{pmatrix} x & y-z & z+3 \end{pmatrix} + \begin{pmatrix} y & 4 & 3 \end{pmatrix} = \begin{pmatrix} 4 & 8 & 16 \end{pmatrix}$

$\begin{pmatrix} x+y & y-z+4 & z+3+3 \end{pmatrix} = \begin{pmatrix} 4 & 8 & 16 \end{pmatrix}$

$\begin{pmatrix} x+y & y-z+4 & z+6 \end{pmatrix} = \begin{pmatrix} 4 & 8 & 16 \end{pmatrix}$

$z+6=16$

$z=16-6$

$\boxed{z=10}$

$y-z+4=8$

$y-10+4=8$

$y-6=8$

$y=8+6$

$\boxed{y=14}$

$x+y=4$

$x+14=4$

$x=4-14$

$\boxed{x=-10}$

---

6) Find x and y if $x\begin{pmatrix} 4 \\ -3 \end{pmatrix} + y\begin{pmatrix} -2 \\ 3 \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \end{pmatrix}$

$x\begin{pmatrix} 4 \\ -3 \end{pmatrix} + y\begin{pmatrix} -2 \\ 3 \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \end{pmatrix}$

$\begin{pmatrix} 4x \\ -3x \end{pmatrix} + \begin{pmatrix} -2y \\ 3y \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \end{pmatrix}$

$\begin{pmatrix} 4x-2y \\ -3x+3y \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \end{pmatrix}$

$4x-2y=4 \longrightarrow$ ① $\times 3$

$-3x+3y=6 \longrightarrow$ ② $\times 4$

$$\begin{array}{r} \cancel{12x}-6y=12 \\ -\cancel{12x}+12y=24 \\ \hline 6y=36 \end{array}$$

$$y = \frac{\cancel{36}\,6}{\cancel{6}}$$

$$\boxed{y = 6}$$

Put $y = 6$ in ①

$$4x - 2y = 4$$

$$4x - 2(6) = 4$$

$$4x - 12 = 4$$

$$4x = 4 + 12$$

$$4x = 16$$

$$x = \frac{\cancel{16}\,4}{\cancel{4}}$$

$$\boxed{x = 4}$$

---

7) Find the non-zero values of x, satisfying the matrix equation.

$$x\begin{pmatrix} 2x & 2 \\ 3 & x \end{pmatrix} + 2\begin{pmatrix} 8 & 5x \\ 4 & 4x \end{pmatrix} = 2\begin{pmatrix} x^2+8 & 24 \\ 10 & 6x \end{pmatrix}$$

$$\begin{pmatrix} 2x^2 & 2x \\ 3x & x^2 \end{pmatrix} + \begin{pmatrix} 16 & 10x \\ 8 & 8x \end{pmatrix} = \begin{pmatrix} 2x^2+16 & 48 \\ 20 & 12x \end{pmatrix}$$

$$\begin{pmatrix} 2x^2+16 & \overset{\checkmark}{12x} \\ 3x+8 & x^2+8x \end{pmatrix} = \begin{pmatrix} 2x^2+16 & \overset{\checkmark}{48} \\ 20 & 12x \end{pmatrix}$$

$$12x = 48$$

$$x = \frac{\cancel{48}\,4}{\cancel{12}} \Rightarrow \boxed{x = 4}$$

8) Solve for x, y:

$$\begin{pmatrix} x^2 \\ y^2 \end{pmatrix} + 2\begin{pmatrix} -2x \\ -y \end{pmatrix} = \begin{pmatrix} 5 \\ 8 \end{pmatrix}$$

$$\begin{pmatrix} x^2 \\ y^2 \end{pmatrix} + \begin{pmatrix} -4x \\ -2y \end{pmatrix} = \begin{pmatrix} 5 \\ 8 \end{pmatrix}$$

$$\begin{pmatrix} x^2-4x \\ y^2-2y \end{pmatrix} = \begin{pmatrix} 5 \\ 8 \end{pmatrix}$$

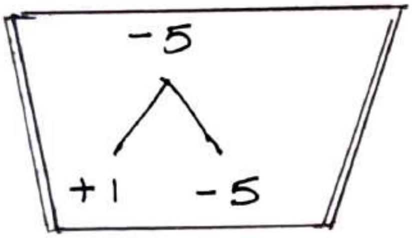
$x^2-4x=5 \longrightarrow$ ①

$\boxed{x^2-4x-5=0}$

$(x+1)(x-5)=0$

$x+1=0$ $\quad$ $x-5=0$

$x=-1$ $\quad$ $x=5$



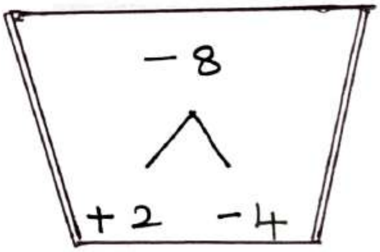
$y^2-2y=8 \longrightarrow$ ②

$\boxed{y^2-2y-8=0}$

$(y+2)(y-4)=0$

$y+2=0$ $\quad$ $y-4=0$

$y=-2$ $\quad$ $y=4$



EXERCISE 3.19

1) Find the order of the product matrix AB if

	(i)	(ii)	(iii)	(iv)	(v)
Order of A	3×3	4×3	4×2	4×5	1×1
order of B	3×3	3×2	2×2	5×1	1×3

(i) A B

3×3 3×3

Same

⇒ Order of AB = 3×3

(ii) A B

4×3 3×2

Same

⇒ Order of AB = 4×2

(iii) A B

4×2 2×2

Same

⇒ Order of AB = 4×2

(iv) A B

4×5 5×1

Same

⇒ Order of AB = 4×1

(v) A B

$\boxed{1\times 1}$ $\boxed{1\times 3}$

Same

$\Rightarrow$ Order of $AB = 1\times 3$

---

2) If A is of order $p\times q$ and B is of order $q\times r$, what is the order of AB and BA?

A B

$\boxed{p\times q}$ $\boxed{q\times r}$

same

$\Rightarrow$ Order of AB

$p\times r$

B A

$\boxed{q\times r}$ $\boxed{p\times q}$

not same

$\Rightarrow$ order of BA is not defined.

---

3) A has 'a' rows and 'a+3' columns. B has 'b' rows and '17-b' columns, and if both products AB and BA exists, find a and b

A B

$\boxed{a \quad a+3}$ $\boxed{b \quad 17-b}$

Same

$\Rightarrow a+3=b$

$a-b=3 \rightarrow ①$

B A

$\boxed{b \quad 17-b}$ $\boxed{a \quad a+3}$

Same

$\Rightarrow 17-b=a$

$a+b=17 \rightarrow ②$

Solving ① & ②

$$\begin{array}{r} a-\cancel{b}=3 \rightarrow ① \\ a+\cancel{b}=17 \rightarrow ② \\ \hline 2a \quad =20 \end{array}$$

$$a = \frac{\cancel{20}\,10}{\cancel{2}}$$

$$\boxed{a = 10}$$

Put $a = 10$ in ②

$$a + b = 17$$

$$10 + b = 17$$

$$b = 17 - 10$$

$$\boxed{b = 7}$$

---

4) If $A = \begin{pmatrix} 2 & 5 \\ 4 & 3 \end{pmatrix}$, $B = \begin{pmatrix} 1 & -3 \\ 2 & 5 \end{pmatrix}$ Find AB, BA and verify AB = BA?

$$A = \begin{pmatrix} 2 & 5 \\ 4 & 3 \end{pmatrix} \quad B = \begin{pmatrix} 1 & -3 \\ 2 & 5 \end{pmatrix}$$

$\boxed{AB}$

$$= \begin{pmatrix} 2 & 5 \\ 4 & 3 \end{pmatrix} \begin{pmatrix} 1 & -3 \\ 2 & 5 \end{pmatrix}$$

$$= \begin{pmatrix} 2+10 & -6+25 \\ 4+6 & -12+15 \end{pmatrix}$$

$$= \begin{pmatrix} 12 & 19 \\ 10 & 3 \end{pmatrix}$$

$\boxed{BA}$

$$= \begin{pmatrix} 1 & -3 \\ 2 & 5 \end{pmatrix} \begin{pmatrix} 2 & 5 \\ 4 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 2-12 & 5-9 \\ 4+20 & 10+15 \end{pmatrix}$$

$$= \begin{pmatrix} -10 & -4 \\ 24 & 25 \end{pmatrix}$$

$$\boxed{AB \neq BA}$$

---

5) Given that $A = \begin{pmatrix} 1 & 3 \\ 5 & -1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & -1 & 2 \\ 3 & 5 & 2 \end{pmatrix}$, $C = \begin{pmatrix} 1 & 3 & 2 \\ -4 & 1 & 3 \end{pmatrix}$ verify $A(B+C) = AB + AC$.

$A = \begin{pmatrix} 1 & 3 \\ 5 & -1 \end{pmatrix} \quad B = \begin{pmatrix} 1 & -1 & 2 \\ 3 & 5 & 2 \end{pmatrix} \quad C = \begin{pmatrix} 1 & 3 & 2 \\ -4 & 1 & 3 \end{pmatrix}$

Verify $\Rightarrow$ $A(B+C) = AB + AC$

L.H.S

$$\boxed{A(B+C)}$$

$$= \begin{pmatrix} 1 & 3 \\ 5 & -1 \end{pmatrix} \left[\begin{pmatrix} 1 & -1 & 2 \\ 3 & 5 & 2 \end{pmatrix} + \begin{pmatrix} 1 & 3 & 2 \\ -4 & 1 & 3 \end{pmatrix} \right]$$

$$= \begin{pmatrix} 1 & 3 \\ 5 & -1 \end{pmatrix} \begin{bmatrix} 2 & 2 & 4 \\ -1 & 6 & 5 \end{bmatrix}$$

$$= \begin{pmatrix} 2-3 & 2+18 & 4+15 \\ 10+1 & 10-6 & 20-5 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & 20 & 19 \\ 11 & 4 & 15 \end{pmatrix} \checkmark$$

R.H.S

$$\boxed{AB + AC}$$

$$= \begin{pmatrix} 1 & 3 \\ 5 & -1 \end{pmatrix} \begin{pmatrix} 1 & -1 & 2 \\ 3 & 5 & 2 \end{pmatrix} + \begin{pmatrix} 1 & 3 \\ 5 & -1 \end{pmatrix} \begin{pmatrix} 1 & 3 & 2 \\ -4 & 1 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 1+9 & -1+15 & 2+6 \\ 5-3 & -5-5 & 10-2 \end{pmatrix} + \begin{pmatrix} 1-12 & 3+3 & 2+9 \\ 5+4 & 15-1 & 10-3 \end{pmatrix}$$

$$= \begin{pmatrix} 10 & 14 & 8 \\ 2 & -10 & 8 \end{pmatrix} + \begin{pmatrix} -11 & 6 & 11 \\ 9 & 14 & 7 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & 20 & 19 \\ 11 & 4 & 15 \end{pmatrix} \checkmark$$

L.H.S = R.H.S

6) Show that the matrices $A = \begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & -2 \\ -3 & 1 \end{pmatrix}$ verify commutative property $AB = BA$.

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & -2 \\ -3 & 1 \end{pmatrix}$$

$$\boxed{AB = BA}$$

$$\begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix}\begin{pmatrix} 1 & -2 \\ -3 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -2 \\ -3 & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1-6 & -2+2 \\ 3-3 & -6+1 \end{pmatrix} = \begin{pmatrix} 1-6 & 2-2 \\ -3+3 & -6+1 \end{pmatrix}$$

$$\begin{pmatrix} -5 & 0 \\ 0 & -5 \end{pmatrix} = \begin{pmatrix} -5 & 0 \\ 0 & -5 \end{pmatrix}$$

$$\boxed{L.H.S = R.H.S}$$

7) Let $A = \begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix}$, $B = \begin{pmatrix} 4 & 0 \\ 1 & 5 \end{pmatrix}$, $C = \begin{pmatrix} 2 & 0 \\ 1 & 2 \end{pmatrix}$ show that (i) $A(BC) = (AB)C$ (ii) $(A-B)C = AC - BC$ (iii) $(A-B)^T = A^T - B^T$.

$$A = \begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix} \quad B = \begin{pmatrix} 4 & 0 \\ 1 & 5 \end{pmatrix} \quad C = \begin{pmatrix} 2 & 0 \\ 1 & 2 \end{pmatrix}$$

(i) $A(BC) = (AB)C$

$$\begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix}\left[\begin{pmatrix} 4 & 0 \\ 1 & 5 \end{pmatrix}\begin{pmatrix} 2 & 0 \\ 1 & 2 \end{pmatrix}\right] = \left[\begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} 4 & 0 \\ 1 & 5 \end{pmatrix}\right]\begin{pmatrix} 2 & 0 \\ 1 & 2 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} 8+0 & 0+0 \\ 2+5 & 0+10 \end{pmatrix} = \begin{pmatrix} 4+2 & 0+10 \\ 4+3 & 0+15 \end{pmatrix}\begin{pmatrix} 2 & 0 \\ 1 & 2 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} 8 & 0 \\ 7 & 10 \end{pmatrix} = \begin{pmatrix} 6 & 10 \\ 7 & 15 \end{pmatrix}\begin{pmatrix} 2 & 0 \\ 1 & 2 \end{pmatrix}$$

$$\begin{pmatrix} 8+14 & 0+20 \\ 8+21 & 0+30 \end{pmatrix} = \begin{pmatrix} 12+10 & 0+20 \\ 14+15 & 0+30 \end{pmatrix}$$

$$\begin{pmatrix} 22 & 20 \\ 29 & 30 \end{pmatrix} = \begin{pmatrix} 22 & 20 \\ 29 & 30 \end{pmatrix}$$

L.H.S = R.H.S

(ii) $(A-B)\,C = AC - BC$

$$\left[\begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix} - \begin{pmatrix} 4 & 0 \\ 1 & 5 \end{pmatrix}\right]\begin{pmatrix} 2 & 0 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} 2 & 0 \\ 1 & 2 \end{pmatrix} - \begin{pmatrix} 4 & 0 \\ 1 & 5 \end{pmatrix}\begin{pmatrix} 2 & 0 \\ 1 & 2 \end{pmatrix}$$

$$\begin{pmatrix} -3 & 2 \\ 0 & -2 \end{pmatrix}\begin{pmatrix} 2 & 0 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 2+2 & 0+4 \\ 2+3 & 0+6 \end{pmatrix} - \begin{pmatrix} 8+0 & 0+0 \\ 2+5 & 0+10 \end{pmatrix}$$

$$\begin{pmatrix} -6+2 & 0+4 \\ 0-2 & 0-4 \end{pmatrix} = \begin{pmatrix} 4 & 4 \\ 5 & 6 \end{pmatrix} - \begin{pmatrix} 8 & 0 \\ 7 & 10 \end{pmatrix}$$

$$\begin{pmatrix} -4 & 4 \\ -2 & -4 \end{pmatrix} = \begin{pmatrix} -4 & 4 \\ -2 & -4 \end{pmatrix}$$

L.H.S = R.H.S

(iii) $(A-B)^T = A^T - B^T$

$$\left[\begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix} - \begin{pmatrix} 4 & 0 \\ 1 & 5 \end{pmatrix}\right]^T = \begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix}^T - \begin{pmatrix} 4 & 0 \\ 1 & 5 \end{pmatrix}^T$$

$$\begin{pmatrix} -3 & 2 \\ 0 & -2 \end{pmatrix}^T = \begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix} - \begin{pmatrix} 4 & 1 \\ 0 & 5 \end{pmatrix}$$

$$\begin{pmatrix} -3 & 0 \\ 2 & -2 \end{pmatrix} = \begin{pmatrix} -3 & 0 \\ 2 & -2 \end{pmatrix}$$

L.H.S = R.H.S

8) If $A = \begin{pmatrix} \cos\theta & 0 \\ 0 & \cos\theta \end{pmatrix}, B = \begin{pmatrix} \sin\theta & 0 \\ 0 & \sin\theta \end{pmatrix}$ then,

show that $A^2 + B^2 = I$

$$A = \begin{pmatrix} \cos\theta & 0 \\ 0 & \cos\theta \end{pmatrix} \quad B = \begin{pmatrix} \sin\theta & 0 \\ 0 & \sin\theta \end{pmatrix}$$

$$\boxed{A^2 + B^2 = I}$$

$$\begin{pmatrix} \cos\theta & 0 \\ 0 & \cos\theta \end{pmatrix}\begin{pmatrix} \cos\theta & 0 \\ 0 & \cos\theta \end{pmatrix} + \begin{pmatrix} \sin\theta & 0 \\ 0 & \sin\theta \end{pmatrix}\begin{pmatrix} \sin\theta & 0 \\ 0 & \sin\theta \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} \cos^2\theta + 0 & 0+0 \\ 0+0 & 0+\cos^2\theta \end{pmatrix} + \begin{pmatrix} \sin^2\theta + 0 & 0+0 \\ 0+0 & 0+\sin^2\theta \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} \cos^2\theta & 0 \\ 0 & \cos^2\theta \end{pmatrix} + \begin{pmatrix} \sin^2\theta & 0 \\ 0 & \sin^2\theta \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} \cos^2\theta + \sin^2\theta & 0 \\ 0 & \cos^2\theta + \sin^2\theta \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\boxed{\text{L.H.S} = \text{R.H.S}}$$

---

9) If $A = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$ Prove that $AA^T = I$

$$A = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$$\boxed{AA^T = I}$$

$$\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}^T = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix}\cos\theta & \sin\theta \\ -\sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}\cos\theta & -\sin\theta \\ \sin\theta & \cos\theta\end{pmatrix} = \begin{pmatrix}1 & 0 \\ 0 & 1\end{pmatrix}$$

$$\begin{pmatrix}\cos^2\theta+\sin^2\theta & -\cos\cancel{\theta\sin\theta}+\sin\cancel{\theta\cos\theta} \\ -\sin\cancel{\theta\cos\theta}+\cos\cancel{\theta\sin\theta} & \sin^2\theta+\cos^2\theta\end{pmatrix} = \begin{pmatrix}1 & 0 \\ 0 & 1\end{pmatrix}$$

$$\begin{pmatrix}1 & 0 \\ 0 & 1\end{pmatrix} = \begin{pmatrix}1 & 0 \\ 0 & 1\end{pmatrix}$$

L.H.S = R.H.S

---

10) Verify that $A^2 = I$ when $A = \begin{pmatrix}5 & -4 \\ 6 & -5\end{pmatrix}$

$A^2 = I$

$$\begin{pmatrix}5 & -4 \\ 6 & -5\end{pmatrix}\begin{pmatrix}5 & -4 \\ 6 & -5\end{pmatrix} = \begin{pmatrix}1 & 0 \\ 0 & 1\end{pmatrix}$$

$$\begin{pmatrix}25-24 & -20+20 \\ 30-30 & -24+25\end{pmatrix} = \begin{pmatrix}1 & 0 \\ 0 & 1\end{pmatrix}$$

$$\begin{pmatrix}1 & 0 \\ 0 & 1\end{pmatrix} = \begin{pmatrix}1 & 0 \\ 0 & 1\end{pmatrix}$$

L.H.S = R.H.S

---

11) If $A = \begin{pmatrix}a & b \\ c & d\end{pmatrix}$ and $I = \begin{pmatrix}1 & 0 \\ 0 & 1\end{pmatrix}$ show that $A^2 - (a+d)A = (bc-ad)I_2$

$$\begin{pmatrix}a & b \\ c & d\end{pmatrix}\begin{pmatrix}a & b \\ c & d\end{pmatrix} - (a+d)\begin{pmatrix}a & b \\ c & d\end{pmatrix} = (bc-ad)\begin{pmatrix}1 & 0 \\ 0 & 1\end{pmatrix}$$

$$\begin{pmatrix}a^2+bc & ab+bd \\ ac+cd & bc+d^2\end{pmatrix} - \begin{pmatrix}a^2+ad & ab+bd \\ ac+cd & ad+d^2\end{pmatrix} = \begin{pmatrix}bc-ad & 0 \\ 0 & bc-ad\end{pmatrix}$$

$$\begin{pmatrix} \cancel{a^2}+bc-\cancel{a^2}-ad & \cancel{ab}+\cancel{bd}-\cancel{ab}-\cancel{bd} \\ \cancel{ac}+\cancel{cd}-\cancel{ac}-\cancel{cd} & bc+\cancel{d^2}-ad-\cancel{d^2} \end{pmatrix} = \begin{pmatrix} bc-ad & 0 \\ 0 & bc-ad \end{pmatrix}$$

$$\begin{pmatrix} bc-ad & 0 \\ 0 & bc-ad \end{pmatrix} = \begin{pmatrix} bc-ad & 0 \\ 0 & bc-ad \end{pmatrix}$$

L.H.S = R.H.S

---

12) If $A = \begin{pmatrix} 5 & 2 & 9 \\ 1 & 2 & 8 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 7 \\ 1 & 2 \\ 5 & -1 \end{pmatrix}$ verify that $(AB)^T = B^T A^T$

$$\left[\begin{pmatrix} 5 & 2 & 9 \\ 1 & 2 & 8 \end{pmatrix}\begin{pmatrix} 1 & 7 \\ 1 & 2 \\ 5 & -1 \end{pmatrix}\right]^T = \begin{pmatrix} 1 & 7 \\ 1 & 2 \\ 5 & -1 \end{pmatrix}^T \begin{pmatrix} 5 & 2 & 9 \\ 1 & 2 & 8 \end{pmatrix}^T$$

$$\begin{pmatrix} 5+2+45 & 35+4-9 \\ 1+2+40 & 7+4-8 \end{pmatrix}^T = \begin{pmatrix} 1 & 1 & 5 \\ 7 & 2 & -1 \end{pmatrix}\begin{pmatrix} 5 & 1 \\ 2 & 2 \\ 9 & 8 \end{pmatrix}$$

$$\begin{pmatrix} 52 & 30 \\ 43 & 3 \end{pmatrix}^T = \begin{pmatrix} 5+2+45 & 1+2+40 \\ 35+4-9 & 7+4-8 \end{pmatrix}$$

$$\begin{pmatrix} 52 & 43 \\ 30 & 3 \end{pmatrix} = \begin{pmatrix} 52 & 43 \\ 30 & 3 \end{pmatrix}$$

L.H.S = R.H.S

---

13) If $A = \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$ show that $A^2 - 5A + 7I_2 = 0$

$$A^2 - 5A + 7I_2 = 0$$

$$\begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix} - 5\begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix} + 7\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = 0$$

$$\begin{pmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{pmatrix} - \begin{pmatrix} 15 & 5 \\ -5 & 10 \end{pmatrix} + \begin{pmatrix} 7 & 0 \\ 0 & 7 \end{pmatrix} = 0$$

$$\begin{pmatrix} 8 & 5 \\ -5 & 3 \end{pmatrix} - \begin{pmatrix} 15 & 5 \\ -5 & 10 \end{pmatrix} + \begin{pmatrix} 7 & 0 \\ 0 & 7 \end{pmatrix} = 0$$

$$\begin{pmatrix} 15 & 5 \\ -5 & 10 \end{pmatrix} - \begin{pmatrix} 15 & 5 \\ -5 & 10 \end{pmatrix} = 0$$

$$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = 0$$

$$0 = 0$$

$$\boxed{L.H.S = R.H.S}$$

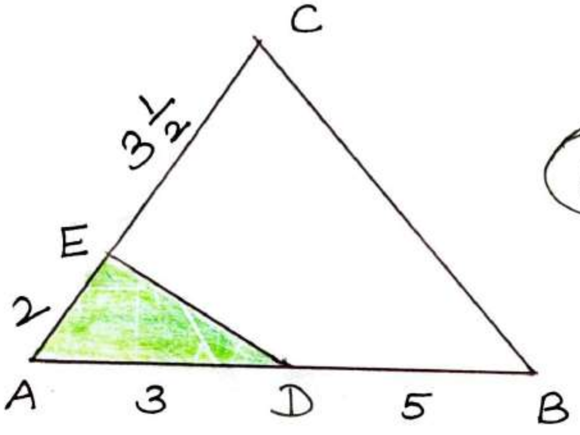
CHAPTER-4

GEOMETRY

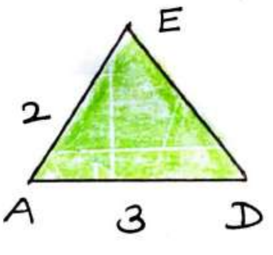
EXERCISE 4.1

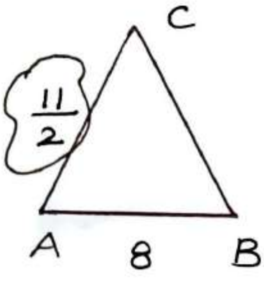
1. Check whether the which triangles are similar and find the value of x.

(i)



Similar?





$$\boxed{AC} = AE + EC$$

$$= 2 + 3\frac{1}{2}$$

$$= 2 + \frac{7}{2} = \frac{4+7}{2} = \boxed{\frac{11}{2}}$$

$$\Rightarrow \boxed{AC = \frac{11}{2}}$$

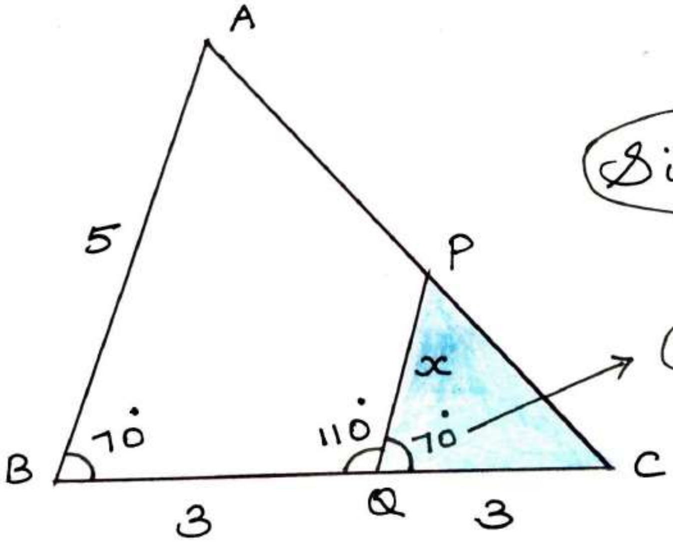
Similar $\Rightarrow \frac{EA}{CA} = \frac{AD}{AB} = \frac{ED}{CB}$

$$\frac{2}{\frac{11}{2}} = \frac{3}{8}$$

$$\frac{4}{11} \neq \frac{3}{8}$$

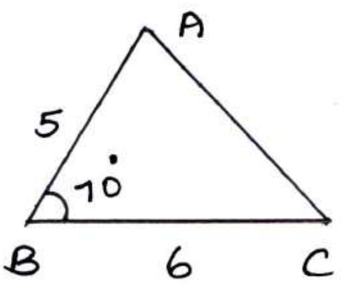
⇒ The triangles are not similar

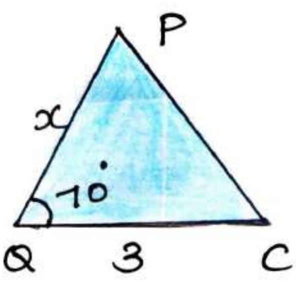
(ii)



Similar?

Angle in a st. line





In $\triangle ABC$ and $\triangle PQC$

$\angle B = \angle Q = 70^\circ$

$\angle C$ is Common

$\therefore$ $\boxed{\triangle ABC \sim \triangle PQC}$ (AA criterion)

Similar $\Rightarrow$ $\frac{AB}{PQ} = \frac{BC}{QC} = \frac{AC}{PC}$

$$\frac{5}{x} = \rightarrow \frac{\cancel{6}^{\,2}}{\cancel{3}_{\,1}}$$

$$5 = 2x$$

$$2x = 5$$

$$x = \frac{5}{2}$$

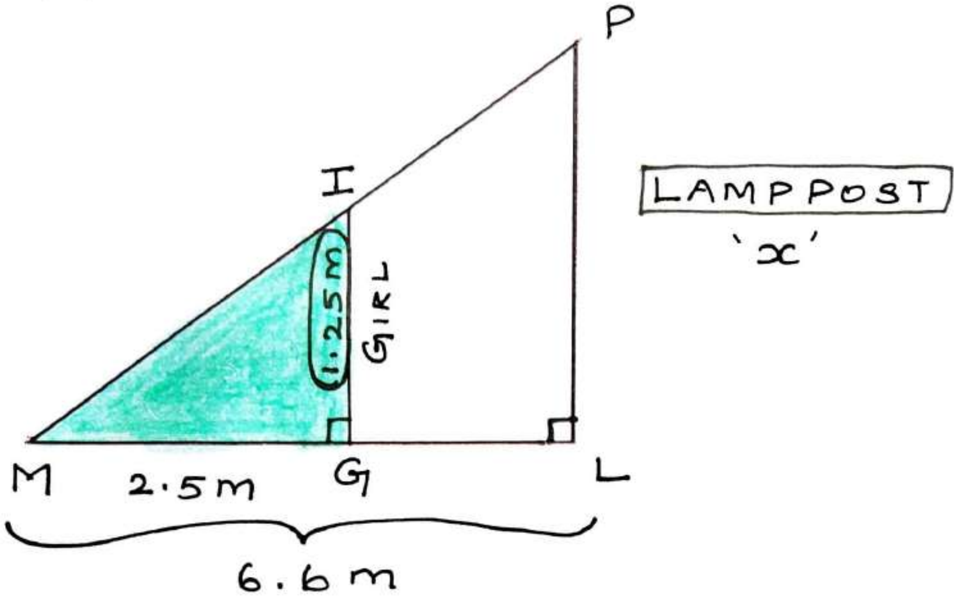
$$\boxed{x = 2.5}$$

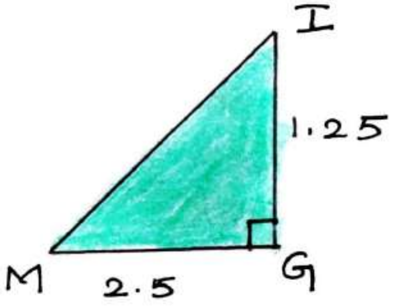
$$\begin{array}{r|l} & 2.5 \\ \hline 2 & 5 \\ & 4 \\ \hline & 10 \\ & 10 \\ \hline & 0 \\ \hline \end{array}$$

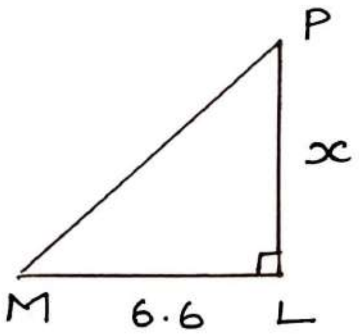
---

2) A girl looks the reflection of the top of the lamp post on the mirror which is 6.6m away from the foot of the lamppost. The Girl whose height is 1.25m is standing 2.5m away from the mirror. Assuming the mirror is placed on the ground facing

the sky and the girl, mirror and the lamppost are in a same line, Find the height of the lamppost.







$\angle G = \angle L = 90^\circ$

$\angle M$ is common

$\therefore \boxed{\triangle IMG \sim \triangle PML}$ (AA criterion)

Similar $\Rightarrow \dfrac{IG}{PL} = \dfrac{MG}{ML}$

$$\frac{1.25}{x} = \frac{2.5}{6.6}$$

$$\frac{1.25\times 100}{x\times 100} = \frac{2.5\times 10}{6.6\times 10}$$

$$\frac{\overset{5}{\cancel{125}}}{\underset{20}{\cancel{100}}\,x} = \frac{\overset{1}{\cancel{25}}}{66}$$

$$66 = 20x$$

$$20x = 66$$

$$x = \frac{\overset{33}{\cancel{66}}}{\underset{10}{\cancel{20}}}$$

$$x = \frac{33}{10}$$

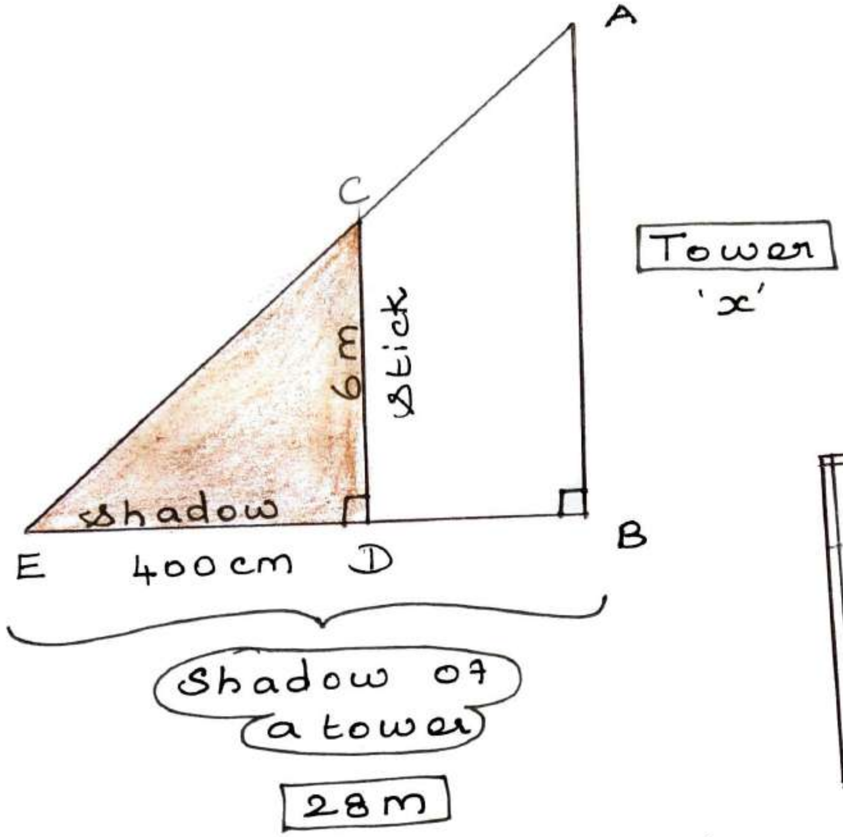
$$\boxed{x = 3.3\,m}$$

⇒ The height of the Lamppost is 3.3 m

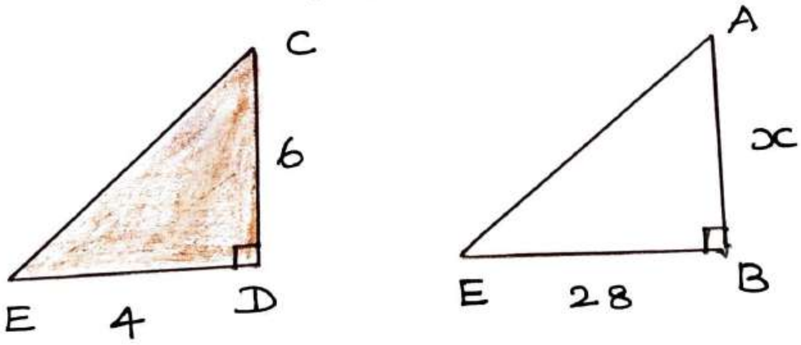
---

3) A vertical stick of length 6 m casts a shadow 400 cm long on the ground and at the same time a tower casts a shadow

28m long. Using similarity find the height of the tower.



$$400 \text{ cm} = \frac{4\cancel{0}\cancel{0}}{1\cancel{0}\cancel{0}} = 4 \text{m}$$



$\angle D = \angle B = 90^\circ$

$\angle E$ is common

$\therefore \boxed{\Delta CED \sim \Delta AEB}$ (AA criterion)

Similar $\Rightarrow \dfrac{ED}{EB} = \dfrac{CD}{AB}$

$$\frac{\cancel{4}^{1}}{\cancel{28}_{7}} = \frac{6}{x}$$

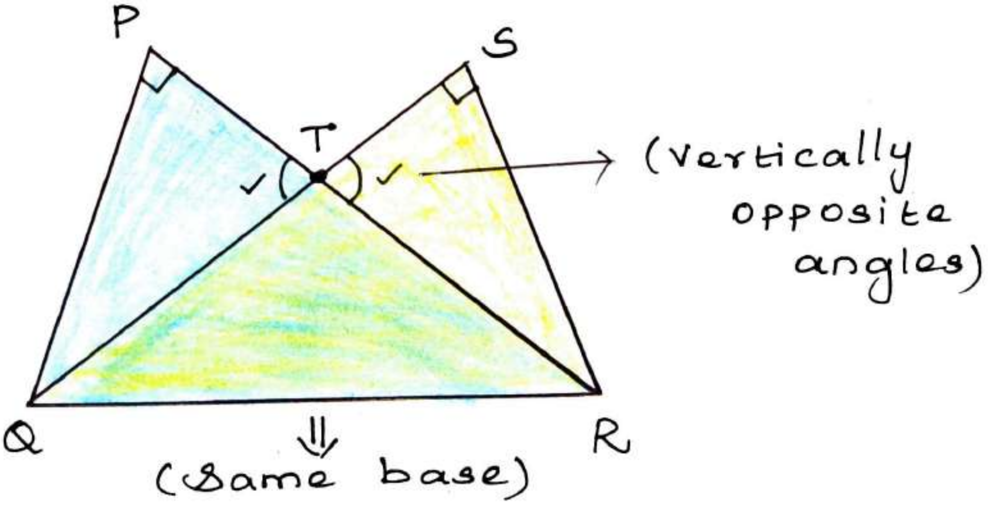
$$x = 6 \times 7$$

$$\boxed{x = 42\,m}$$

⇒ The height of the tower is 42m.

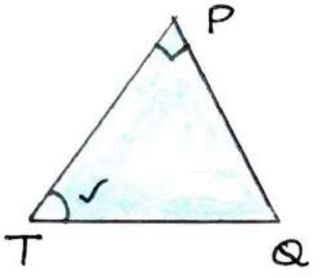
---

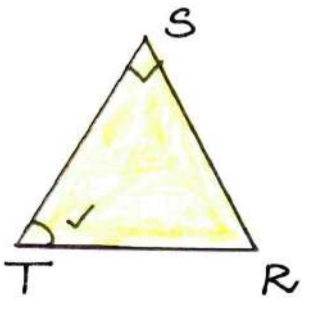
4) Two triangles QPR and QSR, right angled at P and S respectively are drawn on the same base QR and on the same side of QR. If PR and SQ intersect at T, Prove that PT × TR = ST × TQ.



To prove:

$$\boxed{PT \times TR = ST \times TQ}$$





$\angle P = \angle S = 90^\circ$

$\angle T = \angle T$ (vertically opposite angles are equal)

$\therefore \boxed{\Delta PTQ \sim \Delta STR}$ (AA criterion)

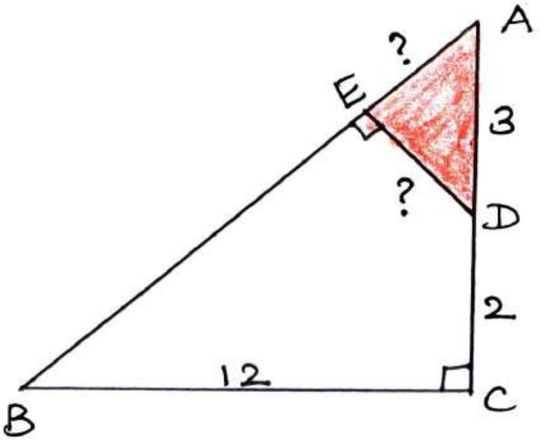
Similar $\Rightarrow$ $\dfrac{PT}{ST} = \dfrac{TQ}{TR} = \dfrac{PQ}{SR}$

$$\boxed{PT \times TR = ST \times TQ}$$

Hence Proved

---

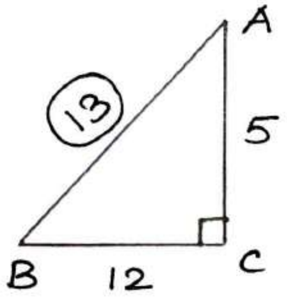
5) In the adjacent figure, ΔABC is right angled at C and $DE \perp AB$. Prove that $\Delta ABC \sim \Delta ADE$ and hence find the lengths of AE and DE.

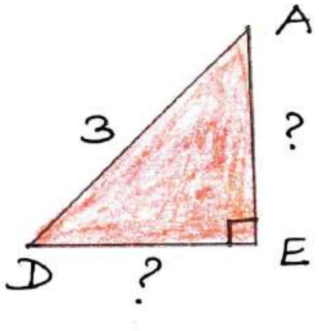


To Prove

$\Rightarrow \triangle ABC \sim \triangle ADE$

$\Rightarrow$ Find AE and DE





$\Downarrow$

$\boxed{AB^2 = BC^2 + AC^2}$ (pythagoras Theorem)

$AB^2 = 12^2 + 5^2$

$AB^2 = 144 + 25$

$AB^2 = 169$

$AB^{\cancel{2}} = 13^{\cancel{2}}$

$\boxed{AB = 13}$

In $\triangle ABC$ and $\triangle ADE$,

$\angle C = \angle E = 90^\circ$

$\angle A$ is Common

$\therefore \boxed{\triangle ABC \sim \triangle ADE}$ (AA criterion)

Similar $\Rightarrow \dfrac{AB}{AD} = \dfrac{BC}{DE} = \dfrac{AC}{AE}$

$$\frac{13}{3} = \frac{12}{DE} = \frac{5}{AE}$$

$$\frac{13}{3} = \frac{12}{DE}$$

$$13 \times DE = 3 \times 12$$

$$DE = \frac{3 \times 12}{13}$$

$$\boxed{DE = \frac{36}{13}}$$

$$\frac{13}{3} = \frac{5}{AE}$$

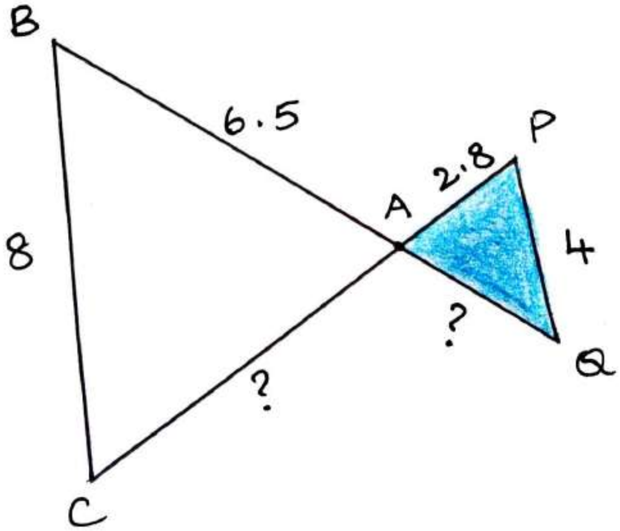
$$13 \times AE = 3 \times 5$$

$$AE = \frac{3 \times 5}{13}$$

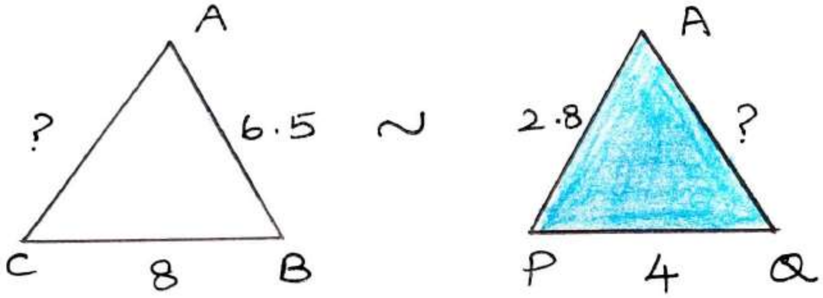
$$\boxed{AE = \frac{15}{13}}$$

---

6) In the adjacent figure, $\triangle ACB \sim \triangle APQ$. If $BC = 8$ cm, $PQ = 4$ cm $BA = 6.5$ cm and $AP = 2.8$ cm. Find CA and AQ.



GIVEN: $\boxed{\Delta ACB \sim \Delta APQ}$



Similar ⟹ $$\frac{AC}{AP} = \frac{CB}{PQ} = \frac{AB}{AQ}$$

$$\frac{AC}{2.8} = \frac{\cancel{8}^{2}}{\cancel{4}} = \frac{6.5}{AQ}$$

$$\frac{AC}{2.8} = 2$$

$$AC = 2 \times 2.8$$

$$\boxed{AC = 5.6\,cm}$$

$$2 = \frac{6.5}{AQ}$$

$$AQ = \frac{6.5}{2}$$

$$\boxed{AQ = 3.25\,cm}$$

---

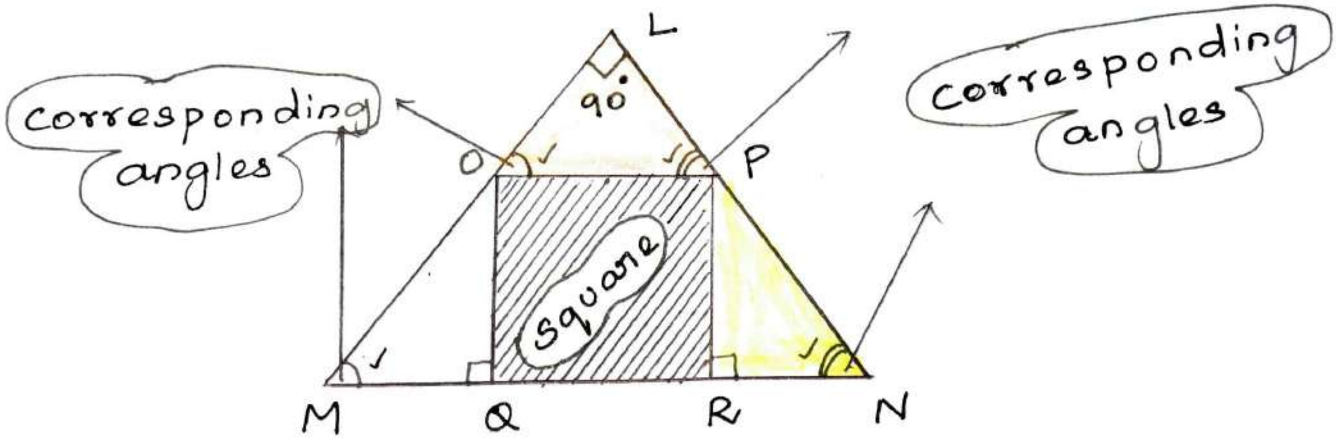
7) If figure OPRQ is a square and $\angle MLN = 90^\circ$. Prove that

(i) $\Delta LOP \sim \Delta QMO$

(ii) $\Delta LOP \sim \Delta RPN$

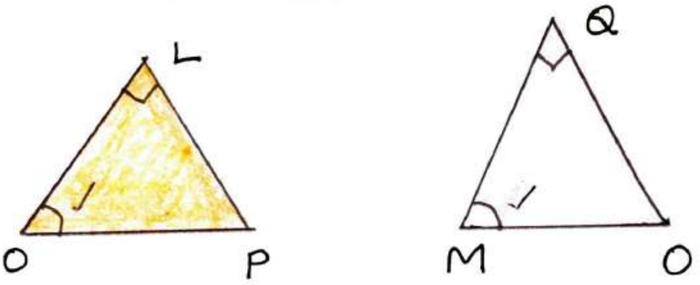
(iii) $\Delta QMO \sim \Delta RPN$

(iv) $QR^2 = MQ \times RN$



TO PROVE:

(i) $\boxed{\Delta LOP \sim \Delta QMO}$?

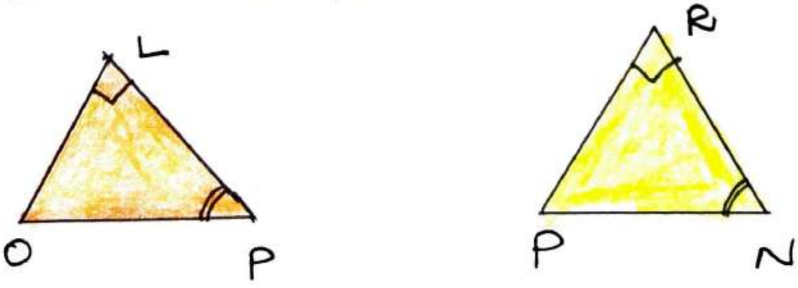


$\angle L = \angle Q = 90^\circ$

$\angle O = \angle M$ (corresponding angles)

$\Rightarrow \boxed{\Delta LOP \sim \Delta QMO}$ (AA criterion)

(ii) $\boxed{\Delta LOP \sim \Delta RPN}$?



$\angle L = \angle R = 90^\circ$

$\angle P = \angle N$ (corresponding angles)

$\Rightarrow \boxed{\Delta LOP \sim \Delta RPN}$ (AA criterion)

(iii) $\boxed{\Delta QMO \sim \Delta RPN}$?

From (i) and (ii), we know

$\Delta LOP \sim \Delta QMO$

$\Delta LOP \sim \Delta RPN$

$\Rightarrow \boxed{\Delta QMO \sim \Delta RPN}$

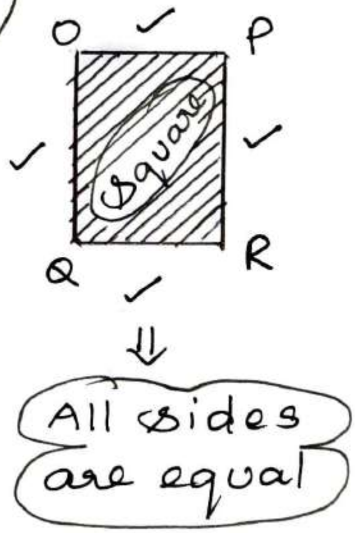
(iv) $\boxed{QR^2 = MQ \times RN}$ ⇐ To prove

From (iii) we know,

$\boxed{\Delta QMO \sim \Delta RPN}$

Similar $\Rightarrow \dfrac{QM}{RP} = \dfrac{MO}{PN} = \dfrac{QO}{RN}$

$$\frac{QM}{QR} = \frac{QR}{RN}$$



$MQ \times RN = QR^2$

$\Rightarrow \boxed{QR^2 = MQ \times RN}$

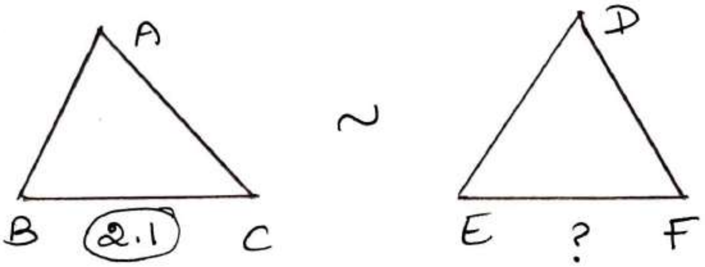
Hence proved.

8) If $\Delta ABC \sim \Delta DEF$ such that area of ΔABC is $9\ cm^2$ and the area of ΔDEF is $16\ cm^2$ and $BC = 2.1 cm$. Find the length of EF.

GIVEN:

$$\boxed{\Delta ABC \sim \Delta DEF}$$

Also, Area of $\Delta ABC = 9\ cm^2$
Area of $\Delta DEF = 16\ cm^2$



$$\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta DEF} \Rightarrow \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2}$$

$$\Rightarrow \frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta DEF} = \frac{BC^2}{EF^2}$$

$$\frac{9}{16} = \frac{(2.1)^2}{EF^2}$$

$$\frac{(3)^{\cancel{2}}}{(4)^{\cancel{2}}} = \frac{(2.1)^{\cancel{2}}}{EF^{\cancel{2}}}$$

$$\frac{3}{4} = \frac{2.1}{EF}$$

$$3 \times EF = 4 \times 2.1$$

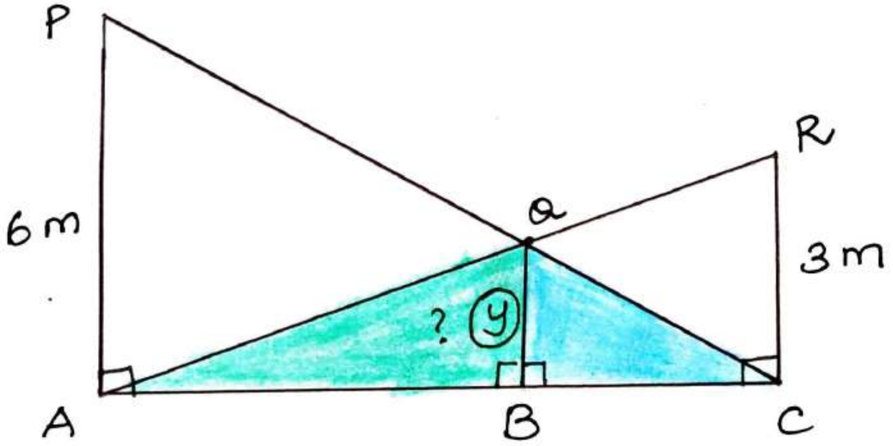
$$EF = \frac{4 \times \cancel{2.1}^{\,0.7}}{\cancel{3}}$$

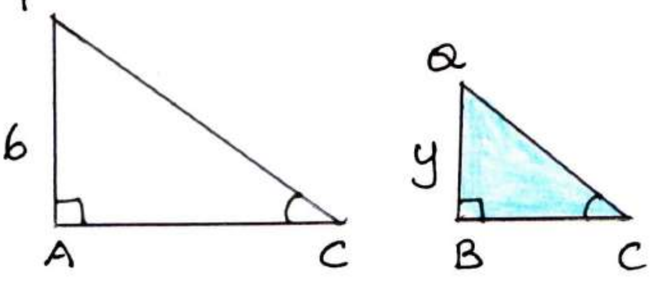
$$EF = 4 \times 0.7$$

$$\boxed{EF = 2.8\,cm}$$

---

9) Two vertical poles of heights 6m and 3m are erected above a horizontal ground AC. Find the value of y.



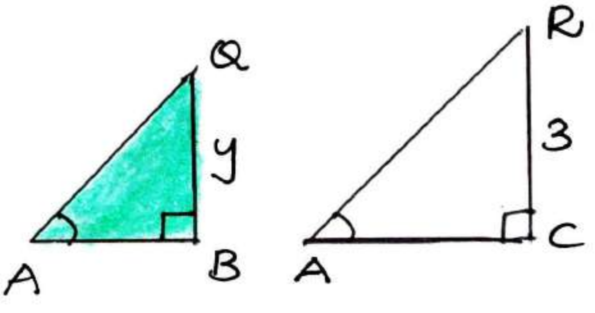


$\angle A = \angle B = 90^\circ$

$\angle C$ is Common

$\therefore \boxed{\triangle PAC \sim \triangle QBC}$

(AA criterion)



$\angle B = \angle C = 90^\circ$

$\angle A$ is Common

$\therefore \boxed{\triangle QBA \sim \triangle RCA}$

(AA criterion)

$$\frac{PA}{QB} = \frac{AC}{BC}$$

$$\frac{6}{y} = \frac{AC}{BC}$$

$$\boxed{6\,BC = (AC)\,y} \rightarrow ①$$

$$\frac{QB}{RC} = \frac{BA}{CA}$$

$$\frac{y}{3} = \frac{AB}{AC}$$

$$\boxed{3\,AB = (AC)\,y} \rightarrow ②$$

From ① and ②

$${}^{2}\cancel{6}\,BC = \cancel{3}\,AB$$

$$\boxed{2\,BC = AB} \rightarrow ③$$

We know, $AC = \underset{\downarrow}{AB} + BC$

$$AC = 2BC + BC \text{ (from ③)}$$

$$\boxed{AC = 3\,BC} \rightarrow ④$$

Put ④ in ①

$$6\,BC = \underset{\downarrow}{(AC)}\,y$$

$$6\,\cancel{BC} = (3\,\cancel{BC})\,y$$

$$6 = 3y$$

$$3y = 6$$

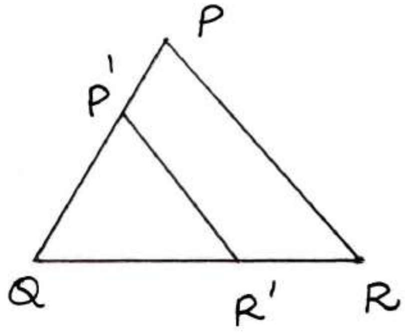
$$y = \frac{\cancel{6}^{\,2}}{\cancel{3}}$$

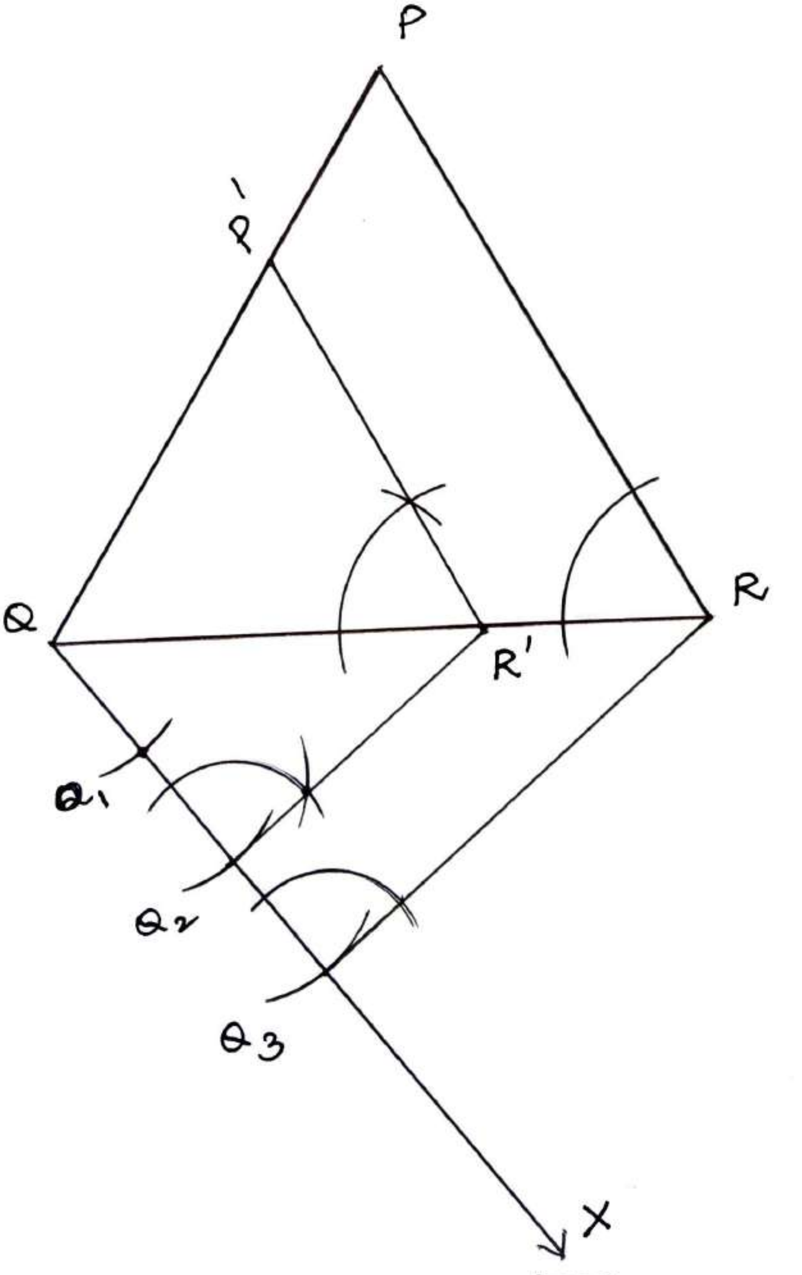
$$\boxed{y = 2m}$$

10) Construct a triangle similar to a given triangle PQR with its sides equal to $\frac{2}{3}$ of the corresponding sides of the triangle PQR.

(Scale factor $\frac{2}{3} < 1$)

ROUGH DIAGRAM





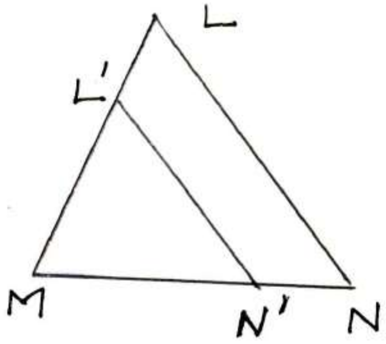
CONSTRUCTION:

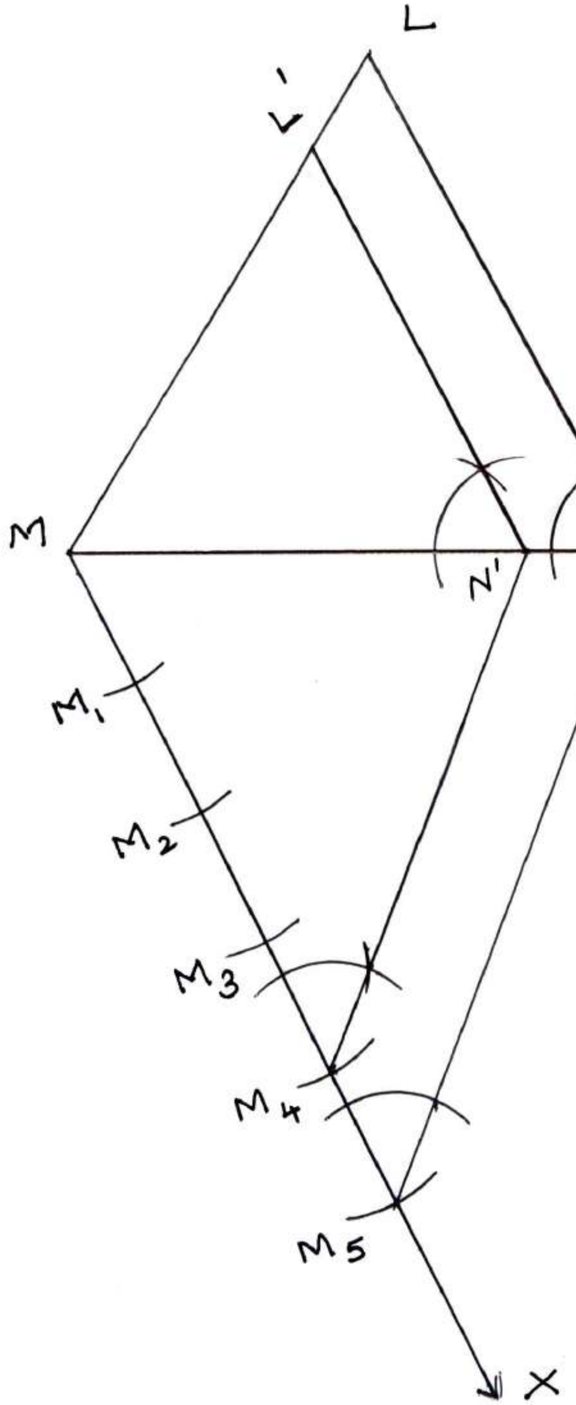
- Construct a Δ PQR with any measurement.
- Draw a ray QX making an acute angle with QR
- Locate 3 points Q_1, Q_2, Q_3 on QX such that $QQ_1 = Q_1Q_2 = Q_2Q_3$.
- Join Q_3R.
- Draw a line through Q_2 parallel to Q_3R to intersect QR at R'
- Draw a line through R' parallel to RP to intersect QP at P'
- Thus $\Delta P'QR'$ is the required triangle.

---

11) Construct a triangle similar to a given triangle LMN with its sides equal to $\frac{4}{5}$ of the corresponding sides of the triangle LMN. (scale factor $\frac{4}{5} < 1$)

ROUGH DIAGRAM





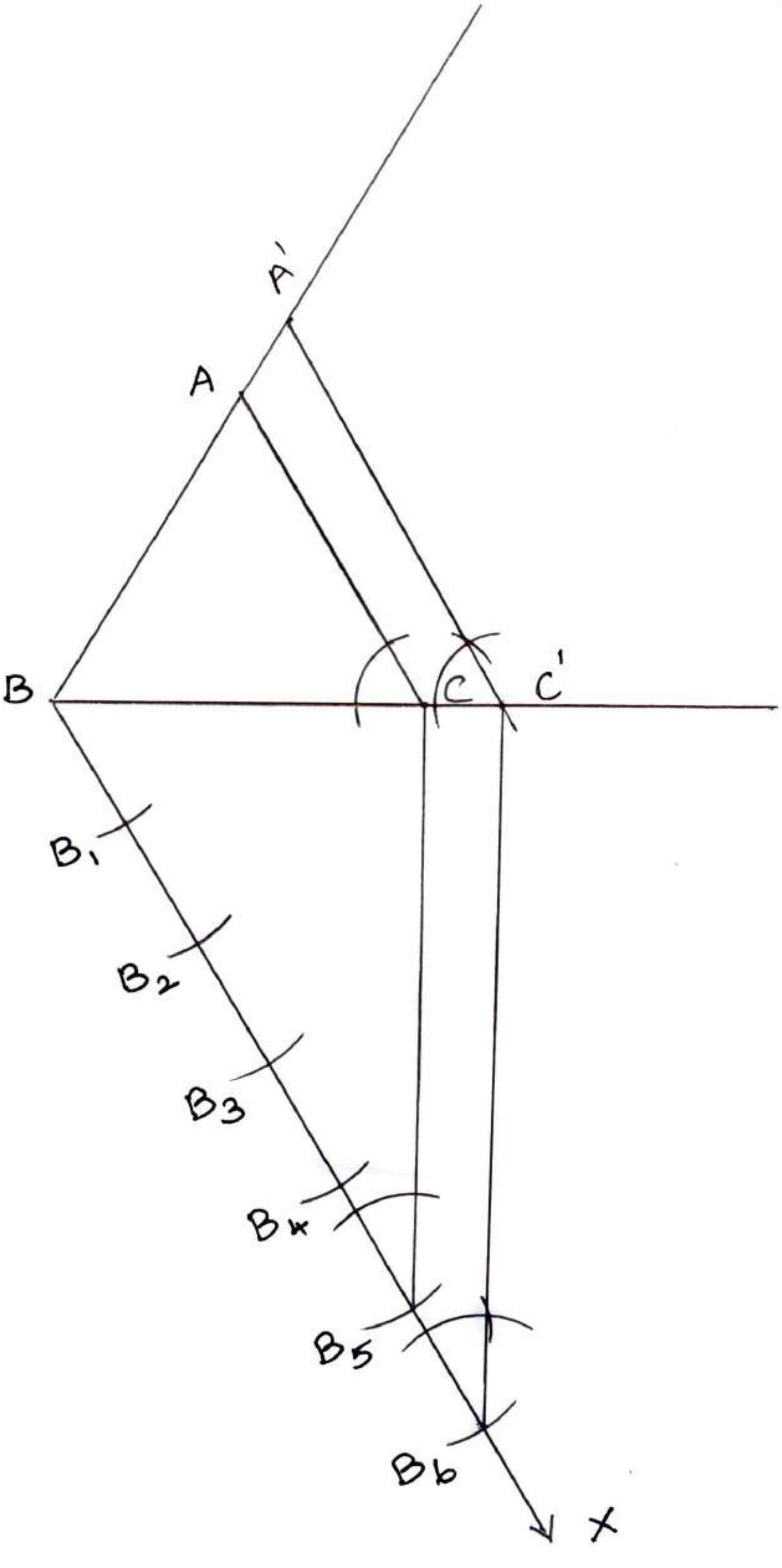
CONSTRUCTION:

- Construct a Δ LMN with any measurements.
- Draw a ray MX making an acute angle with MN.
- Locate 5 points, M_1, M_2, M_3, M_4, M_5 on MX such that $MM_1 = M_1M_2 = M_2M_3 = M_3M_4 = M_4M_5$.
- Join M_5N
- Draw a line through M_4 parallel to M_5N to intersect MN at N'.
- Draw a line through N' parallel to NL to intersect LM at L'
- Thus $\Delta L'MN'$ is the required triangle.

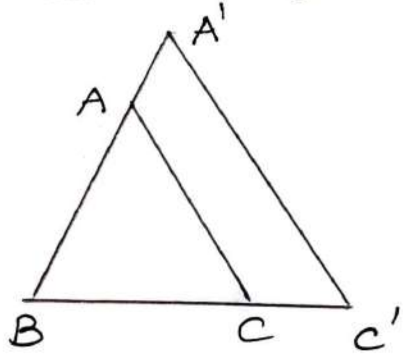
---

12) Construct a triangle similar to a given triangle ABC with its sides equal to $\frac{6}{5}$ of the corresponding sides of the triangle ABC.

(Scale factor $\frac{6}{5} > 1$)



Rough Diagram

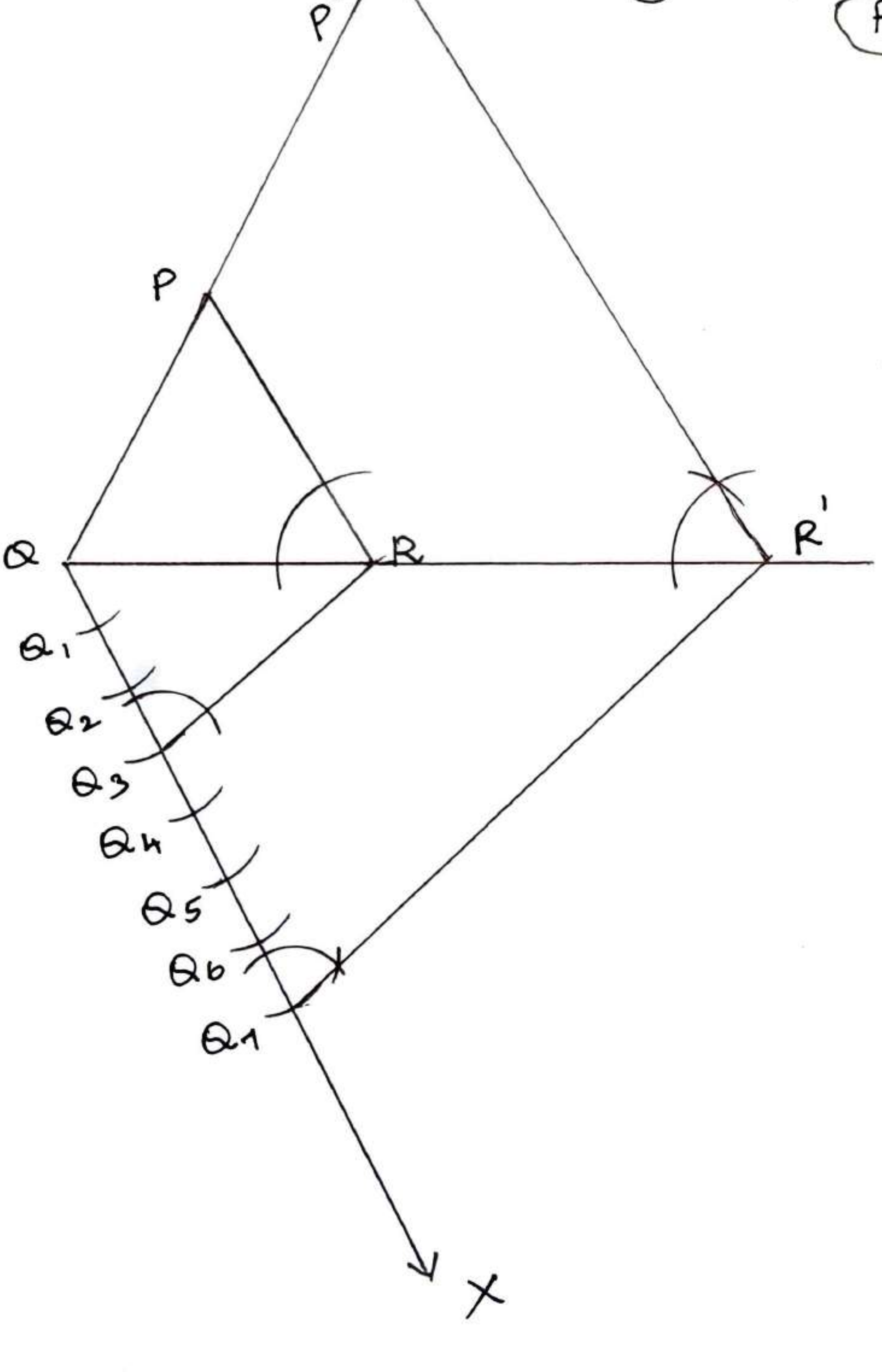


CONSTRUCTION:

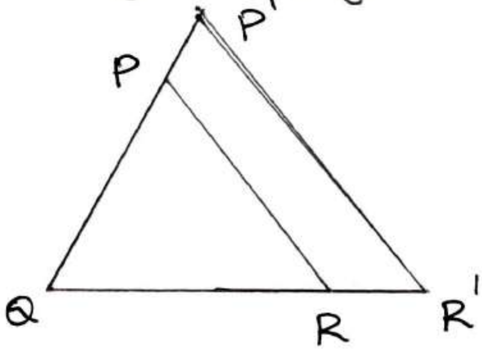
- Draw $\triangle ABC$ with any measurements.
- Draw a ray BX making an acute angle with BC.
- Locate 6 points, $B_1, B_2, B_3, B_4, B_5, B_6$ such that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4 = B_4B_5 = B_5B_6$.
- Join B_5C
- Draw a line through B_6 parallel to B_5C intersecting the extended line segment BC at C'.
- Draw a line through C' Parallel to CA intersecting the extended line segment BA at A'.
- Thus $\triangle A'BC'$ is the required triangle.

13) Construct a triangle similar to a given triangle PQR with its sides equal to $\frac{7}{3}$ of the corresponding sides of the triangle PQR.

(scale factor $\frac{7}{3} > 1$)



Rough Diagram



CONSTRUCTION:

* Construct a Δ PQR with any measurements.

* Draw a ray QX making an acute angle with QR.

* Locate 7 points. $Q_1, Q_2, Q_3, Q_4, Q_5, Q_6, Q_7$ on QX such that $QQ_1 = Q_1Q_2 = Q_2Q_3 = Q_3Q_4 = Q_4Q_5 = Q_5Q_6 = Q_6Q_7$

* Join Q_3R.

* Draw a line through Q_7 parallel to Q_3R to intersect the extended line segment QR at R'.

* Draw a line through R' parallel to RP intersecting the extended line segment QP at P'.

* Thus Δ P'QR' is the required triangle.

BASIC PROPORTIONALITY THEOREM

(OR)

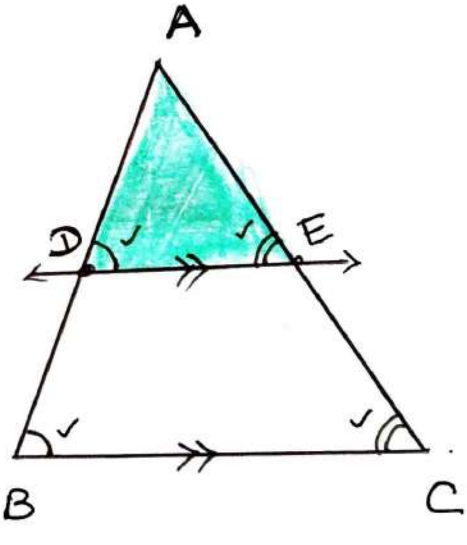
THALES THEOREM

Statement:

If a straight line is drawn parallel to a side of a triangle intersecting the other two sides, then it divides the two sides in the same ratio.

Given:

In Δ ABC, D and E are the points on AB and AC.



To prove:

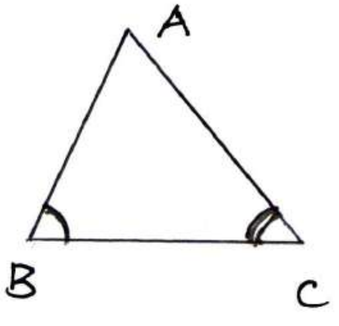
$$\boxed{\frac{AD}{DB} = \frac{AE}{EC}}$$

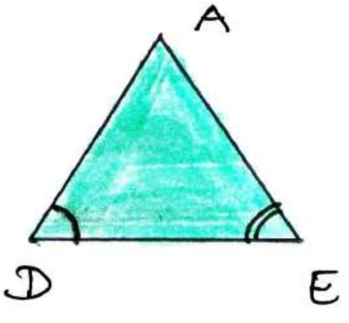
Construction:

Draw a line DE || BC

Proof:

In Δ ABC and Δ ADE,





$\angle ABC = \angle ADE$
$\angle ACB = \angle AED$ } corresponding angles are equal

$\angle A$ is Common.

$\therefore \boxed{\Delta ABC \sim \Delta ADE}$ (AAA criterion)

Similar $\Rightarrow$ $\dfrac{AB}{AD} = \dfrac{BC}{DE} = \dfrac{AC}{AE}$

$$\frac{AB}{AD} = \frac{AC}{AE}$$

$$\frac{AD+DB}{AD} = \frac{AE+EC}{AE}$$

$$\frac{\cancel{AD}}{\cancel{AD}} + \frac{DB}{AD} = \frac{\cancel{AE}}{\cancel{AE}} + \frac{EC}{AE}$$

$$\cancel{1} + \frac{DB}{AD} = \cancel{1} + \frac{EC}{AE}$$

$$\frac{DB}{AD} = \frac{EC}{AE}$$

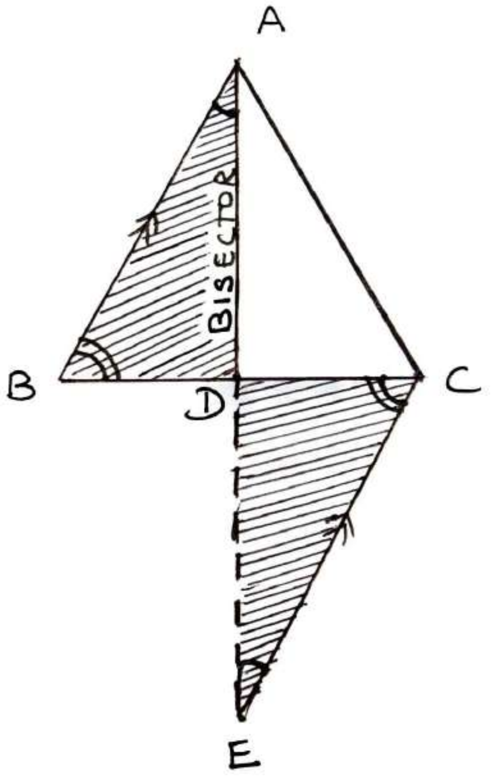
$$\boxed{\frac{AD}{DB} = \frac{AE}{EC}}$$ (Reciprocal)

Hence Proved.

ANGLE BISECTOR THEOREM

Statement:

The internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the corresponding sides containing the angle.



Given:

In $\triangle ABC$, AD is the internal bisector of $\angle A$.

To prove:

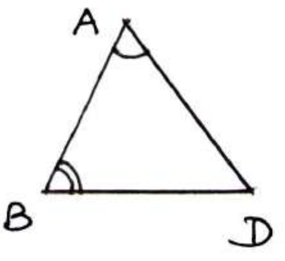
$$\boxed{\frac{AB}{AC} = \frac{BD}{DC}}$$

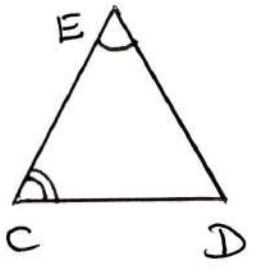
Construction:

Draw $CE \parallel AB$ which meet AD produced at E.

Proof:

In $\triangle ABD$ and $\triangle ECD$





$$\left.\begin{aligned}\angle BAD &= \angle CED\\ \angle ABD &= \angle ECD\end{aligned}\right\}$$ (Alternate angles are equal)

$\angle D$ is common

$\therefore$ $\boxed{\Delta ABD \sim \Delta ECD}$ (AAA criterion)

Similar $\Rightarrow$ $\dfrac{AB}{EC} = \dfrac{BD}{CD} = \dfrac{AD}{ED}$

$$\frac{AB}{EC} = \frac{BD}{CD}$$

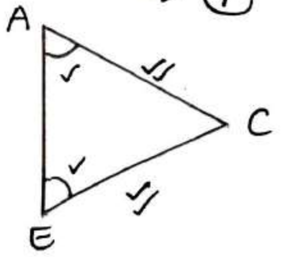
$$\boxed{\frac{AB}{AC} = \frac{BD}{DC}} \text{ (by ①)}$$

Hence Proved.

In ΔAEC,

$\angle CEA = \angle CAE$

$\Rightarrow \boxed{EC = AC} \rightarrow$ ①



EXERCISE 4.2

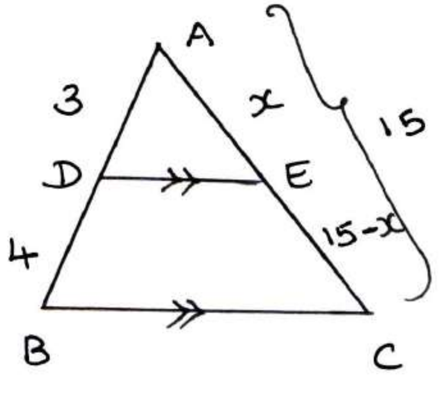
1) In ΔABC, D and E are points on the sides AB and AC respectively such that $DE \parallel BC$

(i) If $\dfrac{AD}{DB} = \dfrac{3}{4}$ and AC = 15 cm. Find AE.

$\boxed{DE \parallel BC}$

$\therefore$ By B.P.T Theorem

$$\boxed{\frac{AD}{DB} = \frac{AE}{EC}}$$



$$\frac{3}{4} = \frac{x}{15-x}$$

$$3(15-x) = 4x$$

$$45 - 3x = 4x$$

$$45 = 4x + 3x$$

$$45 = 7x$$

$$7x = 45$$

$$x = \frac{45}{7}$$

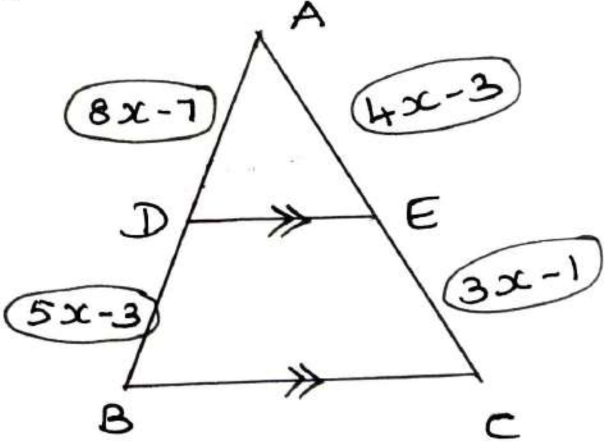
$$x = 6.428\ldots$$

$$\Rightarrow \quad x = 6.43 \text{ cm}$$

$$\Rightarrow \quad \boxed{AE = 6.43 \text{ cm}}$$

$$\begin{array}{r} 6.428\ldots \\ 7\overline{)45} \\ \underline{42} \\ 30 \\ \underline{28} \\ 20 \\ \underline{14} \\ 60 \\ \underline{56} \\ \underline{4} \end{array}$$

(ii) If $AD = 8x-7$, $DB = 5x-3$, $AE = 4x-3$ and $EC = 3x-1$, find 'x'



$$\boxed{DE \parallel BC}$$

$\therefore$ By B.P.T Theorem,

$$\boxed{\frac{AD}{DB} = \frac{AE}{EC}}$$

$$\frac{8x-7}{5x-3} = \frac{4x-3}{3x-1}$$

$$(8x-7)(3x-1) = (4x-3)(5x-3)$$

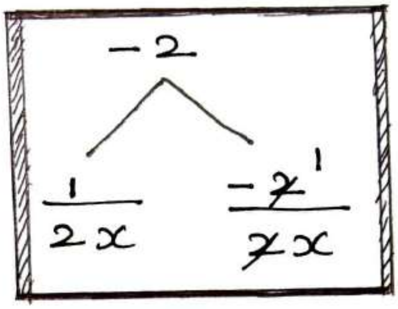
$$24x^2 \underbrace{- 8x - 21x}_{} + 7 = 20x^2 - 12\underbrace{x - 15}_{}x + 9$$

$$24x^2 - 29x + 7 = 20x^2 - 27x + 9$$

$$\underbrace{24x^2 - 20x^2}_{} \underbrace{- 29x + 27x}_{} + \underbrace{7 - 9}_{} = 0$$

$$4x^2 - 2x - 2 = 0$$

$(\div 2)$ $2x^2 - x - 1 = 0$



$\Rightarrow (2x+1)(x-1) = 0$

$2x + 1 = 0$ $\qquad$ $x - 1 = 0$

$2x = -1$ $\qquad$ $\boxed{x = 1}$

$\boxed{x = \frac{-1}{2}}$

↓

omitting the (-ve) value

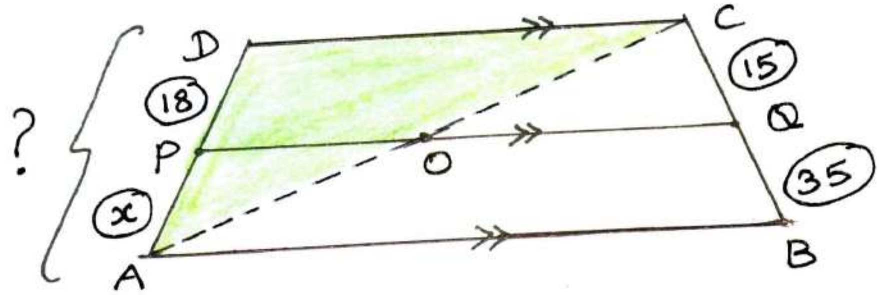
$\boxed{x = 1}$

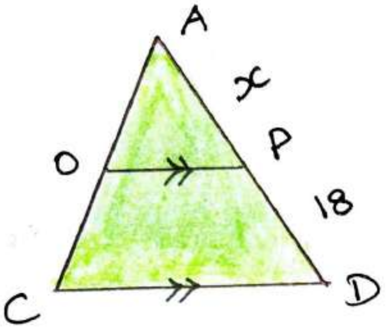
---

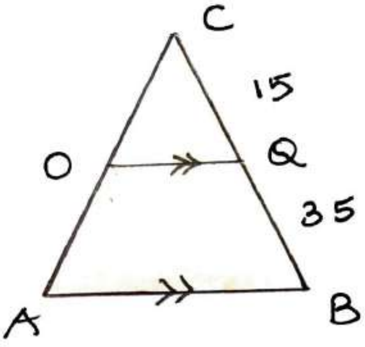
2) ABCD is a trapezium in which AB || DC and P, Q are points on AD and BC respectively, such that PQ || DC if PD = 18 cm, BQ = 35 cm and QC = 15 cm. Find AD.

GIVEN: AB || DC, PQ || DC,

PD = 18 cm, BQ = 35 cm, QC = 15 cm







$\Downarrow$ $\Downarrow$

$$\boxed{\frac{AO}{OC} = \frac{x}{18}} \rightarrow ①$$

$$\frac{CO}{OA} = \frac{15}{35}$$

$$\boxed{\frac{AO}{OC} = \frac{35}{15}} \rightarrow ②$$

(Reciprocal)

From ① and ②

$$\frac{x}{\cancel{18}_{6}} = \frac{\cancel{35}^{7}}{\cancel{15}_{\cancel{5}}}$$

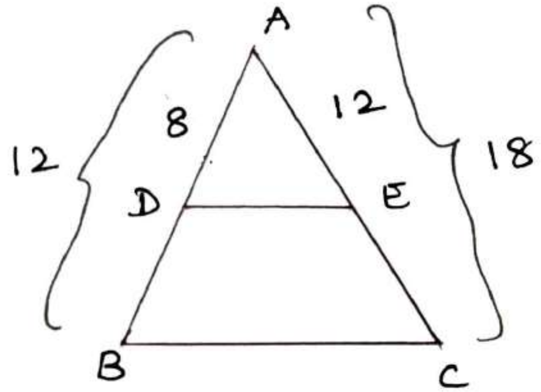
$$x = 7 \times 6$$

$$\boxed{x = 42 \text{ cm}}$$

$$\therefore AD = AP + PD$$
$$= x + 18$$
$$= 42 + 18$$
$$\boxed{AD = 60 \text{ cm}}$$

3) In ΔABC, D and E are points on the sides AB and AC respectively. For each of the following cases show that $DE \parallel BC$.

(i) AB = 12 cm, AD = 8 cm, AE = 12 cm, AC = 18 cm.



$$\boxed{\frac{AD}{DB} = \frac{AE}{EC}}$$

(or)

$$\boxed{\frac{AD}{AB} = \frac{AE}{AC}}$$

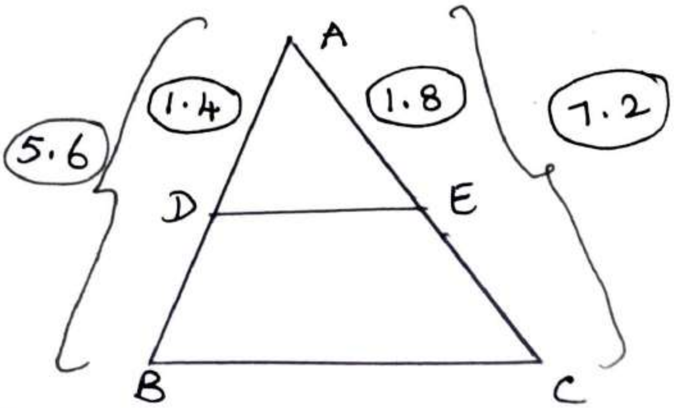
$$\frac{\overset{2}{\cancel{8}}}{\underset{3}{\cancel{12}}} = \frac{\overset{2}{\cancel{12}}}{\underset{3}{\cancel{18}}}$$

$$\boxed{\frac{2}{3} = \frac{2}{3}}$$

$\Rightarrow$ $DE \parallel BC$

(ii) AB = 5.6 cm, AD = 1.4 cm, AC = 7.2 cm

AE = 1.8 cm



$$\Rightarrow \boxed{\frac{AD}{AB} = \frac{AE}{AC}}$$

$$\frac{1.4}{5.6} = \frac{1.8}{7.2}$$

$$\frac{1.4\times10}{5.6\times10} = \frac{1.8\times10}{7.2\times10}$$

$$\frac{\cancel{14}^{\,2}}{\cancel{56}_{\,8}} = \frac{\cancel{18}^{\,3}}{\cancel{72}_{\,12}}$$

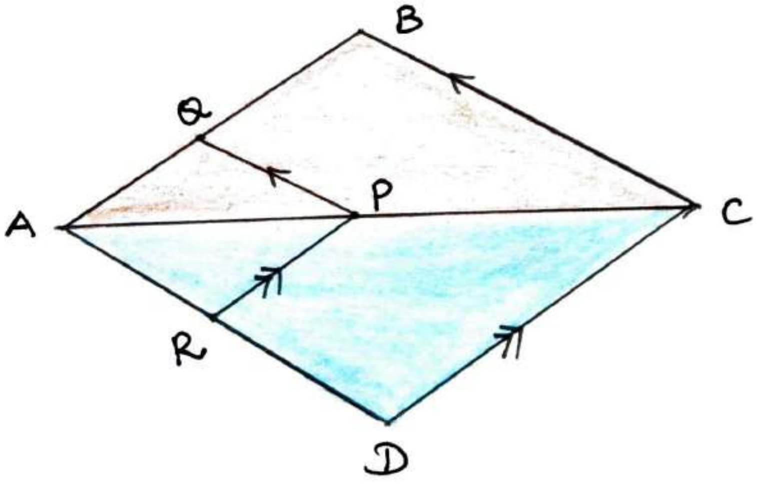
$$\frac{\cancel{2}^{\,1}}{\cancel{8}_{\,4}} = \frac{\cancel{3}^{\,1}}{\cancel{12}_{\,4}}$$

$$\boxed{\frac{1}{4} = \frac{1}{4}}$$

$\Rightarrow$ $DE \parallel BC$

---

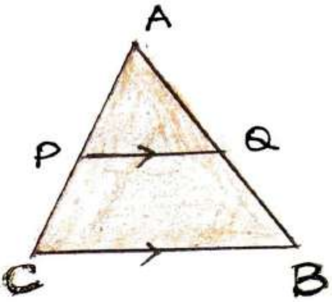
4) In fig, if $PQ \parallel BC$ and $PR \parallel CD$, Prove that (i) $\frac{AR}{AD} = \frac{AQ}{AB}$ (ii) $\frac{QB}{AQ} = \frac{DR}{AR}$



(i) To prove:

$$\boxed{\frac{AR}{AD} = \frac{AQ}{AB}}$$

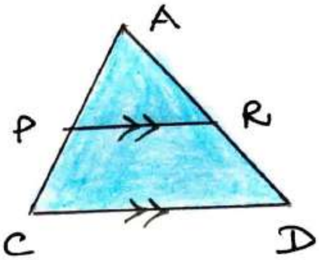
Given: $PQ \parallel BC$



$\therefore$ By BPT Theorem

$$\boxed{\frac{AP}{AC} = \frac{AQ}{AB}} \rightarrow ①$$

Also, $PR \parallel CD$



$\therefore$ By BPT Theorem,

$$\boxed{\frac{AP}{AC} = \frac{AR}{AD}} \rightarrow ②$$

From ① and ②

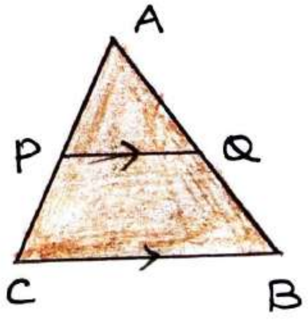
$$\frac{AQ}{AB} = \frac{AR}{AD}$$

$$\Rightarrow \frac{AR}{AD} = \frac{AQ}{AB}$$

Hence Proved.

(ii) To Prove: $\frac{QB}{AQ} = \frac{DR}{AR}$

Given: $PQ \parallel BC$

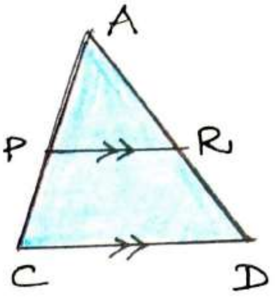


$\therefore$ By BPT Theorem,

$$\boxed{\frac{AP}{PC} = \frac{AQ}{QB}} \rightarrow ①$$

Also, PR || CD

∴ By BPT Theorem,



$$\boxed{\frac{AP}{PC} = \frac{AR}{RD}} \rightarrow ②$$

from ① and ②

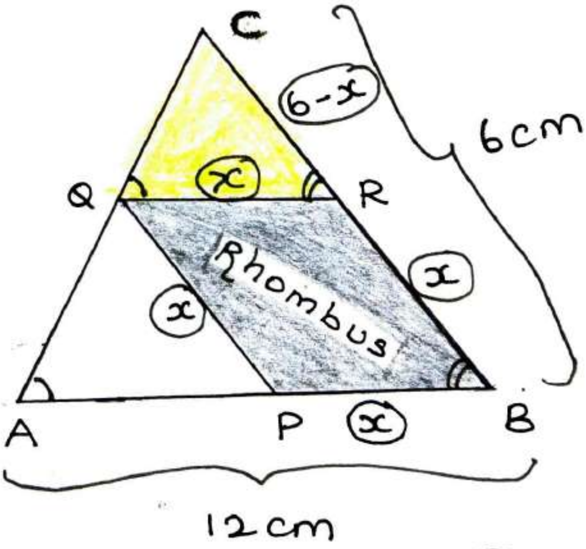
$$\frac{AQ}{QB} = \frac{AR}{RD}$$

$$\Rightarrow \frac{QB}{AQ} = \frac{DR}{AR} \quad \text{(Reciprocal)}$$

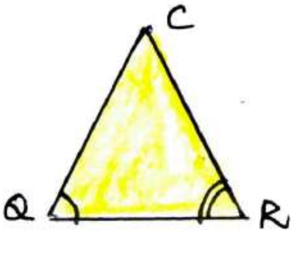
Hence Proved.

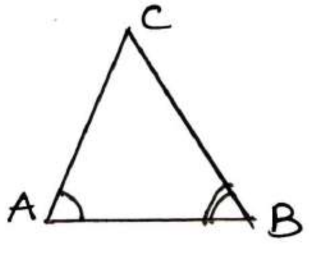
---

5) Rhombus PQRB is inscribed in ΔABC such that $\angle B$ is one of its angle. P, Q and R lie on AB, AC and BC respectively. If AB = 12 cm and BC = 6 cm, Find the sides of PQ, RB of the Rhombus.



In Δ CQR and Δ CAB





$\angle Q = \angle A$
$\angle R = \angle B$ } corresponding angles.

$\angle C$ is common

PQRB → Rhombus
all sides are equal

$\therefore \boxed{\Delta CQR \sim \Delta CAB}$ (AAA criterion)

Similar $\Rightarrow \frac{CQ}{CA} = \frac{QR}{AB} = \frac{CR}{CB}$

$$\boxed{\frac{QR}{AB} = \frac{CR}{CB}}$$

$$\frac{x}{\cancel{12}_{2}} = \frac{6-x}{\cancel{6}_{1}}$$

$$x = 2(6-x)$$

$$x = 12 - 2x$$

$$x + 2x = 12$$

$$3x = 12$$

$$x = \frac{\cancel{12}^{4}}{\cancel{3}}$$

$$\boxed{x = 4}$$

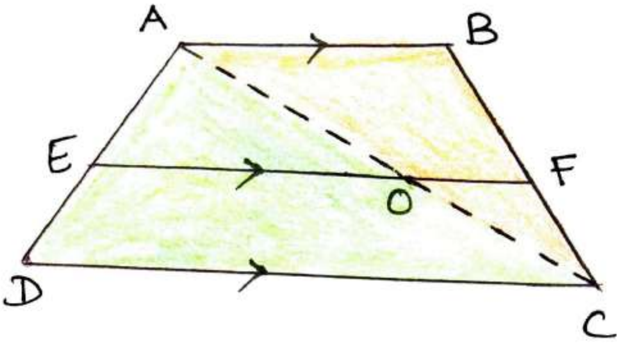
To find:

PQ, RB

$$\Rightarrow \boxed{\begin{array}{l} PQ = 4\text{ cm} \\ RB = 4\text{ cm} \end{array}}$$

---

6) In trapezium ABCD, $AB \parallel CD$, E and F are points on non-parallel sides AD and BC respectively such that $EF \parallel AB$. Show that $\frac{AE}{ED} = \frac{BF}{FC}$.



$\boxed{ABCD \Rightarrow \text{trapezium}}$

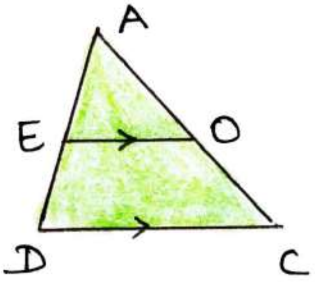
Construction:

Join AC

In ΔADC, $EO \parallel DC$

$\therefore$ By BPT Theorem

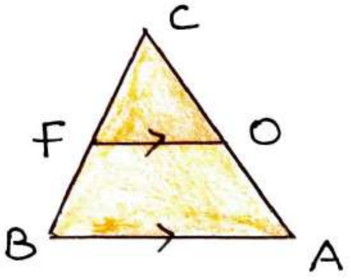
$$\boxed{\frac{AE}{ED} = \frac{AO}{OC}} \rightarrow ①$$



In ΔCBA, $FO \parallel BA$

$\therefore$ By BPT Theorem,

$$\frac{CF}{FB} = \frac{CO}{OA}$$



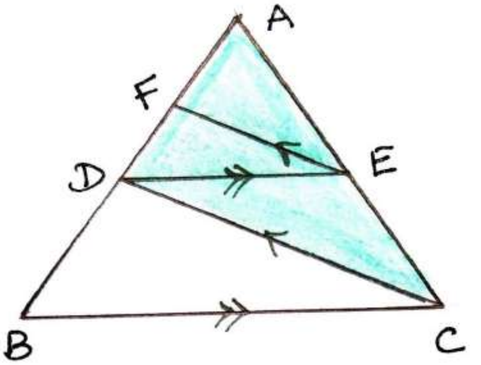
$$\boxed{\frac{BF}{FC} = \frac{AO}{OC}} \rightarrow ② \text{ (Reciprocal)}$$

From ① and ②

$$\frac{AE}{ED} = \frac{BF}{FC}$$

Hence Proved.

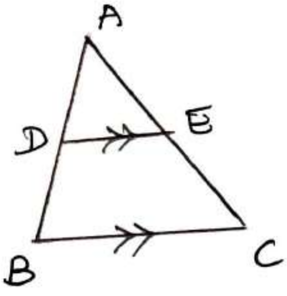
7) In figure DE∥BC and CD∥EF.
Prove that $AD^2 = AB \times AF$.



In Δ ABC, DE∥BC

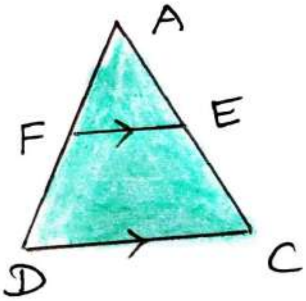
∴ By B.P.T Theorem,

$$\boxed{\frac{AD}{AB} = \frac{AE}{AC}} \rightarrow ①$$



In Δ ADC, CD ∥ EF

$$\boxed{\frac{AF}{AD} = \frac{AE}{AC}} \rightarrow ②$$



From ① and ②

$$\frac{AD}{AB} = \frac{AF}{AD}$$

$$AD^2 = AB \times AF$$

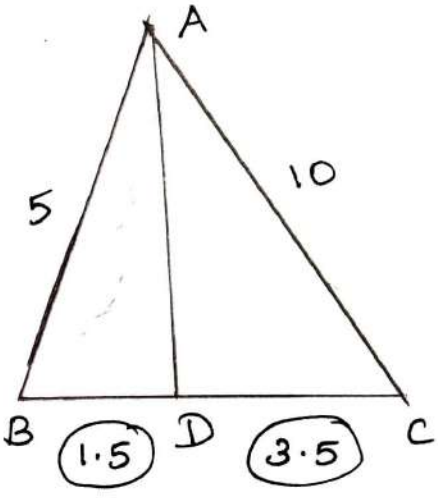
Hence proved.

---

8) check whether AD is bisector of ∠A of Δ ABC in each of the following.

(i) AB = 5 cm , AC = 10 cm, BD = 1.5 cm

CD = 3.5 cm



$$\boxed{\frac{AB}{AC} = \frac{BD}{DC}}$$

$$\frac{5}{10} = \frac{1.5}{3.5}$$

$$\frac{\overset{1}{\cancel{5}}}{\underset{2}{\cancel{10}}} = \frac{1.5\times10}{3.5\times10}$$

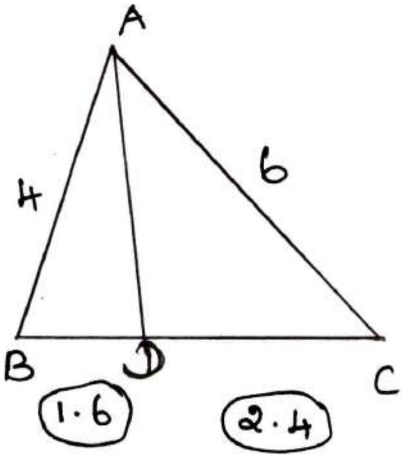
$$\frac{1}{2} = \frac{\cancel{15}^{\,3}}{\cancel{35}_{\,7}}$$

$$\boxed{\frac{1}{2} \neq \frac{3}{7}}$$

$\Rightarrow$ AD is not a bisector of ∠A

(ii) AB = 4 cm , AC = 6 cm,

BD = 1.6 cm , CD = 2.4 cm



$$\boxed{\frac{AB}{AC} = \frac{BD}{DC}}$$

$$\frac{4}{6} = \frac{1.6\times10}{2.4\times10}$$

$$\frac{\overset{2}{\cancel{4}}}{\underset{3}{\cancel{6}}} = \frac{\cancel{16}^{\,2}}{\cancel{24}_{\,3}}$$

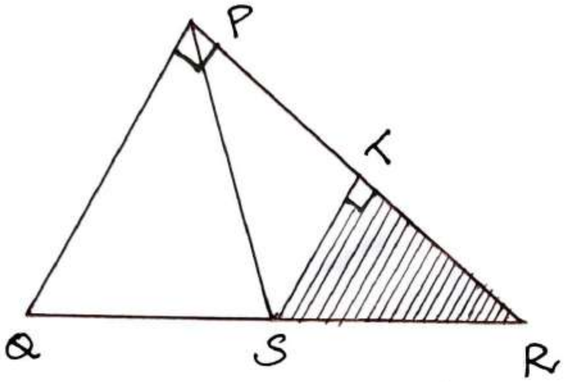
$$\boxed{\frac{2}{3} = \frac{2}{3}}$$

⇒ AD is a bisector of $\angle A$.

---

9) In figure $\angle QPR = 90°$, PS is its bisector.
IF $ST \perp PR$, prove that

$$ST \times (PQ + PR) = PQ \times PR.$$

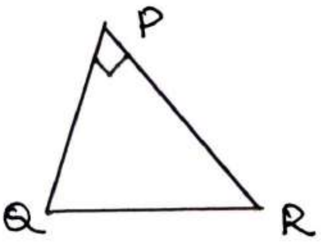


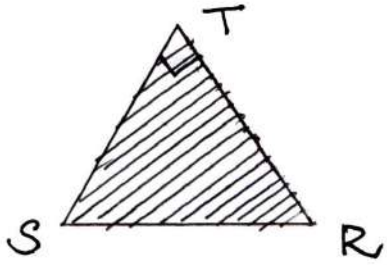
Given: PS is the bisector,

∴ By Angle Bisector Theorem,

$$\boxed{\frac{PQ}{PR} = \frac{QS}{SR}} \rightarrow ①$$

In $\triangle PQR$ and $\triangle TSR$





$\angle P = \angle T = 90°$
$\angle R$ is Common

$\therefore \boxed{\triangle PQR \sim \triangle TSR}$ (AA criterion)

Similar $\rightarrow$ $\dfrac{PQ}{TS} = \dfrac{QR}{SR}$

$$\frac{PQ}{TS} = \frac{QS + SR}{SR}$$

$$\frac{PQ}{TS} = \frac{QS}{SR} + \frac{\cancel{SR}}{\cancel{SR}}$$

$$\Downarrow$$

$$\frac{PQ}{ST} = \frac{PQ}{PR} + 1 \quad (\text{from } ①)$$

$$\frac{PQ}{ST} = \frac{PQ + PR}{PR}$$

$$\Rightarrow \boxed{PQ \times PR = ST\,(PQ + PR)}$$

Hence Proved.

---

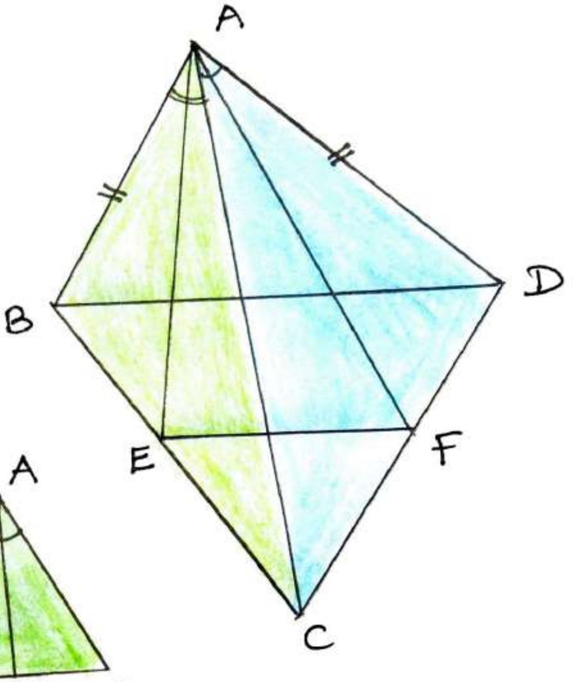
10) ABCD is a quadrilateral in which AB = AD, the bisector of $\angle BAC$ and $\angle CAD$ intersect the sides BC and CD at the points E and F respectively. Prove that EF $\parallel$ BD.

Given:

$\angle BAC$ and $\angle CAD$ $\Rightarrow$ the bisectors intersect the sides BC and CD at E and F.

Also, AB = AD

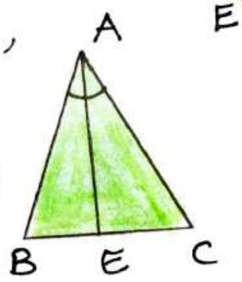
<ins>To prove:</ins> $\boxed{EF \parallel BD}$



AE is the bisector of $\angle BAC$

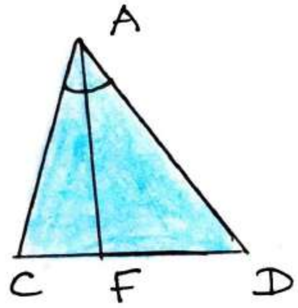
$\therefore$ By ABT Theorem,

$$\boxed{\frac{AB}{AC} = \frac{BE}{EC}} \rightarrow ①$$



AF is the bisector of $\angle CAD$

$$\frac{AC}{AD} = \frac{CF}{FD}$$



$$\frac{AD}{AC} = \frac{DF}{CF} \quad \text{(Reciprocal)}$$

$$\boxed{\frac{AB}{AC} = \frac{DF}{FC}} \rightarrow ② \quad [\because AB = AD]$$

From ① and ②

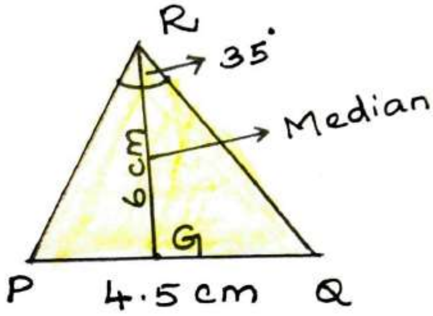
$$\frac{BE}{EC} = \frac{DF}{FC}$$

$$\Rightarrow BD \parallel EF$$

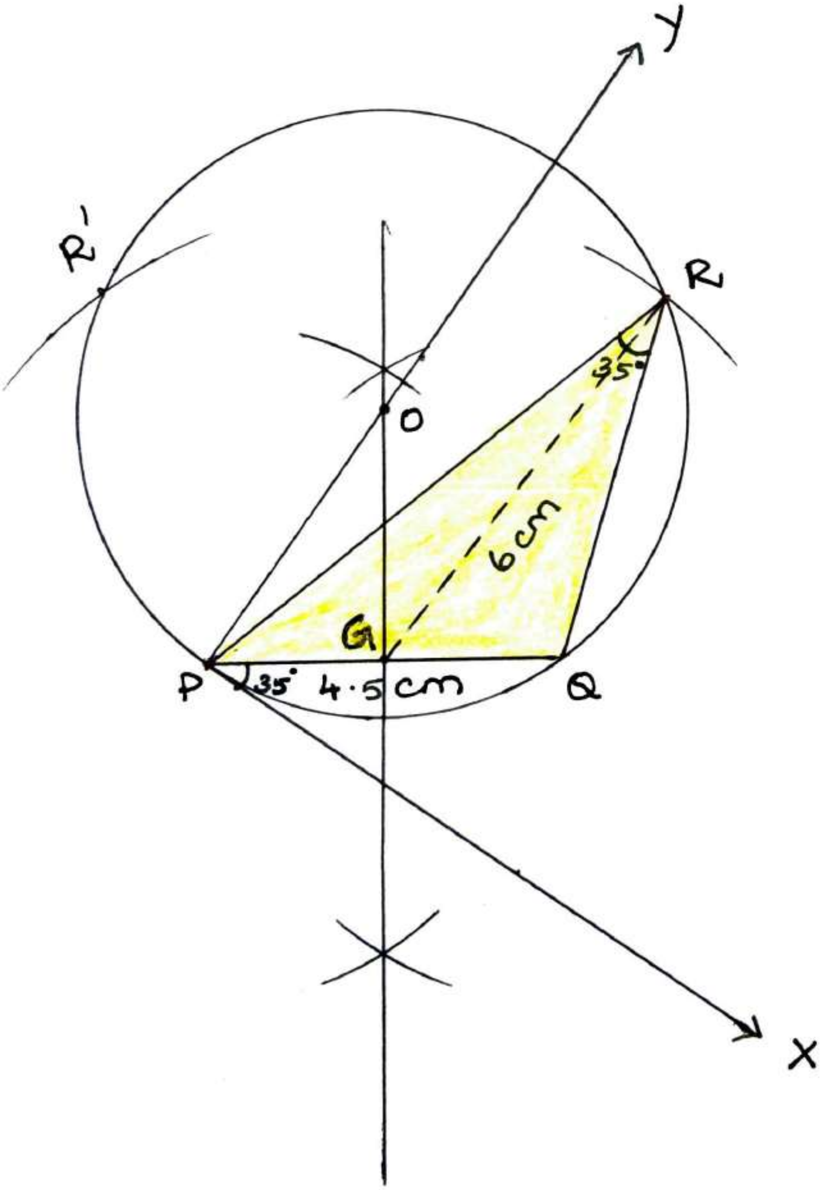
Hence Proved.

11) Construct a Δ PQR which the base PQ = 4.5 cm, ∠R = 35° and the median from R to RG is 6 cm.

ROUGH DIAGRAM



Median 6 cm



CONSTRUCTION:

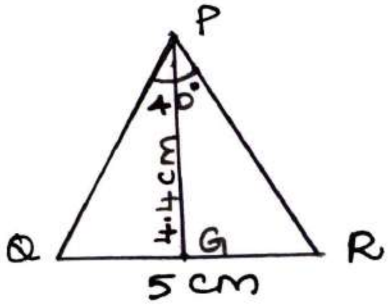
- * Draw PQ = 4.5 cm
- * Below PQ make an angle 35°
- * Draw PY ⊥ PX
- * Draw the perpendicular bisector of PQ which meets PQ at G and PY at 'O'.
- * With 'O' as centre draw a circle.
- * With 'G' as centre, 6 cm radius draw two arcs to meet the circle at R and R'.
- * Join PR and QR.
- * Thus △PQR is the required triangle.

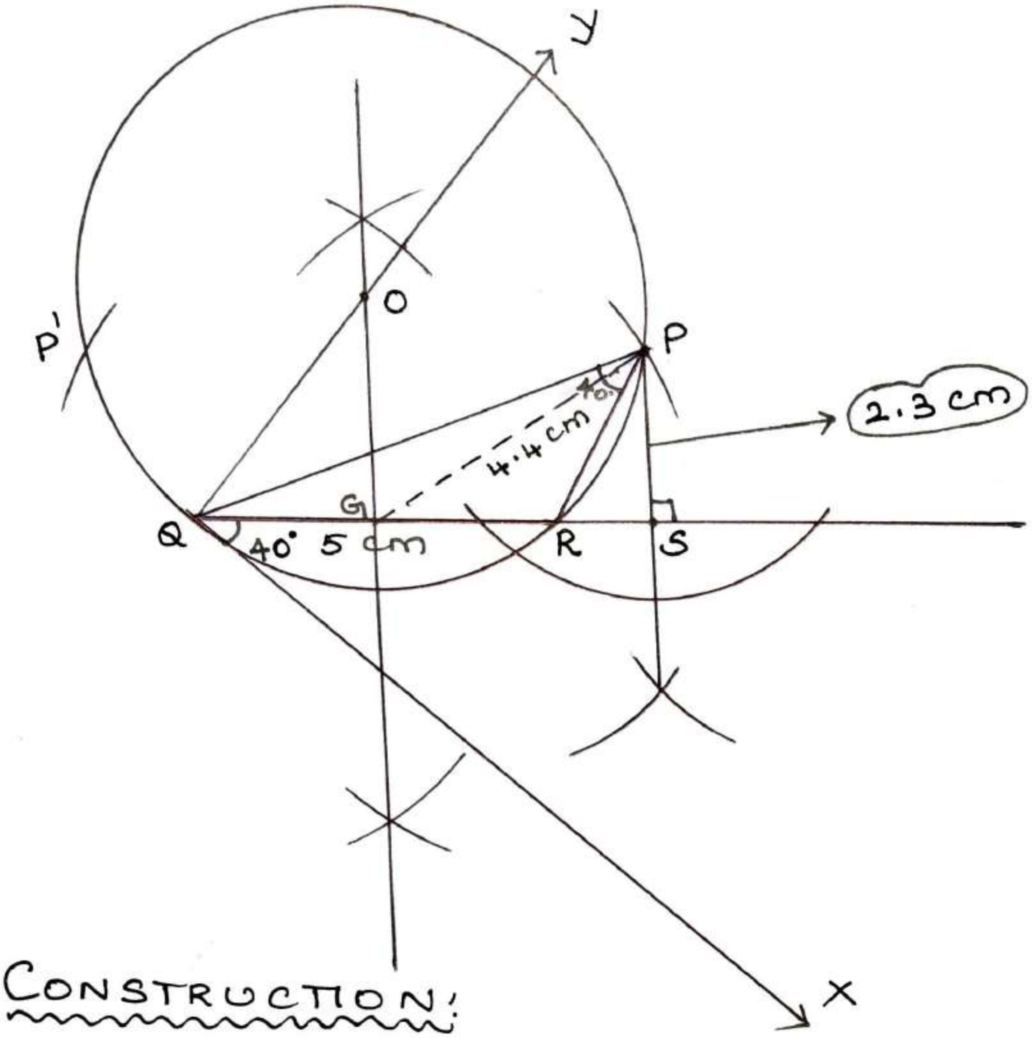
---

12) Construct a △PQR in which QR = 5 cm $\angle P = 40^\circ$ and the median PG from P to QR is 4.4 cm. Find the length of the altitude from P to QR.

ROUGH DIAGRAM

Median = 4.4 cm
Altitude?





CONSTRUCTION:

* Draw QR = 5cm
* Below QR make an angle 40°
* Draw QY ⊥ QX
* Draw the perpendicular bisector of QR to meet at G and O.
* With 'O' as centre draw a circle.
* With 'G' as centre, 4.4cm radius draw two arcs to meet at P and P'.
* Join PQ and PR
* Thus △ PQR is the required triangle.
* Extend QR and draw altitude from P

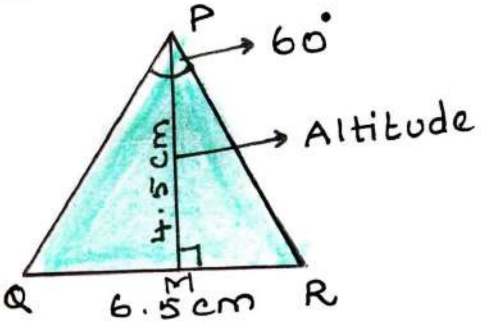
to meet at S.

* The length of the altitude PS = 2.3 cm

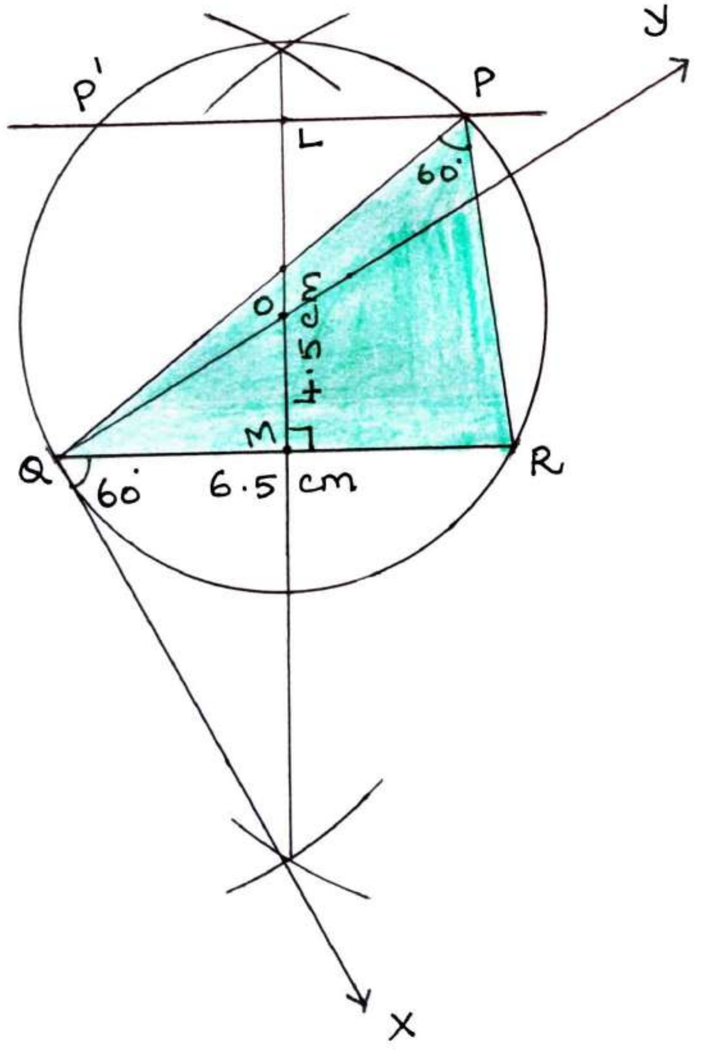
---

13) Construct $\triangle$ PQR, QR = 6.5 cm, $\angle P = 60°$, and the altitude from P to QR is of length 4.5 cm.

ROUGH DIAGRAM



Altitude - 4.5 cm



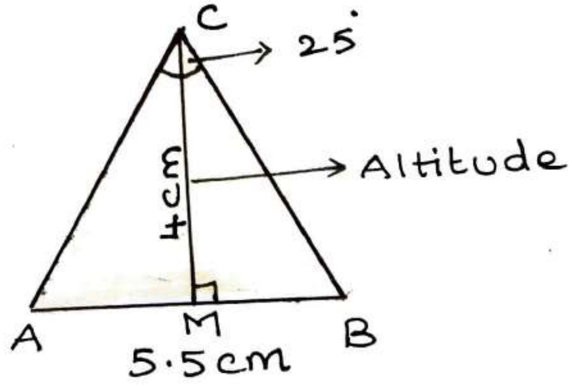
CONSTRUCTION:

- * Draw QR = 6.5cm
- * Below QR make an angle 60°
- * Draw $QY \perp QX$
- * Draw the perpendicular bisector of QR to meet at M and O.
- * With 'O' as centre draw the circle.
- * Mark ML = 4.5 cm
- * Through L draw a line parallel to QR to meet at P and P'.
- * Join PQ and PR
- * Thus $\triangle PQR$ is the required triangle.

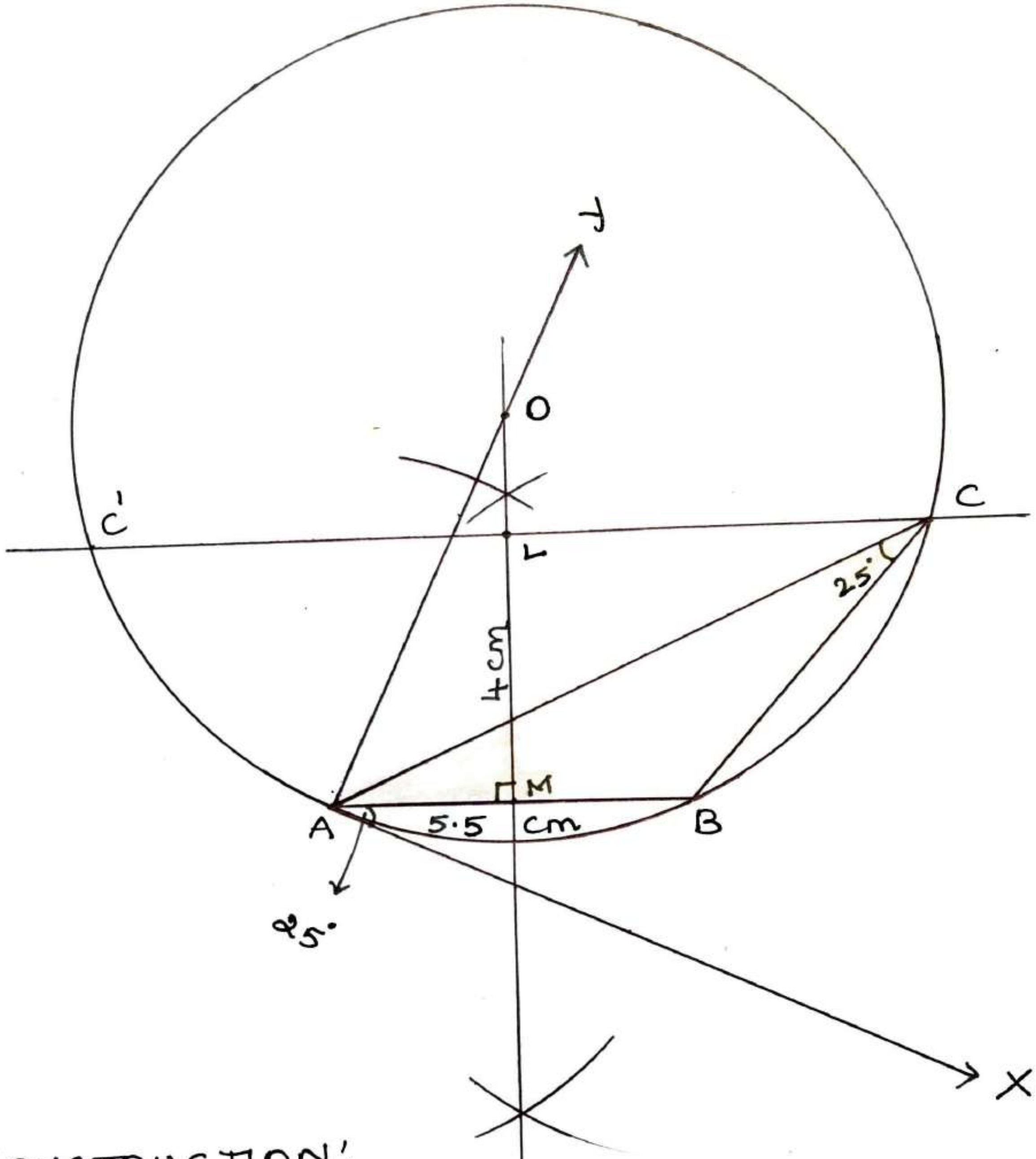
---

14) Construct a $\triangle ABC$ such that AB = 5.5cm, $\angle C = 25^\circ$ and the altitude from C to AB is 4cm.

ROUGH DIAGRAM



Altitude 4cm



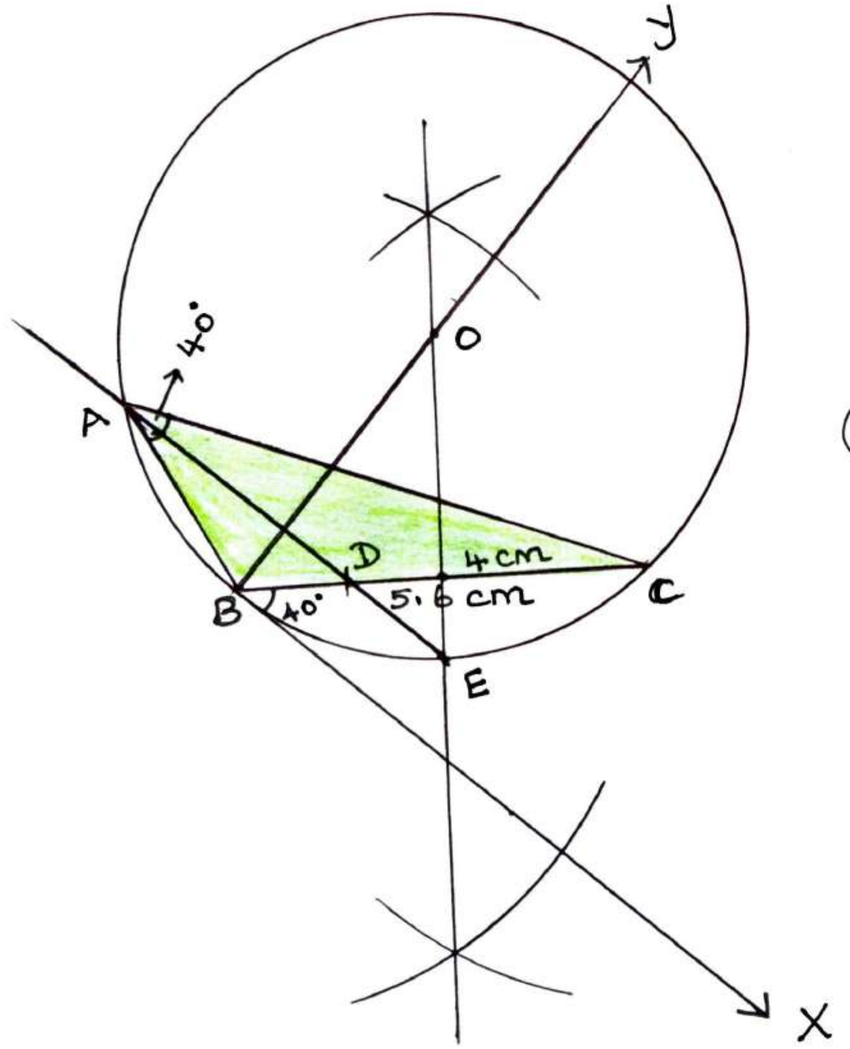
CONSTRUCTION:

- * Draw AB = 5.5cm
- * Below AB make an angle 25°
- * Draw AY ⊥ AX
- * Draw the perpendicular bisector of AB to meet at M and O.
- * With 'O' as centre draw the circle.

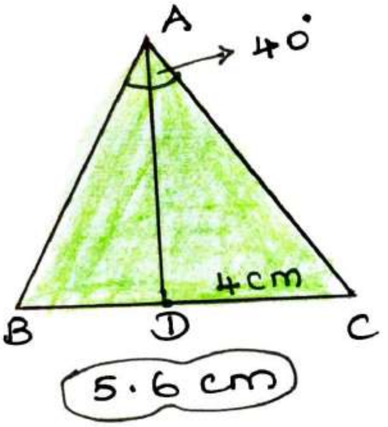
- Mark ML = 4cm
- Through 'L' draw a line parallel to AB to meet at C and C'
- Join AC and BC
- Thus Δ ABC is the required triangle.

---

15) Draw a triangle ABC of base BC = 5.6cm, ∠A = 40° and the bisector of ∠A meets BC at D such that CD = 4cm.



ROUGH DIAGRAM



Bisector
CD = 4cm

CONSTRUCTION:

- * Draw BC = 5.6 cm
- * Below BC make an angle 40°
- * Draw BY ⊥ BX
- * Draw the perpendicular bisector of BC to meet at 'O'.
- * With 'O' as centre draw the circle.
- * Mark CD = 4 cm.
- * Join ED and extend to meet the circle at A.
- * Join AB and AC
- * Thus △ ABC is the required triangle.

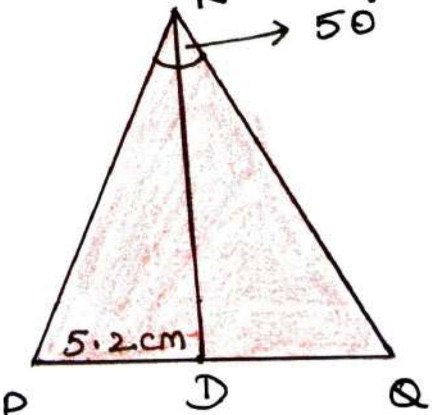
---

16) Draw △ PQR such that PQ = 6.8 cm, vertical angle is 50° and the bisector of the vertical angle meets the base at D where PD = 5.2 cm.

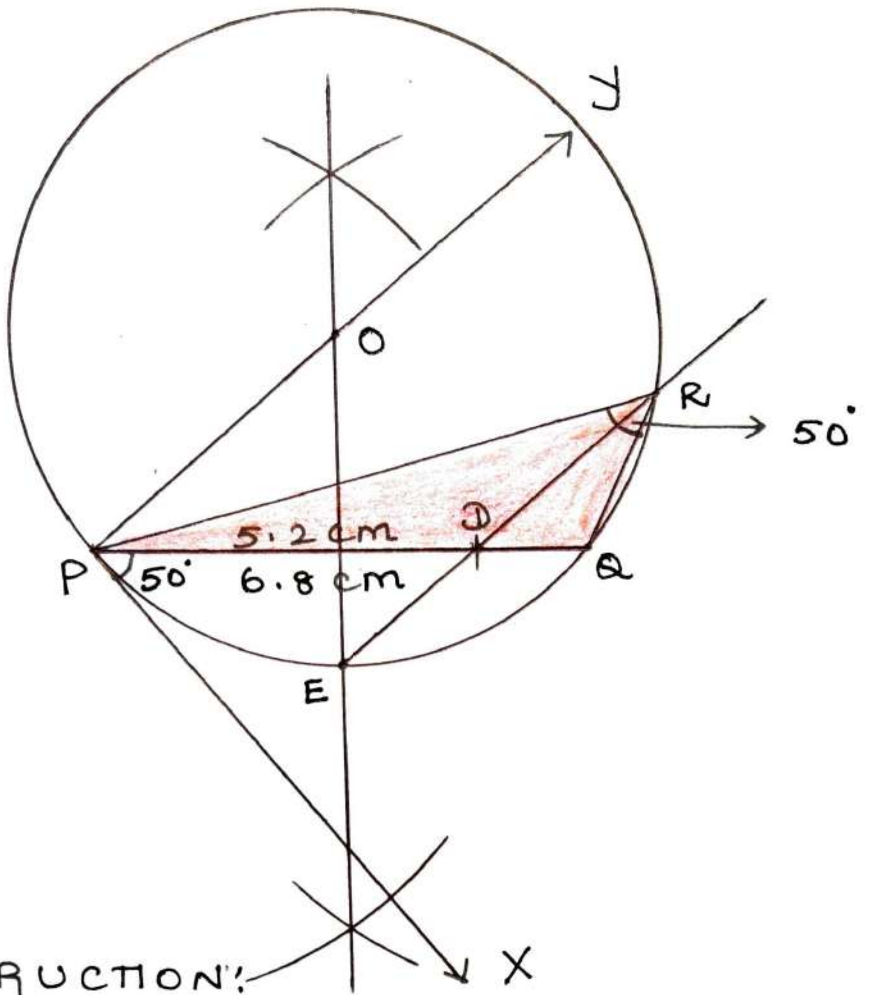
ROUGH DIAGRAM

Bisector

PD = 5.2 cm



6.8 cm



CONSTRUCTION:

- * Draw PQ = 6.8 cm
- * Below PQ make an angle 50°
- * Draw PY ⊥ PX
- * Draw the perpendicular bisector of PQ to meet at 'O'.
- * With 'O' as centre draw the circle.
- * Mark PD = 5.2 cm
- * Join ED and extend to meet the circle at R.
- * Join PR and QR.
- * Thus Δ PQR is the required triangle.

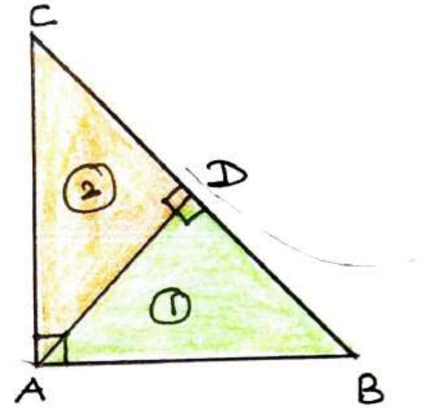
PYTHAGORAS THEOREM

Statement:

In a right angled triangle square on the hypotenuse is equal to the sum of the squares of other two sides.

Given:

In $\triangle ABC$, $\angle A = 90^\circ$



To prove:

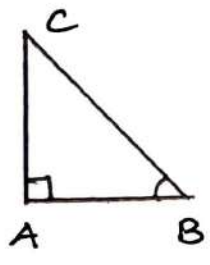
$$BC^2 = AB^2 + AC^2$$

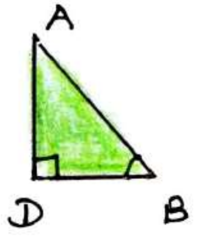
Construction:

Draw $AD \perp BC$

Proof:

In $\triangle ABC$ and $\triangle D\overset{①}{B}A$





$\angle A = \angle D = 90^\circ$

$\angle B$ is common.

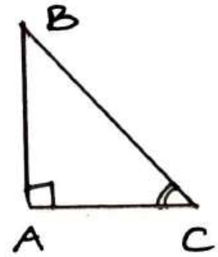
$\therefore \triangle ABC \sim \triangle DBA$

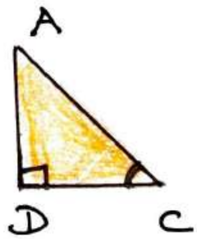
(AA criterion)

$$\Rightarrow \boxed{\frac{AB}{DB} = \frac{BC}{BA}}$$

$$AB^2 = BC \times DB \longrightarrow ①$$

In $\triangle ACB$ and $\triangle D\overset{②}{C}A$





$\angle A = \angle D = 90^\circ$

$\angle C$ is Common.

$\therefore \triangle ACB \sim \triangle DCA$

(AA criterion)

$$\Rightarrow \boxed{\frac{AC}{DC} = \frac{CB}{CA}}$$

$$AC^2 = BC \times DC \longrightarrow ②$$

Adding ① and ②

$AB^2 + AC^2 = (BC \times DB) + (BC \times DC)$

$AB^2 + AC^2 = BC\,[DB + DC]$

$AB^2 + AC^2 = BC\,(BC)$

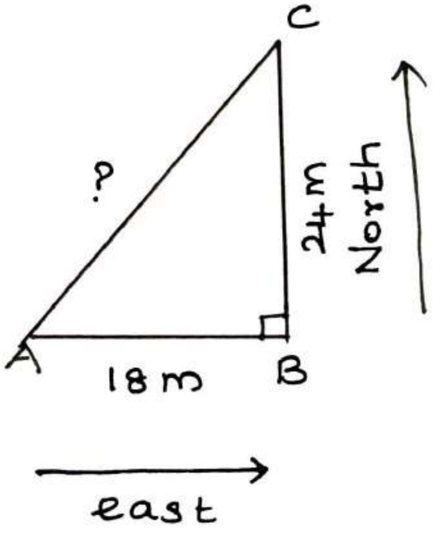
$AB^2 + AC^2 = BC^2$

$\Rightarrow \boxed{BC^2 = AB^2 + AC^2}$

Hence Proved.

EXERCISE 4.3

1) A man goes 18m due east and then 24 m due north. Find the distance of his current position from the starting point?



Man ⇒ 18m due East
24m due North

$\boxed{\text{Distance, } AC \Rightarrow ?}$

Using Pythagoras Theorem

$AC^2 = AB^2 + BC^2$

$AC^2 = 18^2 + 24^2$

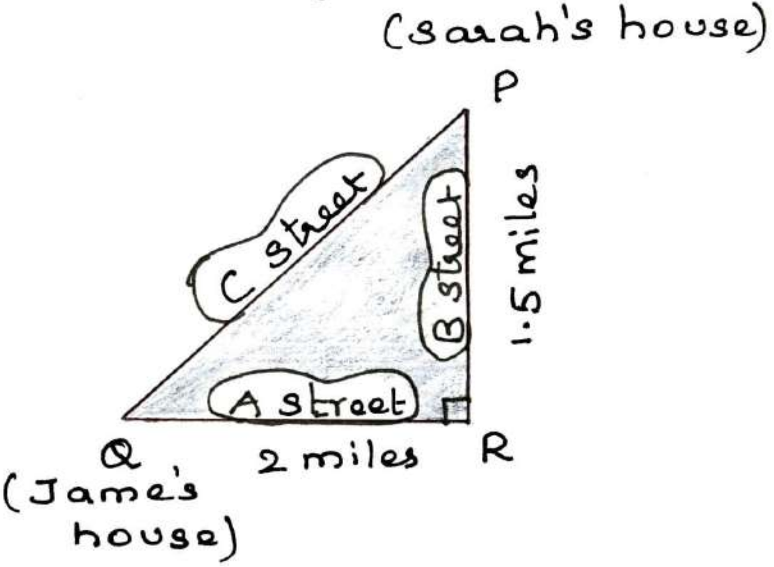
$AC^2 = 324 + 576$

$AC^2 = 900$

$AC^{\cancel{2}} = 30^{\cancel{2}}$

$\boxed{AC = 30m}$

2) There are two paths that one can choose to go from Sarah's house to James house. One way is to take C street, and the other way requires to make B street and then A street. How much shorter is the direct path along 'c' street? (using Figure)



Distance of streets

A and B

= 2 + 1.5 miles

= 3.5 miles

By Pythagoras Thm,

$$PQ^2 = QR^2 + PR^2$$

$$PQ^2 = (2)^2 + (1.5)^2$$

$$PQ^2 = 4 + 2.25$$

$$PQ^2 = 6.25$$

$$PQ^{\cancel{2}} = (2.5)^{\cancel{2}}$$

$$\boxed{PQ = 2.5 \text{ miles}} \Leftarrow \text{C street}$$

∴ Difference between two paths

= 3.5 miles − 2.5 miles = 1 mile

⇒ 1 mile distance shorter is the direct path along c street

3) To get from Point A to point B you must avoid walking through a pond. You must walk 34m south and 41m east. To the nearest meter, how many meters would be saved if it were possible to make a way through the pond?

By Pythagoras Theorem,

$AC^2 = AB^2 + BC^2$

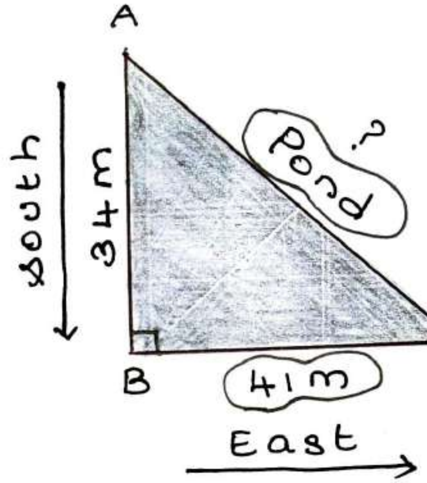
$AC^2 = 34^2 + 41^2$

$AC^2 = 1156 + 1681$

$AC^2 = 2837$

$AC = \sqrt{2837}$

$\boxed{AC = 53.26\text{ m}}$



$\Rightarrow AB + BC = 31 + 41 = \boxed{75\text{ m}}$

$\therefore$ The distance $= 75\text{ m} - 53.26\text{ m}$

$= 21.74\text{ m}$

$\therefore$ To the nearest, one can save 21.74 m through the pond.

---

4) In the rectangle WXYZ, XY + YZ = 17cm, and XZ + YW = 26cm, Calculate the length and breadth of the rectangle?

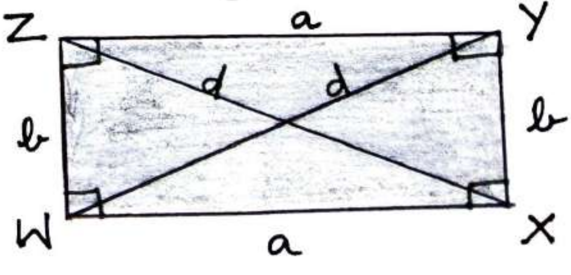
Rectangle

⇓

* opp sides equal

* diagonals equal.

⇐



GIVEN!

$xy + yz = 17\,cm$

and $\boxed{xz + yw = 26\,cm}$

$$\underset{\downarrow}{}\ \ \underset{\downarrow}{}$$

$d + d = 26$

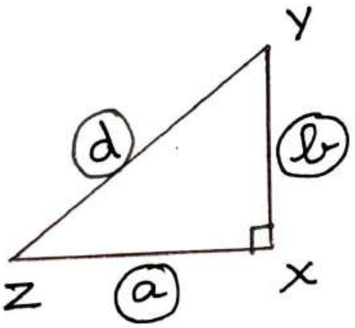
$2d = 26$

$d = \dfrac{\cancel{26}^{13}}{\cancel{2}}$

$\boxed{d = 13}$

In $\Delta\, XYW$,

$d^2 = a^2 + b^2$ $\Longleftarrow$



$13^2 = a^2 + b^2$

$\Rightarrow a^2 + b^2 = 169 \rightarrow ①$

GIVEN!

$XY + YZ = 17$

$b + a = 17 \rightarrow ②$

$b = 17 - a \rightarrow ③$

squaring ② on both sides

$(a+b)^2 = (17)^2$

$\underbrace{a^2 + b^2} + 2ab = 289$

$169 + 2ab = 289$

$2ab = 289 - 169$

$2ab = 120$

$ab = \dfrac{\cancel{120}^{60}}{\cancel{2}}$

$\boxed{56}$

$$\boxed{ab = 60}$$

$$a(17-a) = 60 \quad (\text{by } ③)$$

$$17a - a^2 = 60$$

$$a^2 - 17a + 60 = 0$$

$$(a-12)(a-5) = 0$$

60

$-12 \quad -5$

$a - 12 = 0$	$a - 5 = 0$
✓ $a = 12$	$a = 5$
Put $a = 12$ in ③	Put $a = 5$ in ③
$b = 17 - a$	$b = 17 - a$
$b = 17 - 12$	$b = 17 - 5$
✓ $b = 5$	$\boxed{b = 12}$ ×

⇒ Length of the rectangle $a = 12$ cm

breadth of the rectangle $b = 5$ cm

---

5) The hypotenuse of a right triangle is 6m more than twice of the shortest side. If the third side is 2m less than the hypotenuse, find the sides of the triangle.

⇒ Let shortest side $= x$

⇒ The hypotenuse of a right triangle

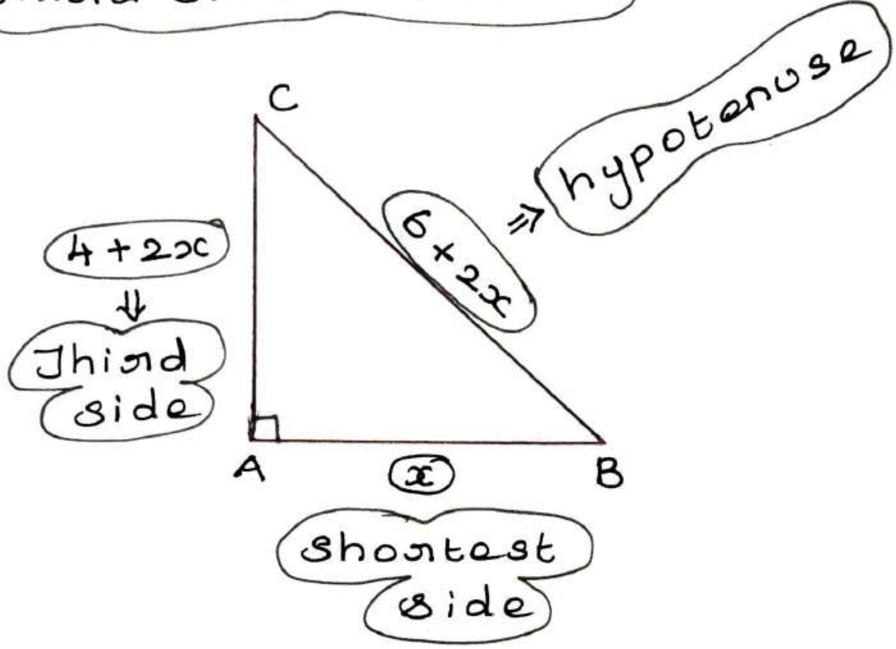
is 6m more than twice of the shortest side.

$\Rightarrow$ hypotenuse $= 6+2x$

The third side is 2m less than the hypotenuse.

$\Rightarrow$ Third side $= (6+2x)-2$

Third side $= 4+2x$.



$\Rightarrow$ By Pythagoras Theorem,

$$BC^2 = AB^2 + AC^2$$

$$(6+2x)^2 = x^2 + (4+2x)^2$$

$$(6)^2+(2x)^2+2(6)(2x) = x^2+(4)^2+(2x)^2+2(4)(2x)$$

$$36+\cancel{4x^2}+24x = x^2+16+\cancel{4x^2}+16x$$

$$36+24x = x^2+16+16x$$

$$x^2+\underbrace{16x-24x}+\underbrace{16-36}=0$$

$$\boxed{x^2-8x-20=0}$$

−20
+2 −10

$$(x+2)(x-10)=0$$

$x+2=0$ | $x-10=0$

× $\boxed{x=-2}$ | $\boxed{x=10}$ ✓

⇒ Shortest Side ⇒ $\boxed{x=10\,m}$

hypotenuse ⇒ $6+2x$

$$=6+2(10)$$

$$=6+20$$

⇒ $\boxed{\text{hypotenuse}=26\,m}$

Third Side ⇒ $4+2x$

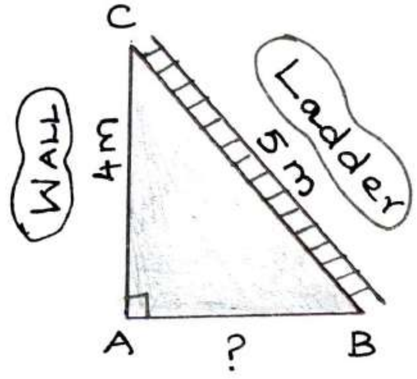
$$=4+2(10)$$

$$=4+20$$

⇒ $\boxed{\text{Third Side}=24\,m}$

---

6) 5m long Ladder is placed leaning towards a vertical wall such that it reaches the wall at a point 4m high, If foot of the Ladder is moved 1.6m towards the wall, then find the distance by which the top of the ladder would slide upwards on the wall.



In Δ ABC,

$$\boxed{BC^2 = AB^2 + AC^2}$$

$$5^2 = AB^2 + 4^2$$

$$25 = AB^2 + 16$$

$$AB^2 = 25 - 16$$

$$AB^2 = 9$$

$$AB^{\cancel{2}} = 3^{\cancel{2}}$$

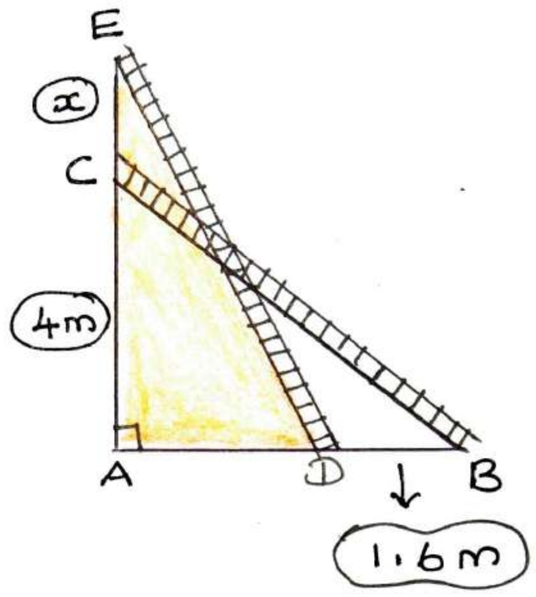
$$AB = 3\,m$$

$\Rightarrow$ $AD = AB - DB$

$= 3\,m - 1.6\,m$

$$AD = 1.4\,m$$

$$\begin{array}{r|l} & 4.8 \\ 4 & 23.04 \\ & 16 \\ 88 & 704 \\ & 704 \\ & 0 \end{array}$$



In Δ ADE,

$$\boxed{ED^2 = EA^2 + AD^2}$$

$$(5)^2 = (4+x)^2 + (1.4)^2$$

$$25 = (4+x)^2 + 1.96$$

$$(4+x)^2 + 1.96 = 25$$

$$(4+x)^2 = 25 - 1.96$$

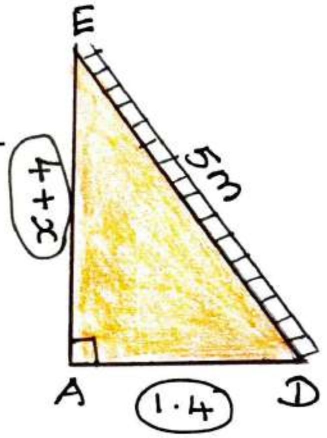
$$(4+x)^2 = 23.04$$

$$4 + x = \sqrt{23.04}$$

$$4 + x = 4.8$$

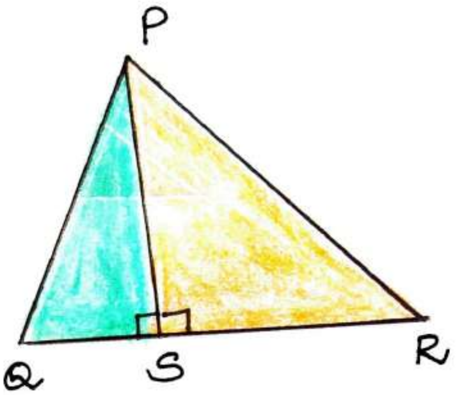
$$x = 4.8 - 4$$

$$x = 0.8\,m$$



$\Rightarrow$ Distance by which the top of the ladder moves upwards is 0.8 m.

7) The perpendicular PS on the base QR of a Δ PQR intersects QR at S, such that QS = 3 SR. Prove that $2PQ^2 = 2PR^2 + QR^2$.



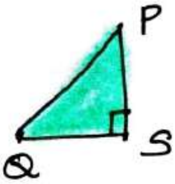
GIVEN: $\boxed{QS = 3SR} \rightarrow ①$

TO PROVE: $\boxed{2PQ^2 = 2PR^2 + QR^2}$

Now,

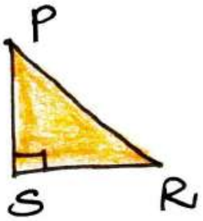
$$QR = QS + SR$$
$$QR = 3SR + SR \text{ (by ①)}$$
$$QR = 4SR \rightarrow ②$$

In Δ PSQ



$$\boxed{PQ^2 = PS^2 + QS^2} \rightarrow ③$$

In Δ PSR,



$$\boxed{PR^2 = PS^2 + SR^2} \rightarrow ④$$

$$PQ^2 - QS^2 = PS^2$$
$$PS^2 = PQ^2 - QS^2$$
$$PS^2 = PQ^2 - (3SR)^2 \text{ [From ①]}$$
$$\boxed{PS^2 = PQ^2 - 9SR^2} \rightarrow ⑤$$

Put ⑤ in ④

$$PR^2 = PS^2 + SR^2$$
$$PR^2 = [PQ^2 - 9SR^2] + SR^2$$
$$PR^2 = PQ^2 - 8SR^2$$

Multiply by 2

$$\Rightarrow 2PR^2 = 2PQ^2 - 16SR^2$$

$$2PR^2 = 2PQ^2 - (4SR)^2$$

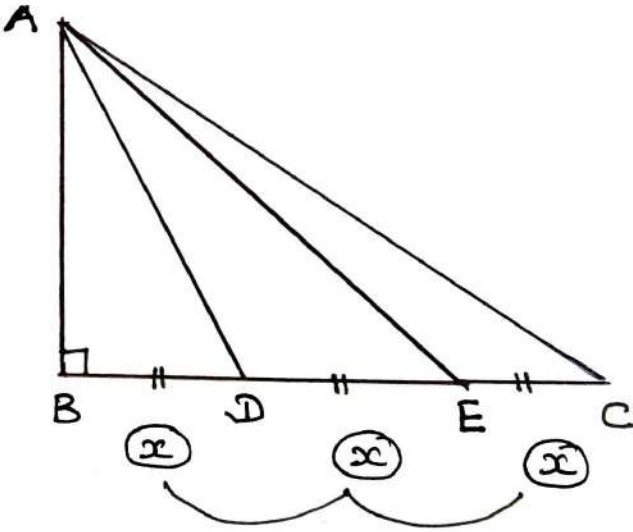
$$2PR^2 = 2PQ^2 - \overset{\downarrow}{QR^2} \quad [\text{From } ②]$$

$$2PQ^2 = 2PR^2 + QR^2$$

Hence Proved.

---

8) In the adjacent figure, ABC is a right angled triangle with right angle at B and points D, E trisect BC. Prove that $8AE^2 = 3AC^2 + 5AD^2$.



$\boxed{D, E \Rightarrow \text{trisect } BC}$

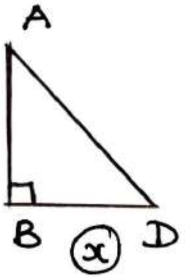
To prove:

$$8AE^2 = 3AC^2 + 5AD^2$$

In △ ABD,

$$AD^2 = AB^2 + BD^2$$

$$\boxed{AD^2 = AB^2 + x^2} \rightarrow ①$$

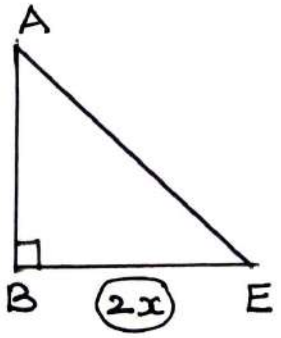


In △ ABE

$$AE^2 = AB^2 + BE^2$$

$$AE^2 = AB^2 + (2x)^2$$

$$\boxed{AE^2 = AB^2 + 4x^2} \rightarrow ②$$



In Δ ABC,

$$AC^2 = AB^2 + BC^2$$

$$AC^2 = AB^2 + (3x)^2$$

$$\boxed{AC^2 = AB^2 + 9x^2} \rightarrow ③$$

Now Multiply ② by 8

$$\boxed{8AE^2 = 8AB^2 + 32x^2} \rightarrow ④ \checkmark$$

Multiply ③ by 3

$$\Rightarrow \boxed{3AC^2 = 3AB^2 + 27x^2} \rightarrow ⑤$$

Multiply ① by 5

$$\Rightarrow \boxed{5AD^2 = 5AB^2 + 5x^2} \rightarrow ⑥$$

Adding ⑤ and ⑥

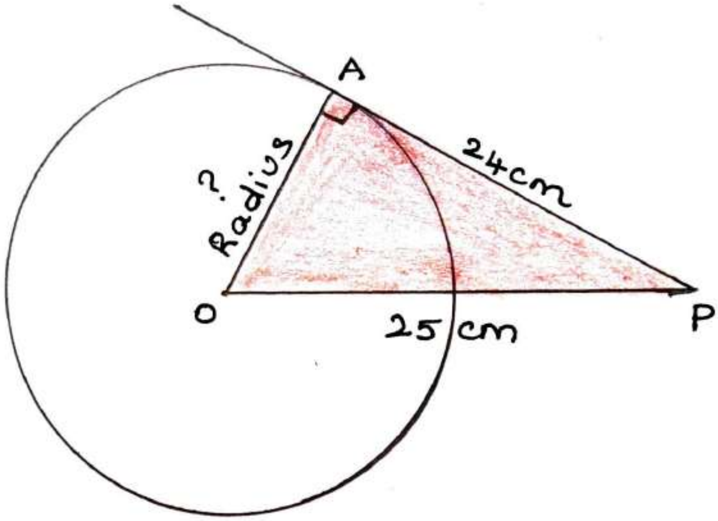
$$\boxed{3AC^2 + 5AD^2 = 8AB^2 + 32x^2} \rightarrow ⑦ \checkmark$$

From ④ and ⑦

$$8AE^2 = 3AC^2 + 5AD^2$$

Ex 4.4

1) The length of the tangent to a circle from a point P, which is 25 cm away from the centre is 24 cm. What is the radius of the circle?



In $\triangle OAP$,

$OP^2 = OA^2 + AP^2$

$25^2 = OA^2 + 24^2$

$625 = OA^2 + 576$

$OA^2 + 576 = 625$

$OA^2 = 625 - 576$

$OA^2 = 49$

$OA^{\cancel{2}} = 7^{\cancel{2}}$

$\boxed{OA = 7\,cm}$

$\Rightarrow$ Radius of the circle = 7cm

---

2) $\triangle LMN$ is a right angled triangle with $\angle L = 90^\circ$. A circle is inscribed in it. The lengths of the sides containing the right angle are 6 cm and 8 cm. Find the radius of the circle.

Tangents:

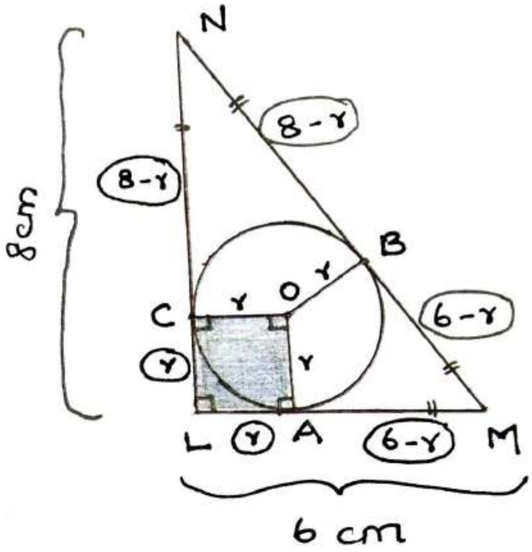
$LA = LC \rightarrow$ ①

$MA = MB \rightarrow$ ②

$NB = NC \rightarrow$ ③

OCLA is a Square.

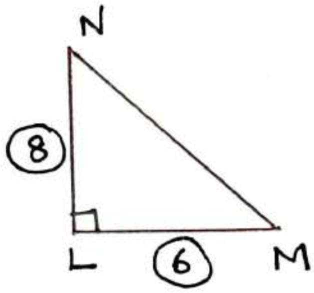
$\Rightarrow CL = LA = r$



$\therefore$ $MA = 6 - r \Rightarrow MB = 6 - r$

$NC = 8 - r \Rightarrow NB = 6 - r$

In $\triangle LMN$,



$MN^2 = LM^2 + LN^2$

$MN^2 = 6^2 + 8^2$

$MN^2 = 36 + 64$

$MN^2 = 100$

$MN^{\cancel{2}} = 10^{\cancel{2}}$

$\boxed{MN = 10\,cm}$

We know that

$MN = NB + BM$

$10 = (8 - r) + (6 - r)$

$10 = 14 - 2r$

$2r = 14 - 10$

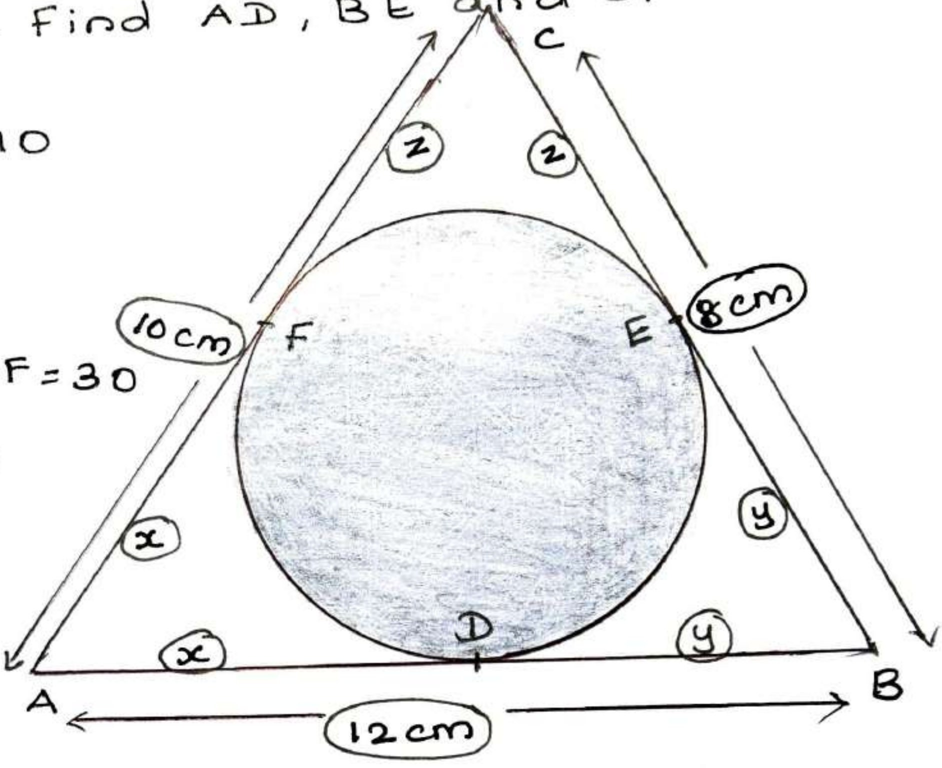
$2r = 4$

$r = \frac{\cancel{4}\,2}{\cancel{2}}$

$\boxed{r = 2\,cm}$

$\Rightarrow$ Radius $= 2\,cm$

3) A circle is inscribed in ΔABC having sides 8 cm, 10 cm and 12 cm as shown in figure. Find AD, BE and CF.



Perimeter = 12 + 8 + 10

$$\Downarrow$$

$$AB + BC + AC = 30$$

$$AD + DB + BE + EC + CF + AF = 30$$

$$x + y + y + z + z + x = 30$$

$$2x + 2y + 2z = 30$$

$$2(x + y + z) = 30$$

$$x + y + z = \frac{\cancel{30}^{15}}{\cancel{2}}$$

$$\Rightarrow \boxed{x + y + z = 15} \rightarrow ①$$

$AB = 12$ cm

$\boxed{x + y = 12}$

$BC = 8$ cm

$\boxed{y + z = 8}$

$AC = 10$ cm

$\boxed{x + z = 10}$

$① \Rightarrow \underbrace{x+y}+z = 15$

$12 + z = 15$

$z = 15 - 12$

$\boxed{z = 3}$

$x + \underbrace{y+z} = 15$

$x + 8 = 15$

$x = 15 - 8$

$\boxed{x = 7}$

$\underbrace{x+y+z} = 15$

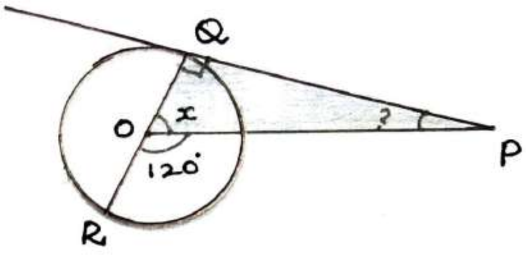
$y + 10 = 15$

$y = 15 - 10$

$\boxed{y = 5}$

$$\Rightarrow \boxed{\begin{array}{l} AD \Rightarrow x = 7\text{ cm} \\ BE \Rightarrow y = 5\text{ cm} \\ CF \Rightarrow z = 3\text{ cm} \end{array}}$$

4) PQ is a tangent drawn from a point P to a circle with centre O and QOR is a diameter of the circle such that $\angle POR = 120^\circ$. Find $\angle OPQ$.



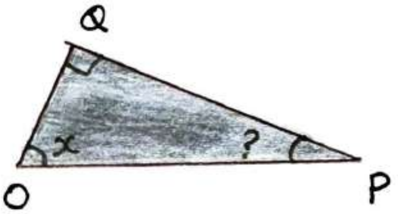
RQ → straight line

$120^\circ + x = 180^\circ$

$x = 180^\circ - 120^\circ$

$\boxed{x = 60^\circ}$

In $\triangle OQP$,



$90^\circ + x + \angle OPQ = 180^\circ$

$90^\circ + 60^\circ + \angle OPQ = 180^\circ$

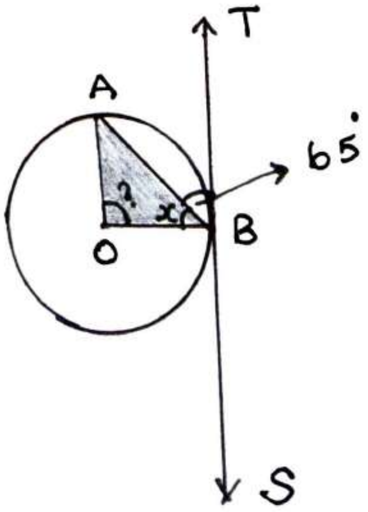
$150^\circ + \angle OPQ = 180^\circ$

$\angle OPQ = 180^\circ - 150^\circ$

$\boxed{\angle OPQ = 30^\circ}$

---

5) A tangent ST to a circle touches it at B. AB is a chord such that $\angle ABT = 65^\circ$. Find $\angle AOB$, where 'O' is the centre of the circle.



We know that

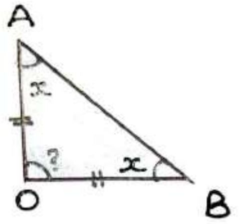
$\angle OBT = 90^\circ$

$\Rightarrow x + 65^\circ = 90^\circ$

$x = 90^\circ - 65^\circ$

$$\boxed{x = 25^\circ}$$

In $\triangle AOB$,



$$x + x + \angle AOB = 180^\circ$$

$$25^\circ + 25^\circ + \angle AOB = 180^\circ$$

$$50^\circ + \angle AOB = 180^\circ$$

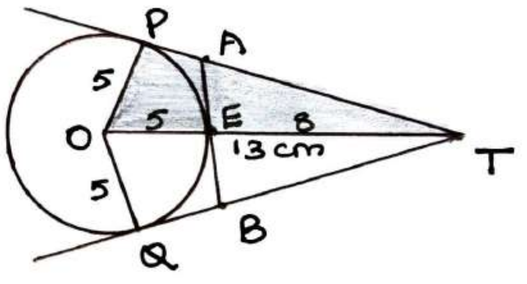
$$\angle AOB = 180^\circ - 50^\circ$$

$$\angle AOB = 130^\circ$$

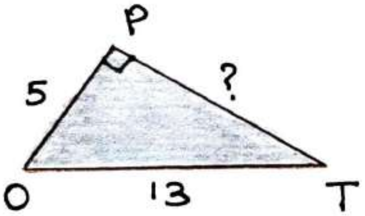
OA, OB → radius

---

6) In figure, O is the centre of the circle with radius 5 cm. T is a point such that OT = 13 cm and OT intersects the circle E. If AB is the tangent to the circle at E, find the length of AB.



In $\triangle OTP$,



$$OT^2 = OP^2 + PT^2$$

$$13^2 = 5^2 + PT^2$$

$$169 = 25 + PT^2$$

$$25 + PT^2 = 169$$

$$PT^2 = 169 - 25$$

$$PT^2 = 144$$

$$PT^{\cancel{2}} = 12^{\cancel{2}}$$

$$\boxed{PT = 12\,cm}$$

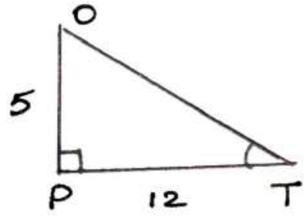
$$\underbrace{OP, OQ, OE}_{\text{Radius } 5cm}$$

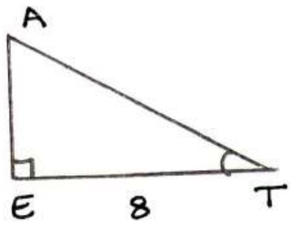
$$\boxed{OT = 13cm}$$

$$\therefore ET = 13cm - 5cm$$

$$\boxed{ET = 8cm}$$

In $\triangle$ OPT and $\triangle$ AET





$\angle P = \angle E = 90^\circ$

$\angle T$ is Common.

$\therefore \boxed{\triangle OPT \sim \triangle AET}$ (AA criterion)

$$\Rightarrow \frac{OP}{AE} = \frac{PT}{ET}$$

$$\frac{5}{AE} = \frac{12}{8}$$

$$5 \times 8 = 12 \times AE$$

$$12 \times AE = 5 \times 8$$

$$AE = \frac{5 \times \cancel{8}^{2}}{\cancel{12}_{3}}$$

$$\boxed{AE = \frac{10}{3}}$$

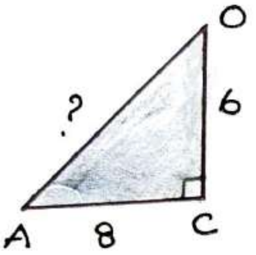
$\Rightarrow$ Length of $AB = AE + EB$

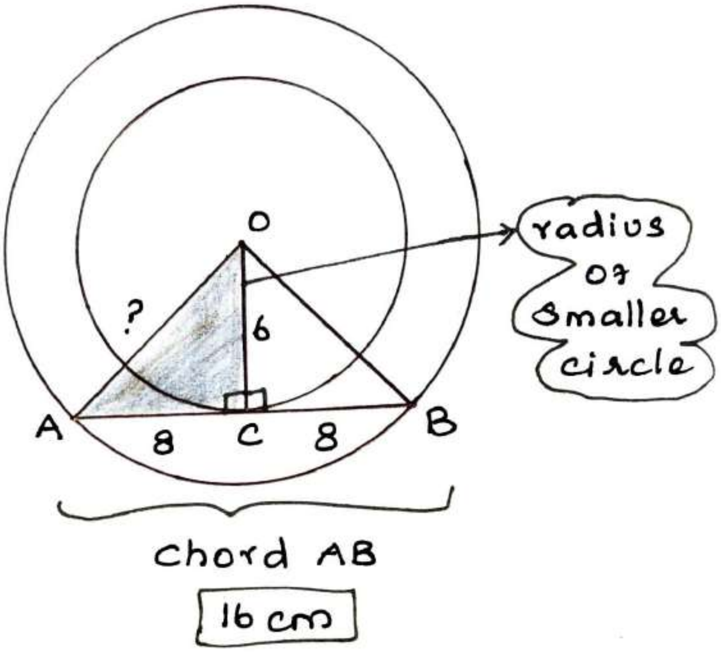
$$= \frac{10}{3} + \frac{10}{3}$$

$$= \frac{10+10}{3}$$

$$AE = \frac{20}{3} \text{ cm}$$

7) In two Concentric circles, a chord of Length 16 cm of Larger circle becomes a tangent to the smaller circle whose radius is 6cm. Find the radius of the larger circle.





$$\boxed{OA^2 = OC^2 + AC^2}$$

$$OA^2 = 6^2 + 8^2$$

$$OA^2 = 36 + 64$$

$$OA^2 = 100$$

$$OA^{\cancel{2}} = 10^{\cancel{2}}$$

$$\boxed{OA = 10\,cm}$$

$\Rightarrow$ Radius of the Larger circle = 10 cm

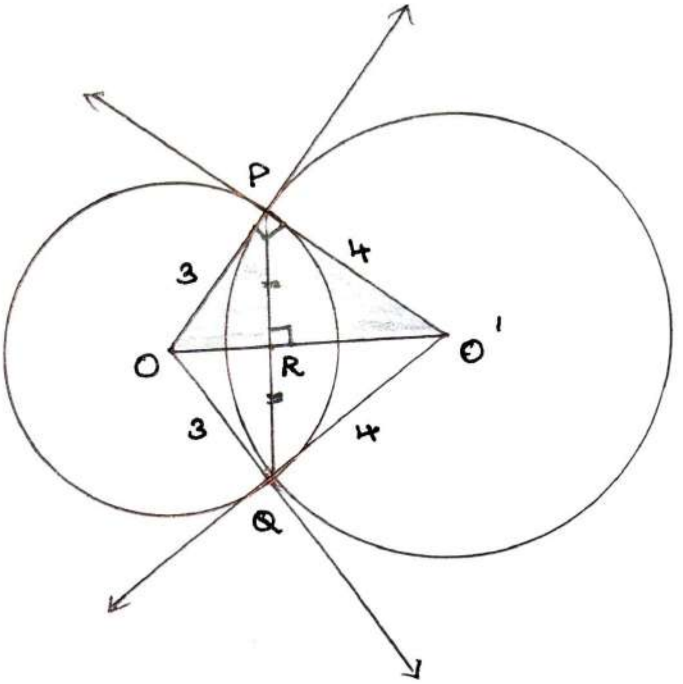
---

8) Two Circles with centres O and O' of radii 3 cm and 4 cm, respectively intersect at two points P and Q, such that OP and O'P are tangents to the two circles. Find the length of the common chord PQ.

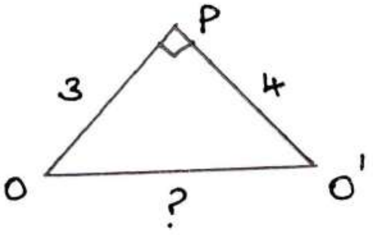
PQ → Common chord

OO' → Perpendicular bisector of PQ

$\Rightarrow \boxed{PR = QR}$



In Δ OPO'



$$(OO')^2 = OP^2 + PO'^2$$

$$(OO')^2 = 3^2 + 4^2$$

$$(OO')^2 = 9 + 16$$

$$(OO')^2 = 25$$

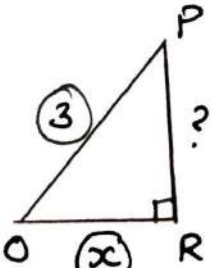
$$(OO')^{\cancel{2}} = 5^{\cancel{2}}$$

$$\boxed{OO' = 5\,cm}$$

$$(a-b)^2 = a^2 + b^2 - 2ab$$

$\Rightarrow$ OR = x , RO' = $5 - x$

In Δ OPR,



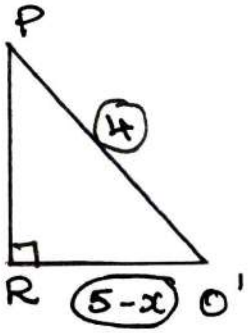
$$OP^2 = OR^2 + PR^2$$

$$3^2 = x^2 + PR^2$$

$$x^2 + PR^2 = 9$$

$$\boxed{PR^2 = 9 - x^2} \longrightarrow ①$$

In Δ PRO'



$$(PO')^2 = PR^2 + (RO')^2$$

$$4^2 = PR^2 + (5-x)^2$$

$$PR^2 + (5-x)^2 = 4^2$$

$$PR^2 + (5)^2 + (x)^2 - 2(5)(x) = 16$$

$$PR^2 + 25 + x^2 - 10x = 16$$

$$PR^2 = 16 - 25 - x^2 + 10x$$

$$\boxed{PR^2 = -9 - x^2 + 10x} \longrightarrow ②$$

From ① and ②

$$9-\cancel{x^2} = -9-\cancel{x^2}+10x$$

$$9+9 = 10x$$

$$18 = 10x$$

$$10x = 18$$

$$x = \frac{18}{10}$$

$$\boxed{x = 1.8\text{ cm}}$$

Put $x = 1.8$ cm in ①

$$PR^2 = 9 - x^2$$

$$PR^2 = 9-(1.8)^2$$

$$PR^2 = 9 - 3.24$$

$$PR^2 = 5.76$$

$$PR^{\cancel{2}} = (2.4)^{\cancel{2}}$$

$$\boxed{PR = 2.4\text{ cm}}$$

⑥

$$\begin{array}{r} 1.8 \times 1.8 \\ \hline 144 \\ 18 \\ \hline 3.24 \\ \hline \end{array}$$

Similarly $\boxed{RQ = 2.4\text{ cm}}$

∴ Length of Common Chord

$$PQ = PR + RQ$$

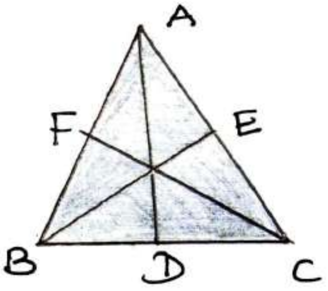
$$= 2.4 + 2.4$$

$$PQ = 4.8\text{ cm}$$

CEVA'S THEOREM: (Without Proof)

In △ ABC, D, E, F be the points on BC, AC and AB, then the cevians AD, BE, CF are congruent if and only if

$$\frac{BD}{DC} \times \frac{CE}{EA} \times \frac{AF}{FB} = 1$$



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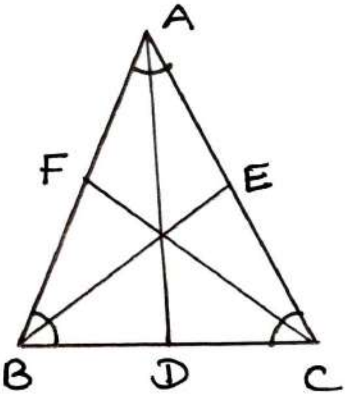
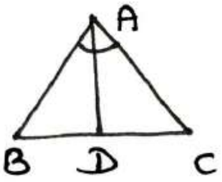
9) Show that the angle bisectors of a triangle are congruent.



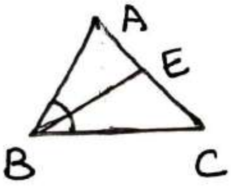
⇒ AD is the bisector of $\angle A$

$$\boxed{\frac{BD}{DC} = \frac{AB}{AC}} \rightarrow ①$$



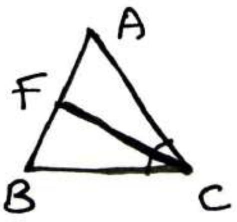
⇒ BE is the bisector of $\angle B$

$$\boxed{\frac{CE}{EA} = \frac{BC}{BA}} \rightarrow ②$$



⇒ CF is the bisector of $\angle C$

$$\boxed{\frac{AF}{FB} = \frac{CA}{CB}} \rightarrow ③$$



Multiplying ①, ②, ③

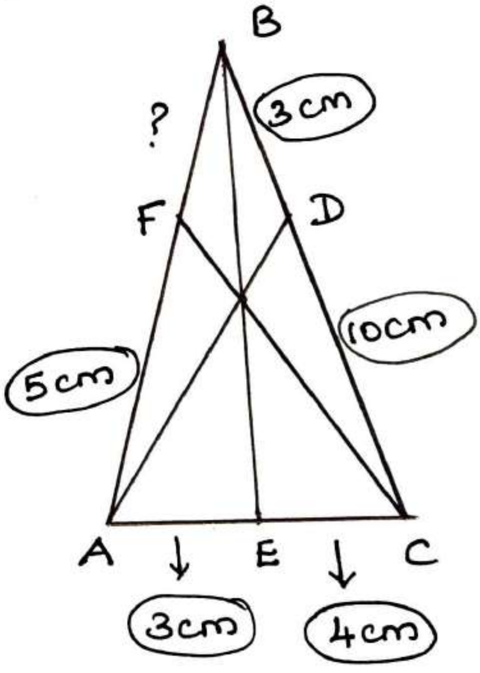
$$\frac{BD}{DC} \times \frac{CE}{EA} \times \frac{AF}{FB} = \frac{A\cancel{B}}{A\cancel{C}} \times \frac{\cancel{B}C}{B\cancel{A}} \times \frac{\cancel{C}A}{C\cancel{B}}$$

$$\boxed{\frac{BD}{DC} \times \frac{CE}{EA} \times \frac{AF}{FB} = 1}$$

∴ By Ceva's Theorem, the angle bisectors of a triangle are congruent.

---

10) An artist has created a triangular stained glass window and has one strip of small length left before completing the window. She needs to figure out the length of left out portions based on the lengths of the other sides as shown in the figure.



By Ceva's Theorem,

$$\frac{AE}{EC} \times \frac{CD}{DB} \times \frac{BF}{FA} = 1$$

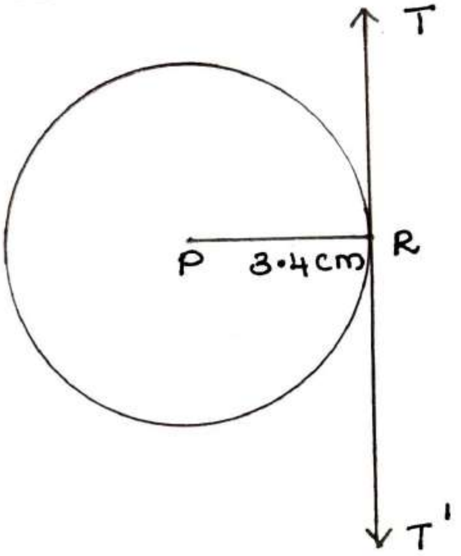
$$\frac{\cancel{3}}{\cancel{4}_{2}} \times \frac{\cancel{10}^{2}}{\cancel{3}} \times \frac{BF}{\cancel{5}} = 1$$

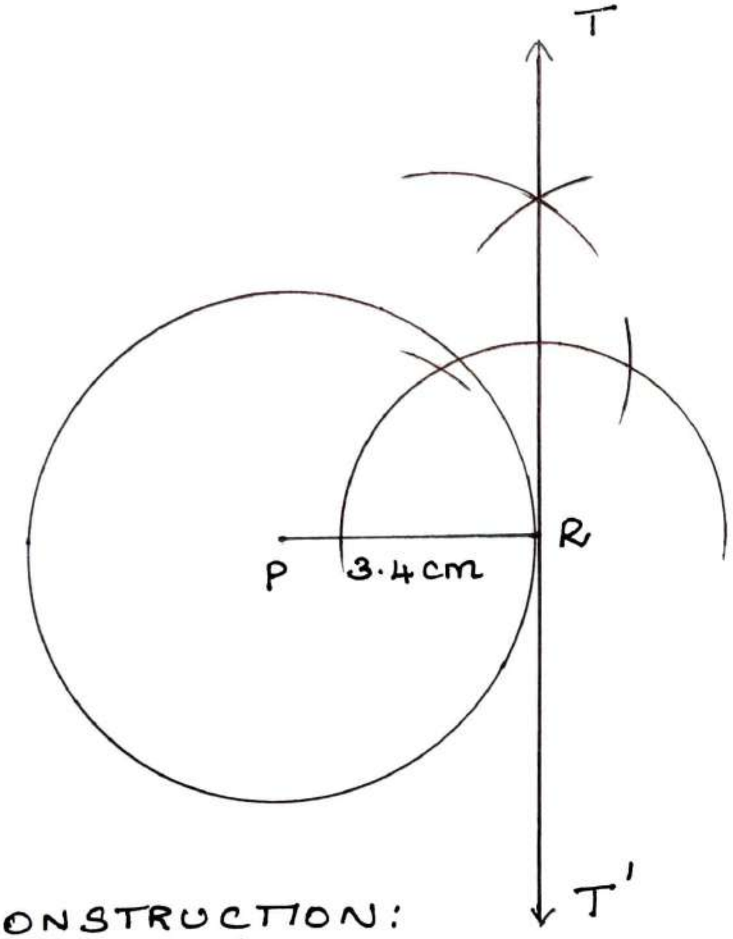
$$\frac{BF}{2} = 1 \rightarrow$$

$$\boxed{BF = 2cm}$$

11) Draw a tangent at any point R on the circle of radius 3.4 cm and centre at P?

ROUGH DIAGRAM

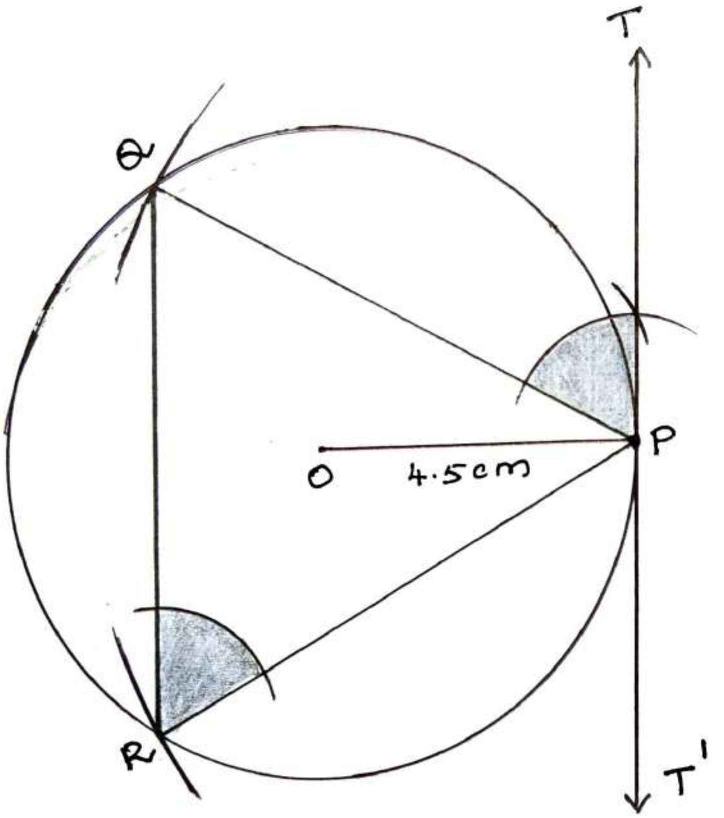




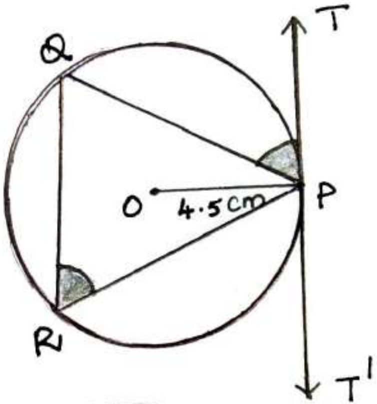
CONSTRUCTION:

- * Draw a circle of radius 3.4cm with 'P' as centre.
- * Take a point 'R' on the circle.
- * Join PR.
- * Draw the perpendicular line TT' to PR which passes through R.
- * TT' is the required tangent.

12) Draw a circle of radius 4.5 cm. Take a point on the circle. Draw the tangent at that point using the alternate segment theorem.



ROUGH DIAGRAM



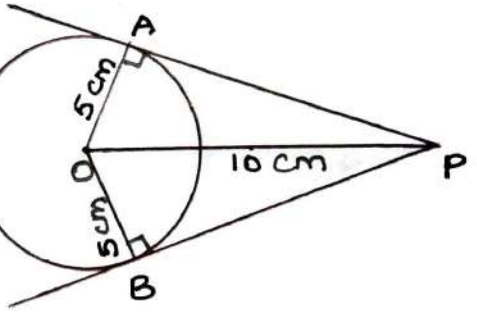
ALTERNATE SEGMENT THEOREM

CONSTRUCTION:

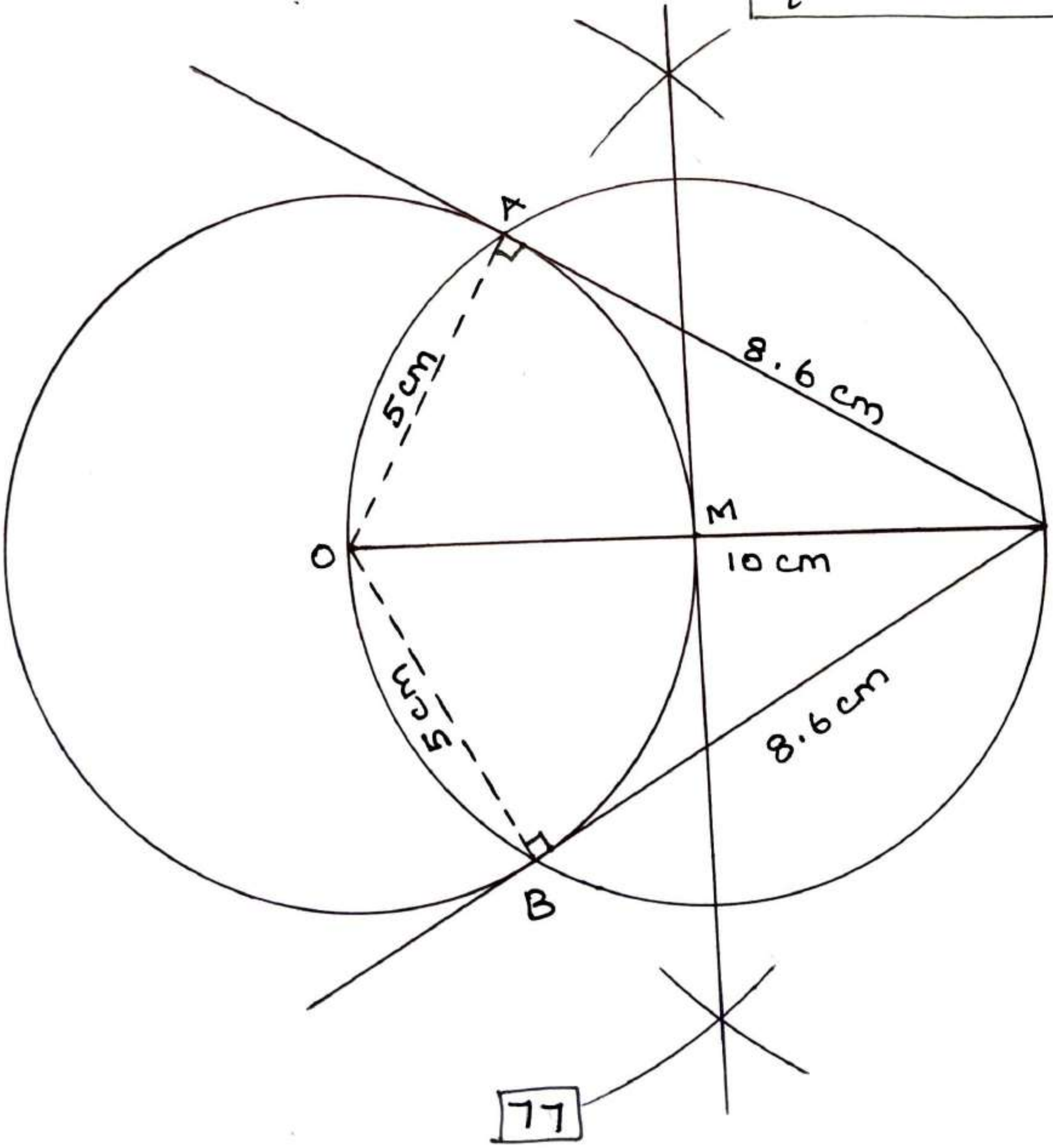
* With 'O' as centre draw a circle of radius 4.5 cm.
* Take a point 'P' on the circle.
* From 'P' draw two arcs to meet at Q and R. Join PQ, PR and QR.
* Through 'P' draw a tangent TT' such that $\angle TPQ = \angle QRP$
* TT' is the required tangent.

13. Draw two tangents from a point which is 10cm away from the centre of a circle of radius 5cm. Also, measure the lengths of the tangents.

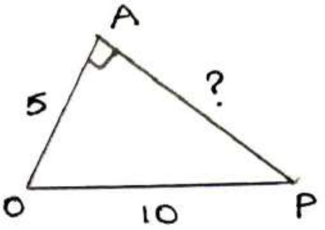
ROUGH DIAGRAM



Radius - 5cm



VERIFICATION:



$OP^2 = OA^2 + PA^2$

$10^2 = 5^2 + PA^2$

$100 = 25 + PA^2$

$25 + PA^2 = 100$

$PA^2 = 100 - 25$

$PA^2 = 75$

$PA = \sqrt{75}$

$\boxed{PA = 8.6 cm}$

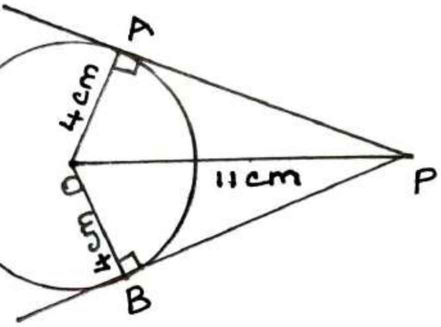
$$\begin{array}{r|l} & 8.6 \\ \hline 8 & 75.00 \\ & 64\downarrow\downarrow \\ \hline 166 & 1100 \\ & 996 \\ \hline & 104 \\ \hline \end{array}$$

CONSTRUCTION:

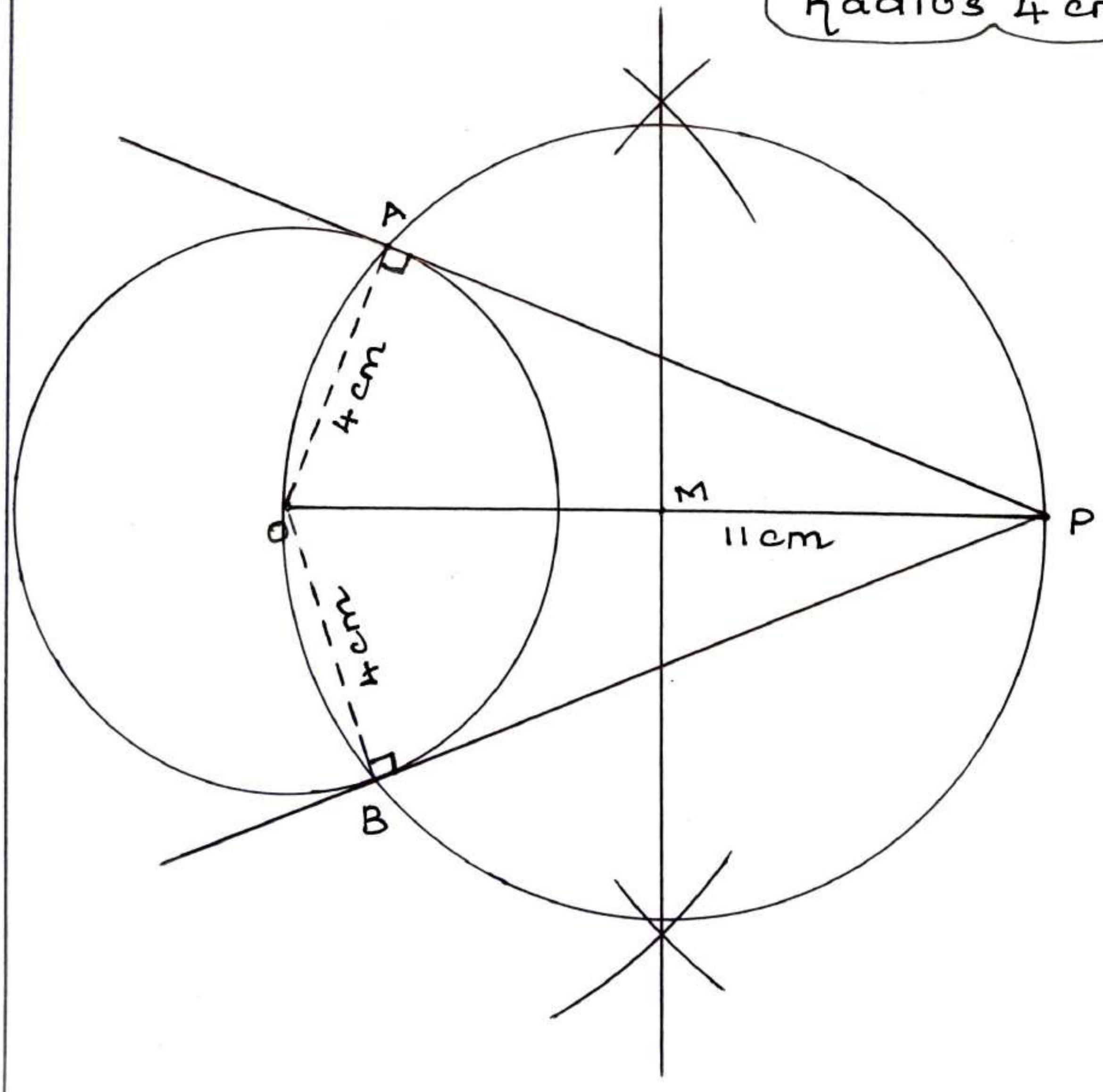
- * With 'O' as centre draw a circle of radius 5 cm.
- * Mark a point 'P' which is 10 cm away from the centre of the circle such that OP = 10 cm.
- * Draw the perpendicular bisector OF OP to meet OP at 'M'.
- * With 'M' as centre draw another circle to meet the previous circle at A and B.
- * Join OA, OB, PA and PB.
- * PA and PB are the two tangents.
- * The length of the tangent PA = 8.6 cm.

14) Take a point which is 11 cm away from the centre of a circle of radius 4cm and draw the two tangents to a circle from that point.

ROUGH DIAGRAM



Radius 4 cm



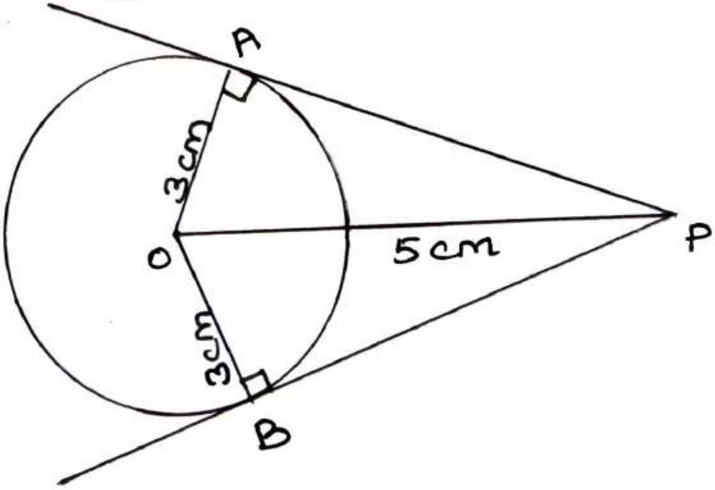
CONSTRUCTION:

- * With 'O' as centre draw a circle of radius 4 cm.
- * Mark a point 'P' which is 11 cm away from the centre of the circle such that OP = 11 cm.
- * Draw the perpendicular bisector of OP to meet OP at 'M'.
- * With 'M' as centre draw another circle to meet the previous circle at A and B.
- * Join OA, OB, PA and PB.
- * PA and PB are the two tangents.

---

15) Draw the two tangents from a point which is 5 cm away from the centre of a circle of diameter 6 cm. Also measure the lengths of the tangents.

ROUGH DIAGRAM

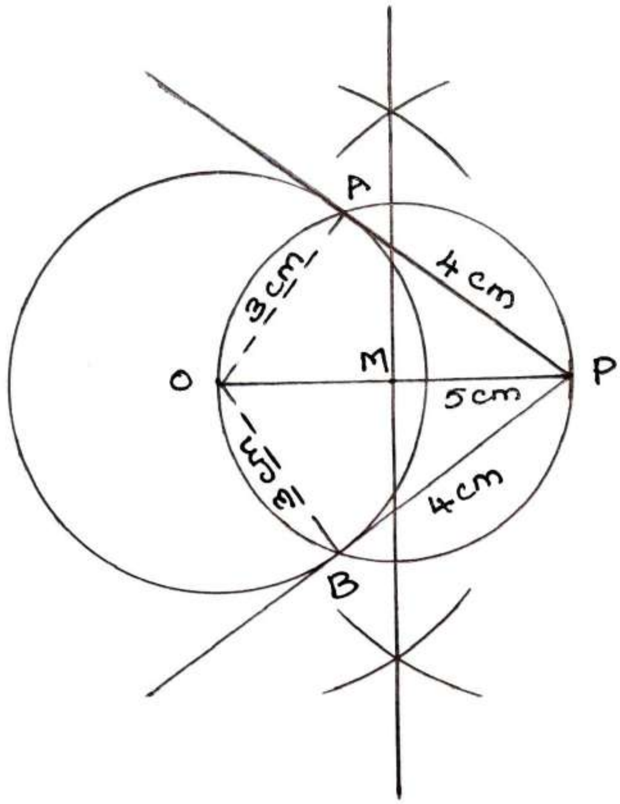


Diameter = 6 cm

⇓

$$\text{Radius} = \frac{\cancel{6}^{3}}{\cancel{2}}$$

$$\Rightarrow \boxed{\text{Radius} = 3\text{ cm}}$$

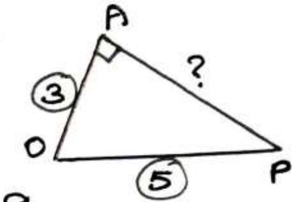


VERIFICATION

$$OP^2 = OA^2 + PA^2$$

$$5^2 = 3^2 + PA^2$$

$$25 = 9 + PA^2$$



$$9 + PA^2 = 25$$

$$PA^2 = 25 - 9$$

$$PA^2 = 16$$

$$PA^{\cancel{2}} = 4^{\cancel{2}}$$

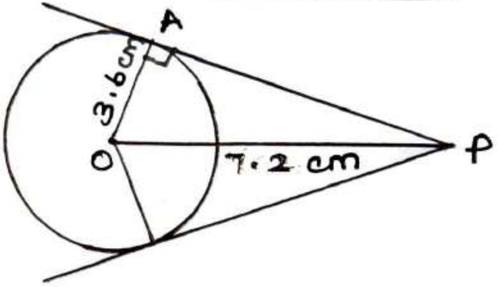
$$\boxed{PA = 4\text{cm}}$$

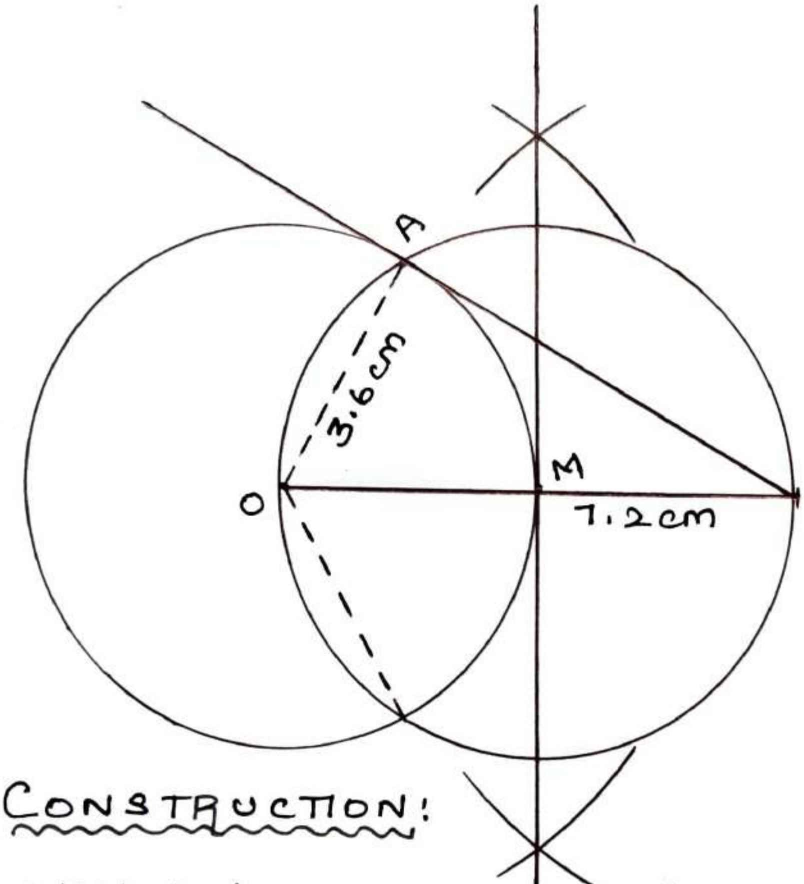
CONSTRUCTION:

- * With 'O' as centre draw a circle of radius 3cm.
- * Mark a point P, which is 5cm away from the centre of the circle such that OP = 5cm.
- * Draw the perpendicular bisector of OP to meet OP at 'M'.
- * With 'M' as centre draw another circle to meet the previous circle at A and B.
- * Join OA, OB, PA and PB.
- * PA and PB are the two tangents.
- * Length of the tangent PA = 4cm.

16) Draw a tangent to the circle from the Point P having radius 3.6 cm, and centre at 'O'. Point 'P' is at a distance 7.2 cm from the centre.

ROUGH DIAGRAM





CONSTRUCTION:

- * With 'O' as centre draw a circle of radius 3.6 cm.
- * Mark a point P, 7.2 cm away from the centre of the circle such that OP = 7.2cm.
- * Draw the perpendicular bisector of OP which meet OP at 'M'.
- * With 'M' as centre draw another circle to meet the previous circle at A.
- * Join OA and PA. PA is the required tangent.

$\boxed{\text{CHAPTER-5}}$

CO-ORDINATE GEOMETRY.

EXERCISE 5.1

1) Find the area of the triangle formed by the points

(i) $(1,-1)$ $(-4,6)$ and $(-3,-5)$

$x_1\ y_1 \qquad x_2\ y_2 \qquad x_3\ y_3$

$\therefore$ Area of the triangle

$$=\frac{1}{2}\begin{pmatrix} x_1 & x_2 & x_3 & x_1\\ y_1 & y_2 & y_3 & y_1\end{pmatrix}$$

$$=\frac{1}{2}\left(\begin{array}{ccccccc}1 & & -4 & & -3 & & 1\\ & \times & & \times & & \times & \\ -1 & & 6 & & -5 & & -1\end{array}\right)$$

$$=\frac{1}{2}\left[(6+20+3)-(4-18-5)\right]$$

$$=\frac{1}{2}\left[(29)-(-19)\right]$$

$$=\frac{1}{2}\left[29+19\right]$$

$$=\frac{1}{\cancel{2}}\left[\overset{24}{\cancel{48}}\right]$$

$$=\boxed{24\text{ sq.units}}$$

(ii) $(-10,-4)$ $(-8,-1)$ and $(-3,-5)$

$x_1\ y_1 \qquad x_2\ y_2 \qquad x_3\ y_3$

$\therefore$ Area of the triangle

$$= \frac{1}{2}\begin{pmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} -10 & \times & -8 & \times & -3 & \times & -10 \\ -4 & & -1 & & -5 & & -4 \end{pmatrix}$$

$$= \frac{1}{2}\left[(10+40+12)-(32+3+50)\right]$$

$$= \frac{1}{2}\left[(62)-(85)\right]$$

$$= \frac{1}{2}\left[62-85\right]$$

$$= \frac{1}{2}\left[-23\right]$$

$$= \frac{-23}{2}$$

$$= \boxed{11.5 \text{ sq.units}}$$

$$\begin{array}{r} 11.5 \\ 2\overline{)\,23} \\ \underline{2\downarrow} \\ 03 \\ \underline{2} \\ 10 \\ \underline{10} \\ \underline{0} \end{array}$$

---

2) Determine whether the set of points are collinear?

(i) $\left(-\frac{1}{2}, 3\right)$ $(-5, 6)$ and $(-8, 8)$

$x_1\ y_1$ $\quad x_2\ y_2$ $\quad x_3\ y_3$

$$\therefore \text{Collinear} = \begin{pmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{pmatrix}$$

$$\frac{-1}{\cancel{2}} \times \overset{3}{\cancel{6}} = -3$$

$$\overset{4}{\cancel{8}} \times -\frac{1}{\cancel{2}} = -4$$

$$= \begin{pmatrix} -\frac{1}{2} & \times & -5 & \times & -8 & \times & -\frac{1}{2} \\ 3 & & 6 & & 8 & & 3 \end{pmatrix}$$

$$= (-3-40-24)-(-15-18-4)$$

$$= (-67) - (-67)$$

$$= -67 + 67$$

$$= 0$$

$\Rightarrow$ It is Collinear

(ii) $(a, b+c)$ $(b, c+a)$ and $(c, a+b)$

$\quad x_1 \quad y_1 \qquad x_2 \quad y_2 \qquad\qquad x_3 \quad y_3$

$\therefore$ Collinear $= \begin{pmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{pmatrix}$

$a(c+a)$

$ac + a^2$

$$= \begin{pmatrix} a & & b & & c & & a \\ b+c & \times & c+a & \times & a+b & \times & b+c \end{pmatrix}$$

$$= (ac + a^2 + ab + b^2 + bc + c^2) - (b^2 + bc + c^2 + ac + a^2 + ab)$$

$$= \cancel{a}c + \cancel{a^2} + \cancel{a}b + \cancel{b^2} + \cancel{b}c + \cancel{c^2} - \cancel{b^2} - b\cancel{c} - \cancel{c^2} - \cancel{a}c - \cancel{a^2} - a\cancel{b}$$

$$= 0$$

$\Rightarrow$ It is Collinear

---

3) Vertices of given triangles are taken in order and their areas are provided aside. In each case, find the value of 'p'.

(i) Vertices	Area (sq. units)
$(0,0)$ $(p,8)$ $(6,2)$	20

$(0,0)\ (P,8)\ (6,2)$

$x_1\,y_1\quad x_2\,y_2\quad x_3\,y_3$

Area of a triangle = 20 sq.units

$$\frac{1}{2}\begin{pmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{pmatrix} = 20$$

$$\frac{1}{2}\begin{pmatrix} 0 & & P & & 6 & & 0 \\ 0 & \times & 8 & \times & 2 & \times & 0 \end{pmatrix} = 20$$

$$\frac{1}{2}\left[(0+2P+0)-(0+48+0)\right] = 20$$

$$\frac{1}{2}\left[(2P)-(48)\right] = 20$$

$$2P-48 = 20\times 2$$

$$2P-48 = 40$$

$$2P = 40+48$$

$$2P = 88$$

$$P = \frac{\cancel{88}^{\,44}}{\cancel{2}}$$

$$\boxed{P = 44}$$

(ii)

Vertices	Area (sq.units)
$(P,P)\ (5,6)\ (5,-2)$	32

$x_1\,y_1\quad x_2\,y_2\quad x_3\,y_3$

Area of a triangle = 32 sq.units

$$\frac{1}{2}\begin{pmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{pmatrix} = 32$$

$$\frac{1}{2}\begin{pmatrix} p & & 5 & & 5 & & p \\ p & \times & 6 & \times & -2 & \times & p \end{pmatrix} = 32$$

$$\frac{1}{2}\left[(6p-10+5p)-(5p+30-2p)\right] = 32$$

$$\frac{1}{2}\left[(11p-10)-(3p+30)\right] = 32$$

$$11p-10-3p-30 = 32\times 2$$

$$8p-40 = 64$$

$$8p = 64+40$$

$$8p = 104$$

$$p = \frac{\cancel{104}^{13}}{\cancel{8}}$$

$$\boxed{p = 13}$$

---

4) In each of the following, find the value of 'a' for which the given points are collinear.

(i) $(2,3)$ $(4,a)$ $(6,-3)$

$x_1\ y_1 \quad x_2\ y_2 \quad x_3\ y_3$

$$\boxed{\text{Collinear} = 0}$$

$$\begin{pmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{pmatrix} = 0$$

$$\begin{pmatrix} 2 & & 4 & & 6 & & 2 \\ 3 & \times & a & \times & -3 & \underset{5}{\times} & 3 \end{pmatrix} = 0$$

$$(2a-\underbrace{12+18})-(12+\underbrace{6a-6})=0$$

$$(2a+6)-(6a+6)=0$$

$$2a+\cancel{6}-6a-\cancel{6}=0$$

$$-4a=0$$

$$a=\frac{0}{-4}$$

$$\boxed{a=0}$$

(ii) $(a,\ 2-2a)\ (-a+1,\ 2a)\ (-4-a,\ 6-2a)$

$x_1\quad y_1\qquad x_2\quad y_2\qquad x_3\quad y_3$

$$\boxed{\text{Collinear}=0}$$

$$\begin{pmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{pmatrix}=0$$

$$\begin{pmatrix} a & & -a+1 & & -4-a & & a \\ 2-2a & \times & 2a & \times & 6-2a & \times & 2-2a \end{pmatrix}=0$$

$$(\underset{\sim}{2a^2}-6\overset{\checkmark}{a}+\underset{\sim}{2a^2}+6-2\overset{\checkmark}{a}-8+8a-2\overset{\checkmark}{a}+\underset{\sim}{2a^2})-$$

$$(-2\overset{\checkmark}{a}+2+2\cancel{a^2}-2\overset{\checkmark}{a}-8\overset{\checkmark}{a}-2\cancel{a^2}+6a-2a^2)=0$$

$$(6a^2-\underbrace{10a+8a}-2)-(-2a^2-\underbrace{12a+6a}+2)=0$$

$$(6a^2-2a-2)-(-2a^2-6a+2)=0$$

$$\underset{\checkmark}{6a^2}-\underset{\sim}{2a}-2+\underset{\checkmark}{2a^2}+\underset{\sim}{6a}-2=0$$

$$8a^2+4a-4=0$$

$$4\left[2a^2+a-1\right]=0$$

$$2a^2+a-1=0$$

$$(a+1)(2a-1)=0$$

$a+1=0$

$\boxed{a=-1}$

$2a-1=0$

$2a=1$

$\boxed{a=\frac{1}{2}}$

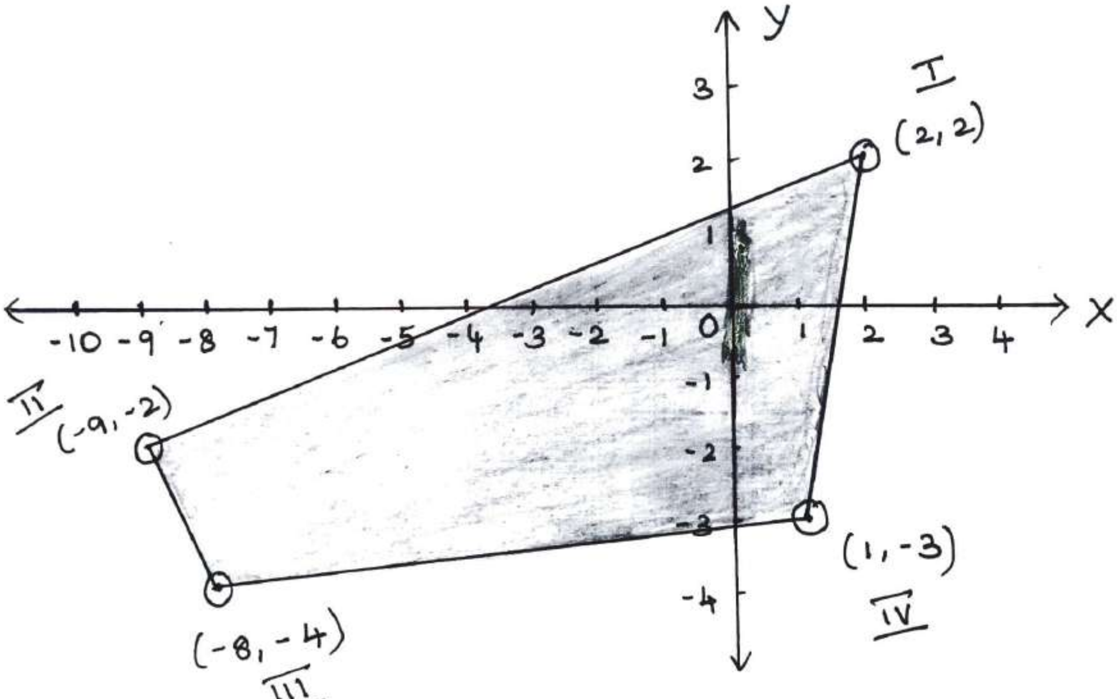
-2

$\frac{\cancel{2}}{\cancel{2}a}$ $\frac{-1}{2a}$

---

5) Find the area of the quadrilateral whose vertices are at

(i) $(-9,-2)$ $(-8,-4)$ $(2,2)$ $(1,-3)$



Arrange the points in anticlockwise direction

$(2,2)$ $(-9,-2)$ $(-8,-4)$ $(1,-3)$

$x_1\, y_1$ $x_2\, y_2$ $x_3\, y_3$ $x_4\, y_4$

∴ Area of a quadrilateral

$$= \frac{1}{2}\begin{pmatrix} x_1 & x_2 & x_3 & x_4 & x_1 \\ y_1 & y_2 & y_3 & y_4 & y_1 \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} 2 & -9 & -8 & 1 & 2 \\ 2 & -2 & -4 & -3 & 2 \end{pmatrix}$$

$$= \frac{1}{2}\left[(-4+\overbrace{36+24+2}) - (-\overbrace{18+16-4-6})\right]$$

$$= \frac{1}{2}\left[(-4+62) - (-28+16)\right]$$

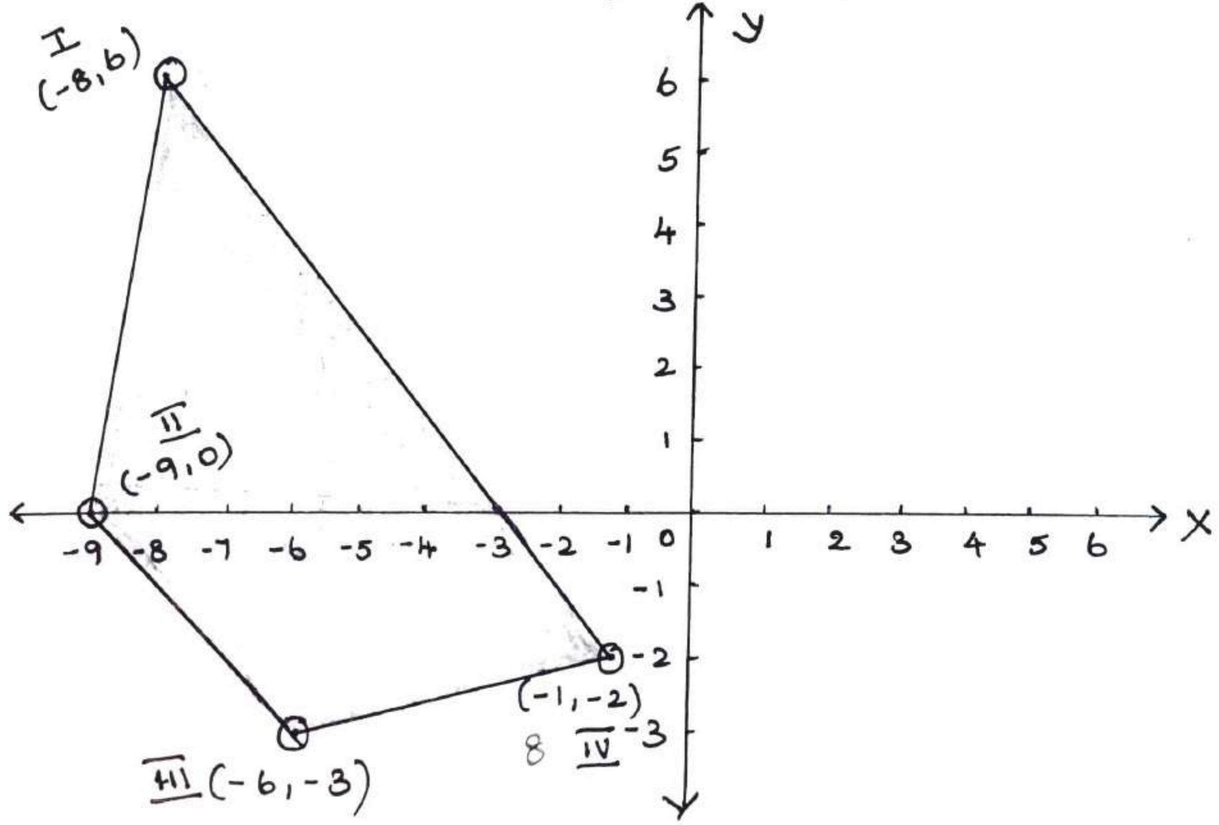
$$= \frac{1}{2}\left[(58) - (-12)\right]$$

$$= \frac{1}{2}\left[58+12\right]$$

$$= \frac{1}{\cancel{2}}\left[\overset{35}{\cancel{70}}\right]$$

$$= \boxed{35 \text{ sq.units}}$$

(ii) (-9,0) (-8,6) (-1,-2) (-6,-3)



$(-8, 6)\ (-9, 0)\ (-6, -3)\ (-1, -2)$

$x_1\ y_1\quad x_2\ y_2\quad x_3\ y_3\quad x_4\ y_4$

$\therefore$ Area of a quadrilateral

$$= \frac{1}{2}\begin{pmatrix} x_1 & x_2 & x_3 & x_4 & x_1 \\ y_1 & y_2 & y_3 & y_4 & y_1 \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} -8 & & -9 & & -6 & & -1 & & -8 \\ 6 & \times & 0 & \times & -3 & \times & -2 & \times & 6 \end{pmatrix}$$

$$= \frac{1}{2}\left[(0 + \overbrace{27 + 12} - 6) - (-54 + 0 + 3 + 16)\right]$$

$$= \frac{1}{2}\left[(39 - 6) - (-54 + 19)\right]$$

$$= \frac{1}{2}\left[(33) - (-35)\right]$$

$$= \frac{1}{2}\left[33 + 35\right]$$

$$= \frac{1}{\cancel{2}}\left[\overset{34}{\cancel{68}}\right]$$

$$= \boxed{34\ \text{Sq.units}}$$

---

6) Find the value of K, if the area of the quadrilateral is 28 sq.units, whose vertices are $(-4, -2)$ $(-3, K)$ $(3, -2)$ and $(2, 3)$

$(-4, -2)\ (-3, K)\ (3, -2)\ (2, 3)$

$x_1\ y_1\quad x_2\ y_2\quad x_3\ y_3\quad x_4\ y_4$

Area of the quadrilateral = 28 Sq.units

$$\frac{1}{2}\begin{pmatrix} x_1 & x_2 & x_3 & x_4 & x_1 \\ y_1 & y_2 & y_3 & y_4 & y_1 \end{pmatrix} = 28$$

$$\frac{1}{2}\begin{pmatrix} -4 & \times & -3 & \times & 3 & \times & 2 & \times & -4 \\ -2 & & k & & -2 & & 3 & & -2 \end{pmatrix} = 28$$

$$\frac{1}{2}\left[(-4k+\overset{\frown}{6+9}-4)-(6+3k-4-12)\right] = 28$$

$$\frac{1}{2}\left[(-4k+\underbrace{15-4})-(3k+\underbrace{6-16})\right] = 28$$

$$\frac{1}{2}\left[(-4k+11)-(3k-10)\right] = 28$$

$$-4k+11-3k+10 = 28\times 2$$

$$-7k+21 = 56$$

$$-7k = 56-21$$

$$-7k = 35$$

$$k = \frac{\cancel{35}^{\,5}}{-\cancel{7}}$$

$$\boxed{k=-5}$$

---

7) If the points A(-3,9) B(a,b) C(4,-5) are collinear and if a+b=1, then find a and b.

$A(-3,9)$ $B(a,b)$ $C(4,-5)$

$x_1\ y_1$ $\qquad x_2\ y_2$ $\qquad x_3\ y_3$

$\boxed{a+b=1} \rightarrow$ ①

$$\boxed{\text{Collinear} = 0}$$

$$\begin{pmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3{}^{10} & y_1 \end{pmatrix} = 0$$

$$\begin{pmatrix} -3 & & a & & 4 & & -3 \\ 9 & \times & b & \times & -5 & \times & 9 \end{pmatrix} = 0$$

$$(-3b - 5a + 36) - (9a + 4b + 15) = 0$$

$$\underset{\checkmark}{-3b} - \underset{\sim\sim}{5a} + 36 - \underset{\sim\sim}{9a} - \underset{\checkmark}{4b} - 15 = 0$$

$$-14a - 7b + 21 = 0$$

÷ by 7 $\quad -2a - b + 3 = 0$

$$\boxed{-2a - b = -3} \rightarrow ②$$

Solving ① and ②

$$a + \cancel{b} = 1 \rightarrow ①$$

$$-2a - \cancel{b} = -3 \rightarrow ②$$

$$\cancel{-}a = \cancel{-}2$$

$$\boxed{a = 2}$$

Put $a = 2$ in ①

$$a + b = 1$$

$$2 + b = 1$$

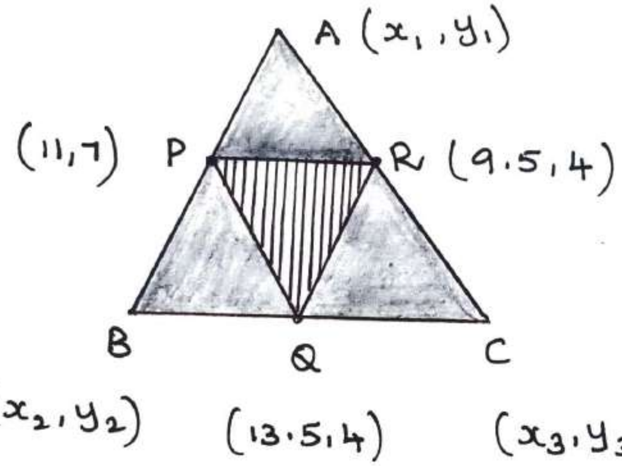
$$b = 1 - 2$$

$$\boxed{b = -1}$$

8) Let P(11, 7) Q(13.5, 4) and R(9.5, 4) be the mid-points of the sides AB, BC and AC respectively of Δ ABC. Find the co-ordinates of the vertices A, B and C. Hence find the area of Δ ABC and compare this with area of Δ PQR.

P(11, 7) Q(13.5, 4) R(9.5, 4)

Midpoints



$$\frac{x_1 + x_2}{2} \longrightarrow = 11$$

$$\boxed{x_1 + x_2 = 22} \longrightarrow ①$$

$$\frac{x_2 + x_3}{2} \longrightarrow = 13.5$$

$$\frac{13.5 \times 2}{27.0}$$

$$\boxed{x_2 + x_3 = 27} \longrightarrow ③$$

$$\frac{x_1 + x_3}{2} \longrightarrow = 9.5$$

$$\frac{9.5 \times 2}{19.0}$$

$$\boxed{x_1 + x_3 = 19} \longrightarrow ⑤$$

$$\frac{y_1 + y_2}{2} \longrightarrow = 7$$

$$\boxed{y_1 + y_2 = 14} \longrightarrow ②$$

$$\frac{y_2 + y_3}{2} \longrightarrow = 4$$

$$\boxed{y_2 + y_3 = 8} \longrightarrow ④$$

$$\frac{y_1 + y_3}{2} \longrightarrow = 4$$

$$\boxed{y_1 + y_3 = 8} \longrightarrow ⑥$$

Adding ①, ③, ⑤

$$2x_1+2x_2+2x_3=68$$

$$2(x_1+x_2+x_3)=68$$

$$x_1+x_2+x_3=\frac{68}{2}$$

$$\therefore \boxed{\underbrace{x_1+x_2}+x_3=34} \rightarrow ⑦$$

$$22+x_3=34$$

$$x_3=34-22$$

$$\boxed{x_3=12}$$

$$\boxed{x_1+\underbrace{x_2+x_3}=34} \rightarrow ⑦$$

$$x_1+27=34$$

$$x_1=34-27$$

$$\boxed{x_1=7}$$

$$\boxed{\underbrace{x_1+x_2+x_3}=34} \rightarrow ⑦$$

$$19+x_2=34$$

$$x_2=34-19$$

$$\boxed{x_2=15}$$

Adding ②, ④, ⑥

$$2y_1+2y_2+2y_3=30$$

$$2(y_1+y_2+y_3)=30$$

$$y_1+y_2+y_3=\frac{30}{2}$$

$$\boxed{\underbrace{y_1+y_2}+y_3=15} \rightarrow ⑧$$

$$14+y_3=15$$

$$y_3=15-14$$

$$\boxed{y_3=1}$$

$$\boxed{y_1+\underbrace{y_2+y_3}=15} \rightarrow ⑧$$

$$y_1+8=15$$

$$y_1=15-8$$

$$\boxed{y_1=7}$$

$$\boxed{\underbrace{y_1+y_2+y_3}=15} \rightarrow ⑧$$

$$y_2+8=15$$

$$y_2=15-8$$

$$\boxed{y_2=7}$$

$$\Rightarrow \underset{x_1\ y_1}{A(7,7)} \quad \underset{x_2\ y_2}{B(15,7)} \quad \underset{x_3\ y_3}{C(12,1)}$$

Area of triangle ABC

$$= \frac{1}{2}\begin{pmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{pmatrix}$$

$$= \frac{1}{2}\left(\begin{matrix} 7 \\ 7 \end{matrix} \times \begin{matrix} 15 \\ 7 \end{matrix} \times \begin{matrix} 12 \\ 1 \end{matrix} \times \begin{matrix} 7 \\ 7 \end{matrix} \right)$$

$$= \frac{1}{2}\left[(49+15+84)-(105+84+7)\right]$$

$$= \frac{1}{2}\left[(148)-(196)\right]$$

$$= \frac{1}{\cancel{2}}(-\overset{24}{\cancel{48}})$$

$$= \boxed{24 \text{ Sq.units}}$$

P (11, 7) Q (13.5, 4) R (9.5, 4)

$x_1\ y_1$ $x_2\ y_2$ $x_3\ y_3$

∴ Area of Δ PQR,

$$= \frac{1}{2}\begin{pmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{pmatrix}$$

$$= \frac{1}{2}\left(\begin{matrix} 11 \\ 7 \end{matrix} \times \begin{matrix} 13.5 \\ 4 \end{matrix} \times \begin{matrix} 9.5 \\ 4 \end{matrix} \times \begin{matrix} 11 \\ 7 \end{matrix} \right)$$

$$= \frac{1}{2}\left[(44+54+66.5)-(94.5+38+44)\right]$$

$$= \frac{1}{2}\left[(164.5)-(176.5)\right]$$

$$= \frac{1}{\cancel{2}}\left[-\overset{6}{\cancel{12}}\right]$$

$$= \boxed{6 \text{ Sq.units}}$$

$$\begin{array}{r} {}^{1\,2} \\ 13.5\times4 \\ \hline 54.\cancel{0} \end{array}$$

$$\begin{array}{r} 9.5\times7 \\ \hline 66.5 \end{array}$$

$$\begin{array}{r} 7\times13.5 \\ \hline 94.5 \end{array}$$

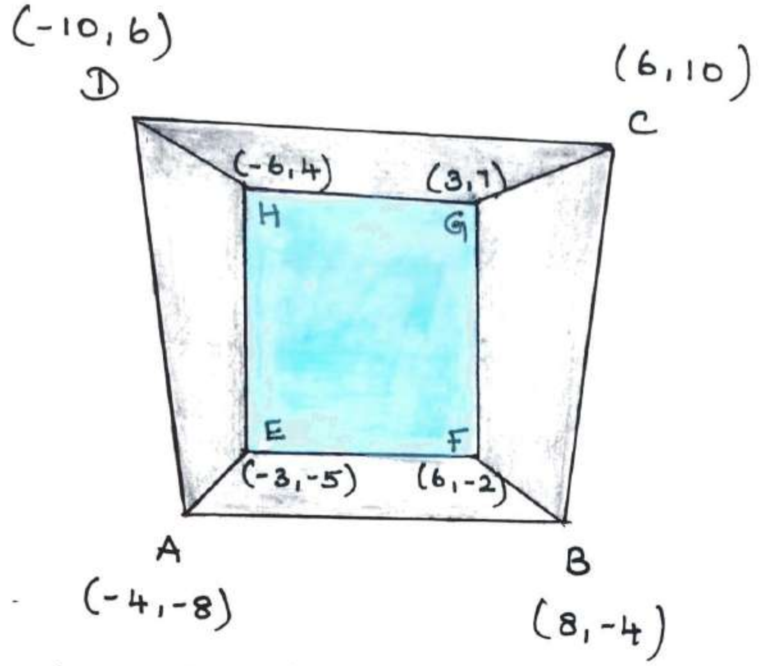
$$\begin{array}{r} 4\times9.5 \\ \hline 38.\cancel{0} \end{array}$$

$$\begin{array}{r} 176.5 \\ 164.5 \\ \hline 12.\cancel{0} \end{array} \ (-)$$

$\Rightarrow$ $\boxed{\text{Area of } \Delta\ ABC = 4(\text{Area of } \Delta\ PQR)}$

---

9) In the figure, the quadrilateral swimming pool shown is surrounded by concrete patio. Find the area of the Patio.



$A(-4,-8)$ $B(8,-4)$ $C(6,10)$ $D(-10,6)$

$x_1\ y_1$ $\quad x_2\ y_2$ $\quad x_3\ y_3$ $\quad x_4\ y_4$

$\Rightarrow$ Area of ABCD

$$= \frac{1}{2}\begin{pmatrix} x_1 & x_2 & x_3 & x_4 & x_1 \\ y_1 & y_2 & y_3 & y_4 & y_1 \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} -4 & & 8 & & 6 & & -10 & & -4 \\ -8 & \times & -4 & \times & 10 & \times & 6 & \times & -8 \end{pmatrix}$$

$$= \frac{1}{2}\left[(16+80+36+80)-(-64-24-100-24)\right]$$

$$= \frac{1}{2}\left[(212)-(-212)\right]$$

$$= \frac{1}{2}\left[212+212\right]$$

$$= \frac{1}{\cancel{2}}\left[\overset{212}{\cancel{424}}\right] = \boxed{212\ \text{sq. units}}$$

$E(-3,-5)$ $F(6,-2)$ $G(3,7)$ $H(-6,4)$
$x_1\ y_1$ $\quad x_2\ y_2$ $\quad x_3\ y_3$ $\quad x_4\ y_4$

$\Rightarrow$ Area of EFGH

$$= \frac{1}{2}\begin{pmatrix} x_1 & x_2 & x_3 & x_4 & x_1 \\ y_1 & y_2 & y_3 & y_4 & y_1 \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} -3 & 6 & 3 & -6 & -3 \\ -5 & -2 & 7 & 4 & -5 \end{pmatrix}$$

$$= \frac{1}{2}\left[(6+42+12+30)-(-30-6-42-12)\right]$$

$$= \frac{1}{2}\left[(90)-(-90)\right]$$

$$= \frac{1}{2}\left[90+90\right]$$

$$= \frac{1}{\cancel{2}}\left[\overset{90}{\cancel{180}}\right]$$

$$= \boxed{90 \text{ sq.units}}$$

$\therefore$ Area of the patio

= Area of ABCD − Area of EFGH

= 212 sq.units − 90 sq.units

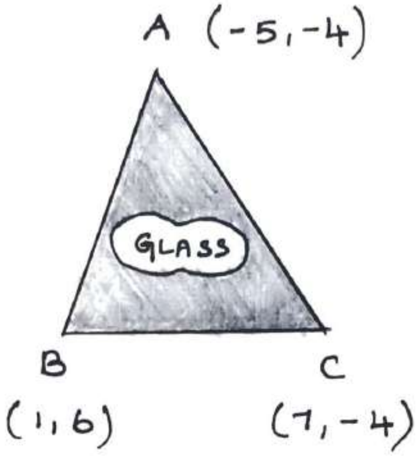
$= \boxed{122 \text{ sq.units}}$

---

10) A triangular shaped glass with vertices at $A(-5,-4)$ $B(1,6)$ $C(7,-4)$ has to be painted. If one bucket of Paint covers 6 square feet, how many buckets of paint will be required to paint the whole glass, if only one

coat of paint is applied.

$A(-5,-4)$ $B(1,6)$ $C(7,-4)$
$x_1\ y_1$ $x_2\ y_2$ $x_3\ y_3$



∴ Area of Δ ABC

$$= \frac{1}{2}\begin{pmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} -5 & 1 & 7 & -5 \\ -4 & 6 & -4 & -4 \end{pmatrix}$$

$$= \frac{1}{2}\left[(-30-4-28)-(-4+42+20)\right]$$

$$= \frac{1}{2}\left[(-62)-(-4+62)\right]$$

$$= \frac{1}{2}\left[(-62)-(58)\right]$$

$$= \frac{1}{2}\left[-62-58\right]$$

$$= \frac{1}{\cancel{2}}\left[-\overset{60}{\cancel{120}}\right]$$

$$= \boxed{60 \text{ sq.units}}$$

GIVEN:
Area of one bucket
= 6 sq. units

∴ No. of buckets of paint required

$$= \frac{\text{Area of } \Delta \text{ ABC}}{\text{Area of one bucket}}$$

$$= \frac{\cancel{60}^{10}}{\cancel{6}}$$

$$= \boxed{10 \text{ buckets}}$$

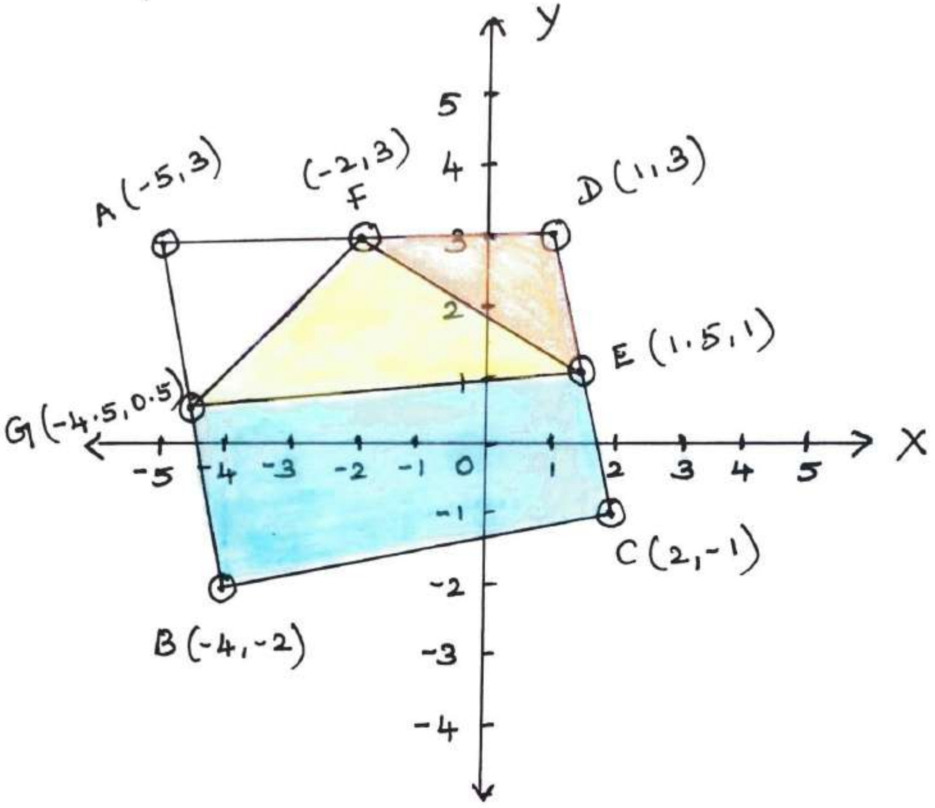
---

11) In the figure, find the area of

(i) triangle AGF

(ii) triangle FED

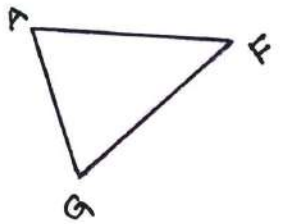
(iii) quadrilateral BCEG.



(i) A (-5, 3) G (-4.5, 0.5) F (-2, 3)

$x_1\ y_1$ $\quad x_2\ \ y_2$ $\quad x_3\ y_3$

$\Rightarrow$ Area of $\triangle$ AGF



$$= \frac{1}{2}\begin{pmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} -5 & & -4.5 & & -2 & & -5 \\ 3 & \times & 0.5 & \times & 3 & \times & 3 \end{pmatrix}$$

$$= \frac{1}{2}\left[(-2.5 - 13.5 - 6) - (-13.5 - 1 - 15)\right]$$

$$= \frac{1}{2}\left[(-22) - (-29.5)\right]$$

$$= \frac{1}{2}\left[-22 + 29.5\right]$$

$$= \frac{1}{2}\left[7.5\right]$$

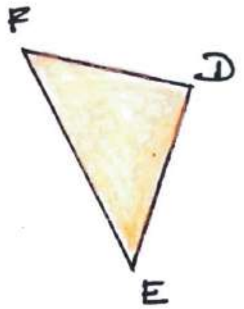
$$= \frac{7.5}{2}$$

$$= \boxed{3.75 \text{ Sq.units}}$$

$$\begin{array}{r|l} & 3.75 \\ \hline 2 & 7.5 \\ & 6\downarrow \\ \hline & 15 \\ & 14 \\ \hline & \;\;10 \\ & \;\;10 \\ \hline & \;\;\;0 \end{array}$$

(ii) $F(-2, 3)$ $E(1.5, 1)$ $D(1, 3)$

$x_1\ y_1$ $\quad x_2\ y_2$ $\quad x_3\ y_3$



$\therefore$ Area of $\triangle$ FED

$$= \frac{1}{2}\begin{pmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} -2 & 1.5 & 1 & -2 \\ 3 & 1 & 3 & 3 \end{pmatrix}$$

$$= \frac{1}{2}\left[(-2 + 4.5 + 3) - (4.5 + 1 - 6)\right]$$

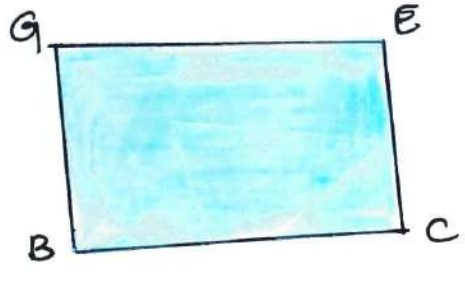
$$= \frac{1}{2}\left[(-2 + 7.5) - (5.5 - 6)\right]$$

$$= \frac{1}{2}\left[(5.5) - (-0.5)\right]$$

$$= \frac{1}{2}\left[5.5 + 0.5\right]$$

$$= \frac{1}{\cancel{2}}\left[\overset{3}{\cancel{6}}\right]$$

$$= \boxed{3 \text{ Sq.units}}$$



(iii) $B(-4, -2)$ $C(-2, -1)$ $E(1.5, 1)$ $G(-4.5, 0.5)$

$x_1\ y_1$ $\quad x_2\ y_2$ $\quad x_3\ y_3$ $\quad x_4\ y_4$

$\therefore$ Area of a quadrilateral BCEG

$$= \frac{1}{2}\begin{pmatrix} x_1 & x_2 & x_3 & x_4 & x_1 \\ y_1 & y_2 & y_3 & y_4 & y_1 \end{pmatrix}$$

$$=\frac{1}{2}\left(\begin{matrix}-4\\-2\end{matrix}\times\begin{matrix}2\\-1\end{matrix}\times\begin{matrix}1.5\\1\end{matrix}\times\begin{matrix}-4.5\\0.5\end{matrix}\times\begin{matrix}-4\\-2\end{matrix}\right)$$

$$=\frac{1}{2}\left[(4+2+0.75+9)-(-4-1.5-4.5-2)\right]$$

$$=\frac{1}{2}\left[(15.75)-(-12)\right]$$

$$=\frac{1}{2}\left[15.75+12\right]$$

$$=\frac{1}{2}\left[27.75\right]$$

$$=\boxed{13.875\ \text{Sq.units}}$$

$$\begin{array}{r}
13.875\\
2\,\big)\,27.75\\
\underline{2\downarrow}\\
7\\
\underline{6\downarrow}\\
17\\
\underline{16\downarrow}\\
15\\
\underline{14}\\
10\\
\underline{10}\\
\underline{0}
\end{array}$$

EXERCISE 5.2

1) What is the slope of a line whose inclination with positive direction of x-axis is

(i) 90°

$\hookrightarrow \boxed{\theta = 90^\circ}$ (Inclination)

$\therefore$ slope $= \tan\theta$

$= \tan 90^\circ$

$\boxed{\text{slope} = \text{n.d}}$

(ii) 0°

$\hookrightarrow \boxed{\theta = 0^\circ}$ (Inclination)

$\therefore$ slope $= \tan\theta$

$= \tan 0^\circ$

$\boxed{\text{slope} = 0}$

---

2) What is the inclination of a line whose slope is

(i) 0

$\hookrightarrow \boxed{\text{slope} = 0}$

slope $= \tan\theta$

$0 = \tan\theta$

$\Rightarrow \boxed{\theta = 0^\circ}$

(ii) 1

$\hookrightarrow \boxed{\text{slope} = 1}$

slope $= \tan\theta$

$1 = \tan\theta$

$\Rightarrow \boxed{\theta = 45^\circ}$

---

3) Find the slope of a line joining the points

(i) $(5, \sqrt{5})$ with the origin.

$(5, \sqrt{5})\ (0,0)$
$x_1\ y_1\quad x_2\ y_2$

$$\text{Slope} = \frac{y_2-y_1}{x_2-x_1}$$

$$= \frac{0-\sqrt{5}}{0-5}$$

$$= \frac{\cancel{-}\sqrt{5}}{\cancel{-}5}$$

$$= \frac{\cancel{\sqrt{5}}}{\cancel{\sqrt{5}}\times\sqrt{5}}$$

$$\boxed{\text{Slope} = \frac{1}{\sqrt{5}}}$$

(ii) $(\sin\theta, -\cos\theta)(-\sin\theta, \cos\theta)$
$x_1\quad y_1\qquad x_2\quad y_2$

$$\text{slope} = \frac{y_2-y_1}{x_2-x_1}$$

$$= \frac{\cos\theta+\cos\theta}{-\sin\theta-\sin\theta}$$

$$= \frac{\cancel{2}\cos\theta}{-\cancel{2}\sin\theta}$$

$$= \frac{\cos\theta}{-\sin\theta}$$

$$\boxed{\text{Slope} = -\cot\theta}$$

---

4) What is the slope of a line perpendicular to the line joining A(5,1) and P where P is the mid-point of the segment joining (4,2) and (-6,4)

$\boxed{P \rightarrow \text{Midpoint}}$

$\boxed{A(5,1)}$

$(4,2)\ (-6,4)$
$x_1\ y_1\quad x_2\ y_2$

$$\text{Midpoint} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$= \left(\frac{4-6}{2}, \frac{2+4}{2}\right)$$

$$= \left(\frac{-\overset{1}{\cancel{2}}}{\cancel{2}}, \frac{\overset{3}{\cancel{6}}}{\cancel{2}}\right)$$

$$\boxed{P = (-1, 3)}$$

$A(5,1)$ $P(-1,3)$

$x_1\ y_1$ $\quad x_2\ y_2$

$$\text{slope} = \frac{y_2-y_1}{x_2-x_1} = \frac{3-1}{-1-5} = \frac{\cancel{2}^{1}}{-\cancel{6}_{3}} = \frac{1}{-3}$$

$$\text{Slope} = \frac{1}{-3} \Rightarrow \boxed{\perp^r \text{ slope} = 3}$$

$$\boxed{\begin{aligned} \text{slope} &= \frac{1}{-3} \\ \perp^r \text{slope} &= \frac{3}{1} \\ &= 3 \end{aligned}}$$

---

5. Show that the given points are Collinear:
$(-3,-4)$ $(7,2)$ and $(12,5)$

$A(-3,-4)$ $B(7,2)$ $C(12,5)$

$A(-3,-4)$ $B(7,2)$

$x_1\ y_1$ $\quad x_2\ y_2$

$$\text{slope} = \frac{y_2-y_1}{x_2-x_1} = \frac{2+4}{7+3} = \frac{\cancel{6}^{3}}{\cancel{10}_{5}}$$

$$\text{slope } AB = \frac{3}{5}$$

$B(7,2)$ $C(12,5)$

$x_1\ y_1$ $\quad x_2\ y_2$

$$\text{slope} = \frac{y_2-y_1}{x_2-x_1} = \frac{5-2}{12-7}$$

$$\text{Slope } BC = \frac{3}{5}$$

$A(-3,-4)$ $C(12,5)$

$x_1\ y_1$ $\quad x_2\ y_2$

$$\text{slope} = \frac{y_2-y_1}{x_2-x_1} = \frac{5+4}{12+3} = \frac{\cancel{9}^{3}}{\cancel{15}_{5}}$$

$$\text{slope } AC = \frac{3}{5}$$

$\Rightarrow$ $\boxed{\text{It is Collinear}}$

6) If the three points $(3,-1)$ $(a,3)$ and $(1,-3)$ are collinear, find the value of 'a'.

$A(3,-1)$ $B(a,3)$ $C(1,-3)$

$A(3,-1)$ $B(a,3)$

x_1, y_1 $x_2\,y_2$

$$\text{Slope} = \frac{y_2-y_1}{x_2-x_1}$$

$$= \frac{3+1}{a-3}$$

$$\boxed{\text{Slope } AB = \frac{4}{a-3}}$$

$B(a,3)$ $C(1,-3)$

$x_1\,y_1$ $x_2\,y_2$

$$\text{Slope} = \frac{y_2-y_1}{x_2-x_1}$$

$$= \frac{-3-3}{1-a}$$

$$\boxed{\text{Slope}_{BC} = \frac{-6}{1-a}}$$

Given ⇒ $\boxed{\text{Collinear}}$

⇓

$$\text{Slope } AB = \text{Slope } BC$$

$$\frac{\cancel{4}^{2}}{a-3} = \frac{-\cancel{6}^{3}}{1-a}$$

$$\frac{2}{a-3} = \frac{-3}{1-a}$$

$$2(1-a) = -3(a-3)$$

$$2-2a = -3a+9$$

$$3a-2a = 9-2$$

$$\boxed{a = 7}$$

---

7) The line through the points $(-2,a)$ and $(9,3)$ has slope $-\frac{1}{2}$. Find the value of 'a'.

$(-2, a) \quad (9, 3)$

$x_1 \; y_1 \quad x_2 \; y_2$

$$\boxed{\text{slope} = -\frac{1}{2}}$$

$$\frac{y_2 - y_1}{x_2 - x_1} = -\frac{1}{2}$$

$$\frac{3 - a}{9 + 2} = -\frac{1}{2}$$

$$\frac{3 - a}{11} = -\frac{1}{2}$$

$$2(3 - a) = -11$$

$$6 - 2a = -11$$

$$-2a = -11 - 6$$

$$\cancel{-}2a = \cancel{-}17$$

$$\boxed{a = \frac{17}{2}}$$

---

8) The line through the points $(-2, 6)$ and $(4, 8)$ is perpendicular to the line through the points $(8, 12)$ and $(x, 24)$. Find the value of x.

$(-2, 6)$ and $(4, 8)$

$x_1 \; y_1 \quad x_2 \; y_2$

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

$(8, 12)$ and $(x, 24)$

$x_1 \; y_1 \quad x_2 \; y_2$

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{8-6}{4+2}$$

$$= \frac{\cancel{2}^{1}}{\cancel{6}\,3}$$

$$\boxed{m_1 = \frac{1}{3}}$$

$$= \frac{24-12}{x-8}$$

$$\boxed{m_2 = \frac{12}{x-8}}$$

Given: $\boxed{\text{Perpendicular}}$

$$\Downarrow$$

$$\boxed{m_1 \times m_2 = -1}$$

$$\frac{\cancel{12}^{4}}{x-8} \times \frac{1}{\cancel{3}} = -1$$

$$\frac{4}{x-8} \rightarrow = -1$$

$$4 = -1(x-8)$$

$$4 = -x+8$$

$$x = 8-4$$

$$\boxed{x = 4}$$

---

9) Show that the given points form a right angled triangle and check whether they satisfies pythagoras theorem.

(i) $A(1,-4)$ $B(2,-3)$ $C(4,-7)$

⊛ $A(1,-4)$ $B(2,-3)$

$x_1\ y_1$ $\quad x_2\ y_2$

$$\text{slope} = \frac{y_2-y_1}{x_2-x_1}$$

⊛ $B(2,-3)$ $C(4,-7)$

$x_1\ y_1$ $\quad x_2\ y_2$

$$\text{slope} = \frac{y_2-y_1}{x_2-x_1}$$

$$= \frac{-3+4}{2-1}$$

$$= \frac{1}{1}$$

$$\boxed{\text{slope } AB = 1}$$

(*) $A(1,-4)$ $C(4,-7)$

$x_1\ y_1$ $\quad x_2\ y_2$

$$\text{slope} = \frac{y_2-y_1}{x_2-x_1}$$

$$= \frac{-7+4}{4-1}$$

$$= \frac{-\cancel{3}\,1}{\cancel{3}}$$

$$\boxed{\text{slope } AC = -1}$$

$$= \frac{-7+3}{4-2}$$

$$= \frac{-\cancel{4}\,2}{\cancel{2}}$$

$$\boxed{\text{slope } BC = -2}$$

$\Rightarrow$ slope AB × slope AC

1×-1

$$\boxed{= -1}$$

$\therefore$ $\boxed{AB \perp AC}$

$\Rightarrow$ It is a Right angled triangle

$A(1,-4)$ $B(2,-3)$

$x_1\ y_1$ $\quad x_2\ y_2$

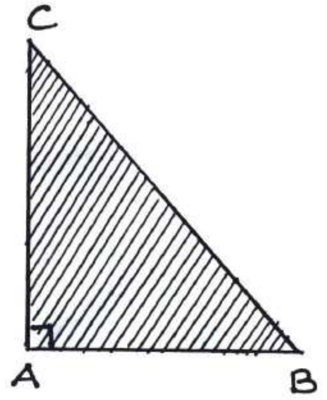
$$AB^2 = (x_2-x_1)^2 + (y_2-y_1)^2$$

$$= (2-1)^2 + (-3+4)^2$$

$$= (1)^2 + (1)^2$$

$$= 1+1$$

$$\boxed{AB^2 = 2}$$



$B(2,-3)$ $C(4,-7)$

$x_1\ y_1$ $\quad x_2\ y_2$

$$BC^2 = (x_2-x_1)^2 + (y_2-y_1)^2$$

$= (4-2)^2 + (-7+3)^2$

$= (2)^2 + (-4)^2$

$= 4 + 16$

$\boxed{BC^2 = 20}$

$A(1, -4)$ $C(4, -7)$

x_1 y_1 x_2 y_2

$AC^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

$= (4-1)^2 + (-7+4)^2$

$= (3)^2 + (-3)^2$

$= 9 + 9$

$\boxed{AC^2 = 18}$

$\Rightarrow$ PYTHAGORAS THEOREM:

$BC^2 = AB^2 + AC^2$

$20 = 2 + 18$

$\boxed{20 = 20}$

$\Rightarrow$ It is satisfied

(iii) $L(0,5)$ $M(9,12)$ and $N(3,14)$

$L(0,5)$ $M(9,12)$

x_1 y_1 x_2 y_2

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

$M(9,12)$ $N(3,14)$

x_1 y_1 x_2 y_2

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

$= \frac{12-5}{9-0}$

$\boxed{\text{Slope } LM = \frac{7}{9}}$

$L(0,5) \quad N(3,14)$

$x_1 \, y_1 \qquad x_2 \, y_2$

$\text{Slope} = \frac{y_2-y_1}{x_2-x_1}$

$= \frac{14-5}{3-0}$

$= \frac{\cancel{9}\,3}{\cancel{3}}$

$\boxed{\text{Slope } LN = 3}$

$= \frac{14-12}{3-9}$

$= \frac{\cancel{2}\,1}{-\cancel{6}\,3}$

$\boxed{\text{Slope } MN = \frac{1}{-3}}$

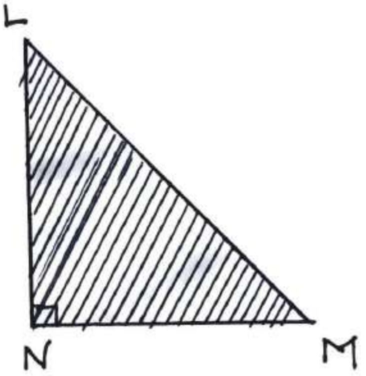
$\Rightarrow$ Slope MN × Slope LN

$= \frac{1}{-\cancel{3}} \times \cancel{3}$

$\boxed{= -1}$

$\therefore \boxed{MN \perp LN}$

$\Rightarrow$ It is a right angled triangle.



$L(0,5) \quad M(9,12)$

$x_1 \, y_1 \qquad x_2 \, y_2$

$LM^2 = (x_2-x_1)^2 + (y_2-y_1)^2$

$= (9-0)^2 + (12-5)^2$

$= (9)^2 + (7)^2$

$= 81 + 49$

$\boxed{LM^2 = 130}$

$M(9,12) \quad N(3,14)$

$x_1 \, y_1 \qquad x_2 \, y_2$

$MN^2 = (x_2-x_1)^2 + (y_2-y_1)^2$

$= (3-9)^2 + (14-12)^2$

$$= (-6)^2 + (2)^2$$

$$= 36 + 4$$

$$\boxed{MN^2 = 40}$$

L (0, 5)   N (3, 14)

$x_1\ y_1$ $x_2\ y_2$

$$LN^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$= (3 - 0)^2 + (14 - 5)^2$$

$$= (3)^2 + (9)^2$$

$$= 9 + 81$$

$$\boxed{LN^2 = 90}$$

⇒ PYTHAGORAS THEOREM

$$LM^2 = MN^2 + LN^2$$

$$130 = 40 + 90$$

$$\boxed{130 = 130}$$

⇒ $\boxed{\text{It is satisfied}}$

---

10) Show that the given points form a parallelogram.

A (2.5, 3.5) B (10, −4) C (2.5, −2.5) D (−5, 5)

A (2.5, 3.5) B (10, −4)

$x_1\ y_1$ $x_2\ y_2$

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-4 - 3.5}{10 - 2.5}$$

B (10, −4) C (2.5, −2.5)

$x_1\ y_1$ $x_2\ y_2$

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-2.5 + 4}{2.5 - 10}$$

$$= \frac{-\cancel{7.5}^{\,1}}{\cancel{7.5}}$$

$\boxed{\text{Slope } AB = -1}$

$C(2.5, -2.5)\ D(-5, 5)$

$x_1\ \ y_1\qquad x_2\ y_2$

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{5 + 2.5}{-5 - 2.5}$$

$$= \frac{\cancel{7.5}^{\,1}}{-\cancel{7.5}}$$

$\boxed{\text{Slope } CD = -1}$

$$\boxed{\text{Slope } BC = \frac{1.5}{-7.5}}$$

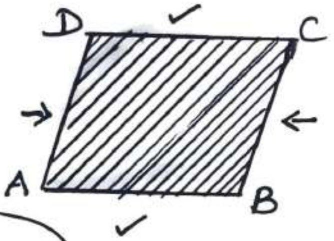
$A(2.5, 3.5)\ D(-5, 5)$

$x_1\ \ y_1\qquad x_2\ y_2$

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{5 - 3.5}{-5 - 2.5}$$

$$\boxed{\text{Slope } AD = \frac{1.5}{-7.5}}$$



$\therefore$ Slope AB = Slope CD

Slope BC = Slope AD

$\Rightarrow$ $\boxed{\text{It is a parallelogram}}$

---

11) If the points $A(2,2)$ $B(-2,-3)$ $C(1,-3)$ and $D(x,y)$ form a parallelogram, then Find the value of x and y.

$A(2,2)\ B(-2,-3)$

$x_1\ y_1\qquad x_2\ y_2$

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-3 - 2}{-2 - 2}$$

$$\boxed{\text{Slope } AB = \frac{\cancel{-}5}{\cancel{-}4}}$$

$C(1,-3)\ D(x,y)$

$x_1\ y_1\qquad x_2\ y_2$

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

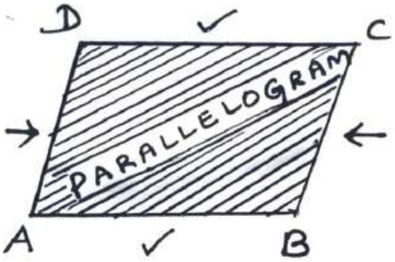
$$= \frac{y + 3}{x - 1}$$

$$\boxed{\text{Slope } CD = \frac{y+3}{x-1}}$$

GIVEN ⇒ $\boxed{\text{PARALLELOGRAM}}$

$$\Downarrow$$

Slope AB = Slope CD



$$\frac{5}{4} = \frac{y+3}{x-1}$$

$$5(x-1) = 4(y+3)$$

$$5x-5 = 4y+12$$

$$5x-4y = 12+5$$

$$\boxed{5x-4y=17} \rightarrow ①$$

B $(-2,-3)$ C $(1,-3)$
x_1 y_1 x_2 y_2

$$\text{Slope} = \frac{y_2-y_1}{x_2-x_1}$$

$$= \frac{-3+3}{1+2}$$

$$= \frac{0}{3}$$

$$\boxed{\text{Slope BC} = 0}$$

A $(2,2)$ D (x,y)
x_1 y_1 x_2 y_2

$$\text{Slope} = \frac{y_2-y_1}{x_2-x_1}$$

$$= \frac{y-2}{x-2}$$

$$\boxed{\text{Slope AD} = \frac{y-2}{x-2}}$$

⇒ $\boxed{\text{Slope BC} = \text{Slope AD}}$

$$0 = \frac{y-2}{x-2}$$

$$0 = y-2$$

$$y-2=0$$

$$\boxed{y=2}$$

Put $y=2$ in ①

$$5x - 4y = 17$$
$$5x - 4(2) = 17$$
$$5x - 8 = 17$$
$$5x = 17 + 8$$
$$5x = 25$$
$$x = \frac{\cancel{25}}{\cancel{5}}\ 5$$
$$\boxed{x = 5}$$

---

12) Let $A(3,-4)$, $B(9,-4)$, $C(5,-7)$ and $D(7,-7)$. Show that ABCD is a trapezium.

$A(3,-4)$ $B(9,-4)$
$x_1\ y_1 \qquad x_2\ y_2$

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{-4+4}{9-3}$$
$$= \frac{0}{6}$$

Slope AB = 0

$C(5,-7)$ $D(7,-7)$
$x_1\ y_1 \qquad x_2\ y_2$

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{-7+7}{7-5}$$
$$= \frac{0}{2}$$

Slope CD = 0

$B(9,-4)$ $C(5,-7)$
$x_1\ y_1 \qquad x_2\ y_2$

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{-7+4}{5-9}$$
$$= \frac{\cancel{-}3}{\cancel{-}4}$$

Slope BC = $\frac{3}{4}$

$A(3,-4)$ $D(7,-7)$
$x_1\ y_1 \qquad x_2\ y_2$

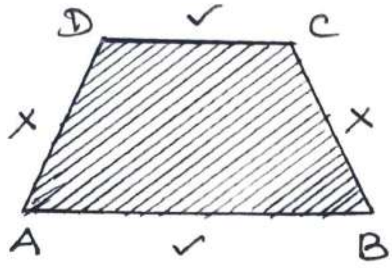
$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{-7+4}{7-3}$$

Slope AD = $\frac{-3}{4}$

Slope AB = Slope CD

Slope BC ≠ Slope AD

⇒ It is a trapezium



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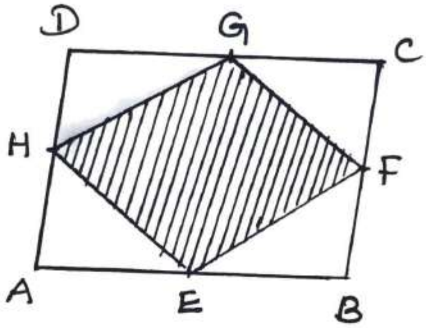
13) A quadrilateral has vertices at $A(-4,-2)$ $B(5,-1)$ $C(6,5)$ and $D(-7,6)$. Show that the mid-points of its sides form a parallelogram.

$A(-4,-2)$ $B(5,-1)$ $C(6,5)$ $D(-7,6)$



$A(-4,-2)$ $B(5,-1)$

x_1 y_1 $\quad$ x_2 y_2

$$\text{Midpt} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$= \left(\frac{-4+5}{2}, \frac{-2-1}{2}\right)$$

$$\boxed{E = \left(\frac{1}{2}, -\frac{3}{2}\right)}$$

$B(5,-1)$ $C(6,5)$

x_1 y_1 $\quad$ x_2 y_2

$$\text{M.P} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$= \left(\frac{5+6}{2}, \frac{-1+5}{2}\right)$$

$$= \left(\frac{11}{2}, \frac{\overset{2}{\cancel{4}}}{\cancel{2}}\right)$$

$$\boxed{F = \left(\frac{11}{2}, 2\right)}$$

$C(6,5)$ $D(-7,6)$

x_1 y_1 $\quad$ x_2 y_2

$$\text{M.P} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$= \left(\frac{6-7}{2}, \frac{5+6}{2}\right)$$

$$\boxed{G = \left(\frac{-1}{2}, \frac{11}{2}\right)}$$

$A(-4, -2)\ D(-7, 6)$

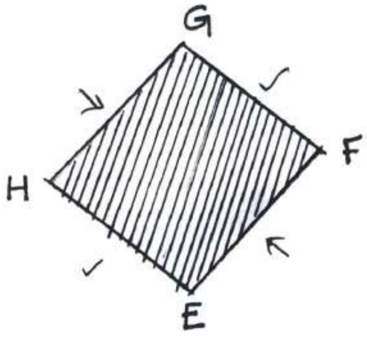
$x_1\ y_1 \qquad x_2\ y_2$

$$M.P = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$= \left(\frac{-4-7}{2}, \frac{-2+6}{2}\right)$$

$$= \left(\frac{-11}{2}, \frac{\overset{2}{\cancel{4}}}{\cancel{2}}\right)$$

$$\boxed{H = \left(\frac{-11}{2}, 2\right)}$$



$E\left(\frac{1}{2}, \frac{-3}{2}\right)\ F\left(\frac{11}{2}, 2\right)$

$x_1\ y_1 \qquad x_2\ y_2$

$$\text{slope} = \frac{y_2-y_1}{x_2-x_1}$$

$$= \frac{2+\frac{3}{2}}{\frac{11}{2}-\frac{1}{2}}$$

$$= \frac{\frac{4+3}{\cancel{2}}}{\frac{11-1}{\cancel{2}}}$$

$$\boxed{\text{slope } EF = \frac{7}{10}}$$

$G\left(\frac{-1}{2}, \frac{11}{2}\right)\ H\left(\frac{-11}{2}, 2\right)$

$x_1\ y_1 \qquad x_2\ y_2$

$$\text{slope} = \frac{y_2-y_1}{x_2-x_1}$$

$$= \frac{2-\frac{11}{2}}{\frac{-11}{2}+\frac{1}{2}}$$

$$= \frac{\frac{4-11}{\cancel{2}}}{\frac{-11+1}{\cancel{2}}}$$

$$= \frac{+7}{+10}$$

$$\boxed{\text{slope } GH = \frac{7}{10}}$$

$F\left(\frac{11}{2}, 2\right)$ $G\left(\frac{-1}{2}, \frac{11}{2}\right)$

$x_1 \quad y_1 \qquad x_2 \quad y_2$

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{\frac{11}{2} \underset{\rightarrow}{-} 2}{\frac{-1}{2} - \frac{11}{2}}$$

$$= \frac{\frac{11-4}{\cancel{2}}}{\frac{-1-11}{\cancel{2}}}$$

$$\boxed{\text{slope } FG = \frac{7}{-12}}$$

$E\left(\frac{1}{2}, \frac{-3}{2}\right)$ $H\left(\frac{-11}{2}, 2\right)$

$x_1 \quad y_1 \qquad x_2 \quad y_2$

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{\underset{\nwarrow}{2} + \frac{3}{2}}{\frac{-11}{2} - \frac{1}{2}}$$

$$= \frac{\frac{4+3}{\cancel{2}}}{\frac{-11-1}{\cancel{2}}}$$

$$\boxed{\text{slope } EH = \frac{7}{-12}}$$

$\Rightarrow$ Slope EF = slope GH

Slope FG = slope EH

$\therefore$ EFGH is a parallelogram

EXERCISE 5.3

1) Find the equation of a straight line Passing through the midpoint of a line segment joining the points (1, -5) and (4, 2) and parallel to (i) X-axis (ii) Y-axis.

eqn?

$\boxed{\text{Midpoint}} \Rightarrow (1, -5)$ and $(4, 2)$

$x_1 \; y_1 \qquad x_2 \; y_2$

$$M.P = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$= \left(\frac{1+4}{2}, \frac{-5+2}{2}\right)$$

$$= \left(\frac{5}{2}, \frac{-3}{2}\right)$$

(i) Parallel to X-axis:

The equation of line parallel to X-axis is $y = b$

It passes through $\left(\frac{5}{2}, -\frac{3}{2}\right)$

where $\boxed{b = -\frac{3}{2}}$

$\therefore$ The equation is $y = -\frac{3}{2}$

$$\Rightarrow \boxed{y + \frac{3}{2} = 0}$$

(ii) Parallel to Y-axis:

The equation of line parallel to y-axis is $x = c$

It passes through $\left(\frac{5}{2}, -\frac{3}{2}\right)$

where $\boxed{c = \frac{5}{2}}$

$\therefore$ The equation is $x = \frac{5}{2}$

$$\Rightarrow \boxed{x - \frac{5}{2} = 0}$$

---

2) The equation of a straight line is $2(x-y)+5=0$. Find its slope, inclination and intercept on the y-axis.

$$2(x-y)+5=0$$

$$2x-2y+5=0$$

$$-2y=-2x-5$$

$$2y=2x+5 \text{ (Multiply by '–')}$$

$$y=\frac{2x+5}{2}$$

$$y=\frac{\cancel{2}x}{\cancel{2}}+\frac{5}{2}$$

$$y=x+\frac{5}{2}$$

We know, $y=mx+c$ [compare]

$m=1 \Rightarrow$ slope $=1$

$c=\frac{5}{2} \Rightarrow$ y-Intercept $=\frac{5}{2}$

$$\text{Slope} = \tan\theta$$
$$\downarrow$$
$$1 = \tan\theta$$
$$\therefore \boxed{\theta = 45^\circ} \Rightarrow \boxed{\text{Inclination} = 45^\circ}$$

3) Find the equation of a line whose inclination is 30° and making an intercept −3 on the y-axis.

$\boxed{\text{eqn?}}$

$$\boxed{\text{Inclination is } 30^\circ} \Rightarrow \boxed{\theta = 30^\circ}$$
$$\boxed{\text{Intercept on y-axis is } -3} \Rightarrow \boxed{c = -3}$$

$$\text{Slope} = \tan\theta$$
$$= \tan 30^\circ$$
$$\text{Slope} = \frac{1}{\sqrt{3}}$$
$$\Downarrow$$
$$\boxed{m = \frac{1}{\sqrt{3}}}$$

$\boxed{\text{eqn?}}$ $y = mx + c$

$$y = \frac{1}{\sqrt{3}}x + (-3)$$
$$\nwarrow \qquad \longrightarrow$$
$$\sqrt{3}y = x - 3\sqrt{3}$$
$$\boxed{x - \sqrt{3}y - 3\sqrt{3} = 0}$$

4) Find the slope and y-Intercept of $\sqrt{3}x + (1-\sqrt{3})y = 3$

$$(1-\sqrt{3})y = 3 - \sqrt{3}x$$
$$y = \frac{3 - \sqrt{3}x}{1-\sqrt{3}}$$
$$y = \frac{3}{1-\sqrt{3}} - \frac{\sqrt{3}}{1-\sqrt{3}}x$$

$$\boxed{y = \frac{-\sqrt{3}}{1-\sqrt{3}}x + \frac{3}{1-\sqrt{3}}}$$

We know, $y = mx + c$ [comparing]

$$\Rightarrow m = \frac{-\sqrt{3}}{1-\sqrt{3}} \times \frac{(1+\sqrt{3})}{(1+\sqrt{3})}$$

$$(a-b) \times (a+b)$$

$$\boxed{\sqrt{3} \times \sqrt{3} = 3}$$

$$\boxed{(a-b)(a+b) = a^2 - b^2}$$

$$= \frac{-\sqrt{3}-3}{(1)^2-(\sqrt{3})^2}$$

$$= \frac{-\sqrt{3}-3}{1-3}$$

$$= \frac{\cancel{-}(\sqrt{3}+3)}{\cancel{-}2}$$

$$\Rightarrow \boxed{\text{slope} = \frac{\sqrt{3}+3}{2}}$$

$$\Rightarrow c = \frac{3}{1-\sqrt{3}} \times \frac{(1+\sqrt{3})}{(1+\sqrt{3})}$$

$$= \frac{3+3\sqrt{3}}{(1)^2-(\sqrt{3})^2}$$

$$= \frac{3+3\sqrt{3}}{1-3}$$

$$\Rightarrow \boxed{y\text{-Intercept} = \frac{3+3\sqrt{3}}{-2}}$$

---

5) Find the value of 'a', if the line through $(-2, 3)$ and $(8, 5)$ is perpendicular to $y = ax + 2$

$a = ?$

Perpendicular

$(-2, 3)\quad(8, 5)$

$x_1\ y_1\quad x_2\ y_2$

$$\boxed{\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}}$$

$$= \frac{5-3}{8+2}$$

$$= \frac{\cancel{2}^{1}}{\cancel{10}_{5}}$$

$$m_1 = \frac{1}{5}$$

$y = ax + 2$

$ax - y + 2 = 0$

$$\boxed{\text{slope} = -\frac{\text{co}x}{\text{co}y}}$$

$$= \frac{\cancel{-}a}{\cancel{-}}$$

$$m_2 = \frac{a}{1}$$

Perpendicular

$\Downarrow$

$$\boxed{m_1 \times m_2 = -1}$$

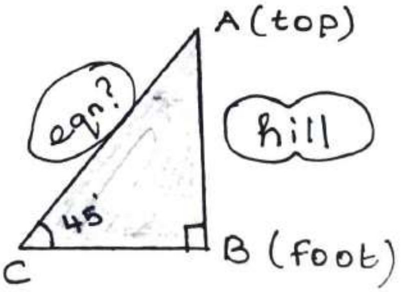
$$\frac{1}{5} \times \frac{a}{1} = -1$$

$$\frac{a}{5} = -1$$

$$a = -5$$

---

6) The hill in the form of a right triangle has its foot at (19, 3). The inclination of the hill to the ground is 45°. Find the equation of the hill joining the foot and top.



Foot of the hill is (19, 3)

$\Rightarrow \boxed{B(19, 3)}$

Inclination is 45°

$$\Rightarrow \boxed{\theta = 45^\circ}$$

$$\text{Slope} = \tan\theta$$

$$= \tan 45^\circ$$

$$\boxed{\text{Slope} = 1}$$

$$\Downarrow$$

$$\boxed{m = 1} \quad \underset{x_1 \;\; y_1}{(19, 3)}$$

eqn?

$$y - y_1 = m(x - x_1)$$

$$y - 3 = 1(x - 19)$$

$$y - 3 = x - 19$$

$$x - y - 19 + 3 = 0$$

$$\boxed{x - y - 16 = 0}$$

---

7) Find the equation of a line through the given pair of points.

(i) $\underset{x_1 \;\; y_1}{\left(2, \frac{2}{3}\right)}$ and $\underset{x_2 \;\; y_2}{\left(\frac{-1}{2}, -2\right)}$

eqn?

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - \frac{2}{3}}{-2 - \frac{2}{3}} = \frac{x - 2}{-\frac{1}{2} - 2}$$

$$\frac{\frac{3y - 2}{\cancel{3}}}{\frac{-6 - 2}{\cancel{3}}} = \frac{x - 2}{\frac{-1 - 4}{2}}$$

$$\frac{3y-2}{\cancel{-}8} = \frac{2(x-2)}{\cancel{-}5}$$

$$\frac{3y-2}{8} = \frac{2x-4}{5}$$

$$5(3y-2) = 8(2x-4)$$

$$15y-10 = 16x-32$$

$$16x-15y-32+10=0$$

$$\boxed{16x-15y-22=0}$$

(ii) $(2,3)$ and $(-7,-1)$

$x_1\ y_1$ $\qquad$ $x_2\ y_2$

eqn?

$$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$$

$$\frac{y-3}{-1-3} = \frac{x-2}{-7-2}$$

$$\frac{y-3}{\cancel{-}4} = \frac{x-2}{\cancel{-}9}$$

$$9(y-3) = 4(x-2)$$

$$9y-27 = 4x-8$$

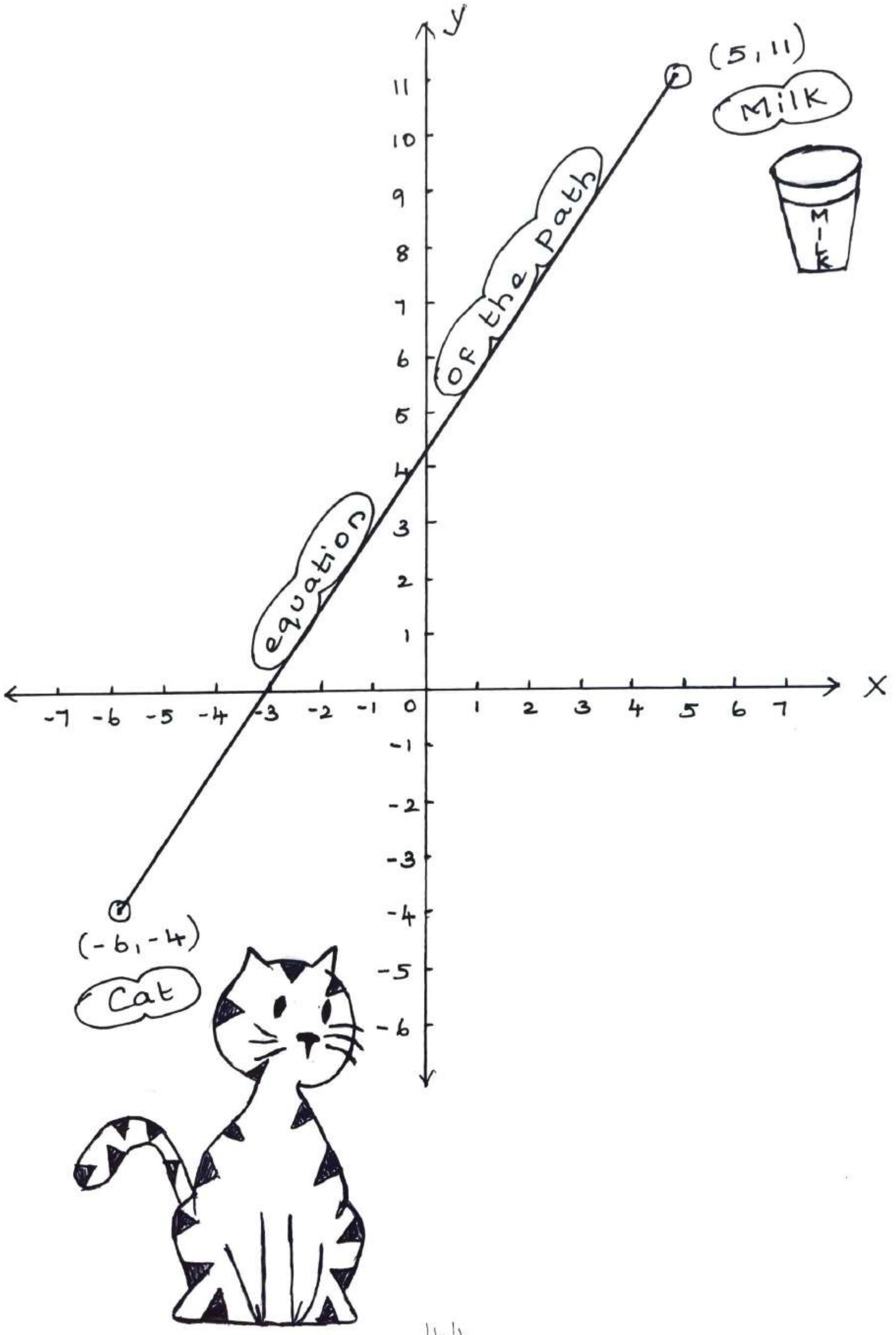
$$4x-9y-8+27=0$$

$$\boxed{4x-9y+19=0}$$

---

8) A cat is located at a point $(-6,-4)$ in xy plane. A bottle of milk is kept at $(5,11)$. The cat wish to consume the

milk travelling through shortest possible distance. Find the equation of the path it needs to take the milk.



Cat $(-6,-4)$ Milk $(5,11)$

$x_1 \ y_1$ $x_2 \ y_2$

eqn?

$$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$$

$$\frac{y+4}{11+4} = \frac{x+6}{5+6}$$

$$\frac{y+4}{15} = \frac{x+6}{11}$$

$$11(y+4) = 15(x+6)$$

$$11y+44 = 15x+90$$

$$15x-11y+90-44=0$$

$$\boxed{15x-11y+46=0}$$

---

9) Find the equation of the median and altitude of Δ ABC through A where the vertices are $A(6,2)$ $B(-5,-1)$ and $C(1,9)$

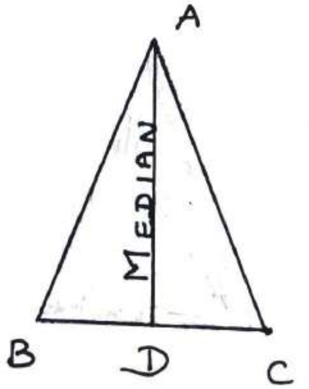
eqn?

(i) Median [through A]

$A(6,2)$ $B(-5,-1)$ $C(1,9)$

$B(-5,-1)$ $C(1,9)$

$x_1 \ y_1$ $x_2 \ y_2$



$$\text{Midpt} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$= \left(\frac{-5+1}{2}, \frac{-1+9}{2}\right)$$

$$= \left(\frac{-\cancel{4}^{2}}{\cancel{2}}, \frac{\cancel{8}^{4}}{\cancel{2}}\right)$$

$\boxed{D = (-2, 4)}$ $A(6, 2)$

$x_1\ y_1$ $x_2\ y_2$

eqn?

$$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$$

$$\frac{y-4}{2-4} = \frac{x+2}{6+2}$$

$$\frac{y-4}{-\cancel{2}_{1}} = \frac{x+2}{\cancel{8}\,4}$$

$$4(y-4) = -1(x+2)$$

$$4y-16 = -x-2$$

$$x+4y-16+2=0$$

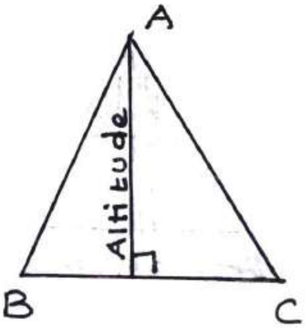
$$\boxed{x+4y-14=0}$$

(ii) Altitude: [through A]

$B(-5, -1)$ $C(1, 9)$

$x_1\ y_1$ $x_2\ y_2$

$$\text{Slope} = \frac{y_2-y_1}{x_2-x_1} = \frac{9+1}{1+5} = \frac{\cancel{10}^{5}}{\cancel{6}_{3}}$$



$$\Rightarrow \boxed{\text{Slope} = \frac{5}{3}}$$

$\perp^r$ slope $= -\dfrac{3}{5}$

$\Downarrow$

$\boxed{m = -\dfrac{3}{5}}$ $A\,(6, 2)$
$\quad\quad\quad\quad\quad\quad x_1, y_1$

eqn:

$$y - y_1 = m(x - x_1)$$

$$y - 2 = -\frac{3}{5}(x - 6)$$

$$5(y-2) = -3(x-6)$$

$$5y - 10 = -3x + 16$$

$$3x + 5y - 10 - 18 = 0$$

$$\boxed{3x + 5y - 28 = 0}$$

---

10) Find the equation of a straight line which has slope $-\dfrac{5}{4}$ and passing through the point $(-1, 2)$

eqn?

$\boxed{\text{slope} = -\dfrac{5}{4}}$

$\Downarrow$

$\boxed{m = -\dfrac{5}{4}}$ $(-1, 2)$
$\quad\quad\quad\quad\quad\quad x_1, y_1$

eqn

$$y - y_1 = m(x - x_1)$$

$$y - 2 = -\frac{5}{4}(x + 1)$$

$$4(y-2) = -5(x+1)$$

$$4y-8=-5x-5$$

$$5x+4y-8+5=0$$

$$\boxed{5x+4y-3=0}$$

11) You are downloading a song. The percent y (in decimal form) of mega bytes remaining to get downloaded in 'x' seconds is given by $y=-0.1x+1$.

(i) Find the total MB of the song.

(ii) After how many seconds will 75% of the song gets downloaded?

(iii) After how many seconds the song will be downloaded completely?

$$\boxed{y=-0.1x+1}$$

(i) Total MB of the Song:

$\Rightarrow$ Time is Zero

$\Rightarrow \boxed{x=0}$

$$\therefore\ y=-0.1(0)+1$$

$$\boxed{y=1\text{ MB}}$$

(ii) Time when 75% of song gets downloaded:

$\Rightarrow \boxed{25\%\text{ remaining}}$

$$25\%=\frac{25}{100}$$

$\Rightarrow \boxed{y=0.25}$

$$\boxed{y = -0.1x + 1}$$

$$0.25 = -0.1x + 1$$

$$0.1x = 1 - 0.25$$

$$0.1x = 0.75$$

$$x = \frac{0.75}{0.1}$$

$$x = \frac{0.75}{0.1} \times \frac{10}{10}$$

$$x = \frac{7.5}{1}$$

$$\boxed{x = 7.5}$$

⇒ Required time = 7.5 seconds

(iii) Time to download Completely:

⇒ Remaining % = 0

$$\Rightarrow \boxed{y = 0}$$

$$y = -0.1x + 1$$

$$0 = -0.1x + 1$$

$$0.1x = 1$$

$$x = \frac{1}{0.1} \times \frac{10}{10}$$

$$x = \frac{10}{1}$$

$$\boxed{x = 10}$$

⇒ Required time is 10 seconds

12) Find the equation of a line whose intercepts on the x and y axes are given below:

(i) $4, -6$

x - Intercept

$$\boxed{a=4}$$

y - Intercept

$$\boxed{b=-6}$$

eqn:

$$\frac{x}{a}+\frac{y}{b}=1$$

$$\frac{x}{4}+\frac{y}{-6}=1$$

$$\frac{x}{4}-\frac{y}{6}=1$$

$$\frac{x\times3}{4\times3}-\frac{y\times2}{6\times2}=1$$

$$\frac{3x-2y}{12 \rightarrow}=1$$

$$3x-2y=12$$

$$\boxed{3x-2y-12=0}$$

$$\begin{array}{r|l} 2 & 4,6 \\ 2 & 2,3 \\ 3 & 1,3 \\ & 1,1 \end{array}$$

L.C.M = 12

(ii) $-5, \frac{3}{4}$

x - Intercept

$$\boxed{a=-5}$$

y - Intercept

$$\boxed{b=\frac{3}{4}}$$

eqn:

$$\frac{x}{a}+\frac{y}{b}=1$$

$$\frac{x}{-5}+\frac{y}{3/4}=1$$

$$\frac{-5}{5}+\frac{4y}{3}=1$$

$$\frac{-x\times3}{5\times3}+\frac{4y\times5}{3\times5}=1$$

$$\frac{-3x+20y}{15 \rightarrow}=1$$

$$-3x+20y=15$$

$$\boxed{3x-20y+15=0}$$

---

13) Find the intercepts made by the following lines on the co-ordinate axes.

(i) $\boxed{3x - 2y - 6 = 0}$

x - Intercepts

$\Downarrow$

$\boxed{y = 0}$

$3x - 2y - 6 = 0$

$3x - 2(0) - 6 = 0$

$3x - 6 = 0$

$3x = 6$

$x = \frac{\cancel{6}^{2}}{\cancel{3}}$

$x = 2$

y - Intercept

$\Downarrow$

$\boxed{x = 0}$

$3x - 2y - 6 = 0$

$3(0) - 2y - 6 = 0$

$-2y = 6$

$y = \frac{\cancel{6}^{3}}{-\cancel{2}}$

$y = -3$

(ii) $\boxed{4x + 3y + 12 = 0}$

x - Intercept $\Rightarrow \boxed{y = 0}$

$4x + 3y + 12 = 0$

$4x + 3(0) + 12 = 0$

$4x + 12 = 0$

$4x = -12$

$x = \frac{-\cancel{12}^{3}}{\cancel{4}}$

$x = -3$

y - Intercept $\Rightarrow \boxed{x = 0}$

$4x + 3y + 12 = 0$

$4(0) + 3y + 12 = 0$

$3y + 12 = 0$

$3y = -12$

$y = \frac{-\cancel{12}^{4}}{\cancel{3}}$

$y = -4$

---

14) Find the equation of a straight line

(i) passing through (1, -4) and has

intercepts which are in the ratio 2:5

eqn?

$(1, -4)$
$x \quad y$

Intercepts are in the ratio 2:5
2a, 5a

Intercept

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$\frac{1}{2a} + \frac{(-4)}{5a} = 1$$

$$\frac{1 \times 5}{2a \times 5} - \frac{4 \times 2}{5a \times 2} = 1$$

$$\frac{5-8}{10a \rightarrow} = 1$$

$$-3 = 10a$$

$$10a = -3$$

$$\boxed{a = \frac{-3}{10}}$$

x-Intercept $\Rightarrow 2a = 2\left(\frac{-3}{10}\right) = -\frac{\cancel{6}^{3}}{\cancel{10}_{5}} = \boxed{\frac{-3}{5}}$

y-Intercept $\Rightarrow 5a = 5\left(\frac{-3}{10}\right) = -\frac{\cancel{15}^{3}}{\cancel{10}_{2}} = \boxed{\frac{-3}{2}}$

eqn:

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$\frac{x}{\frac{-3}{5}} + \frac{y}{\frac{-3}{2}} = 1$$

$$\frac{5x}{-3} + \frac{2y}{-3} = 1$$

$$\frac{-5x}{3} - \frac{2y}{3} = 1$$

$$\frac{-5x-2y}{3} = 1$$

$$-5x-2y = 3$$

$$\boxed{5x+2y+3=0}$$

(ii) passing through (-8, 4) and making equal intercepts on the co-ordinate axes.

Equal Intercepts
a, a

$(-8, 4)$
$x \quad y$

Intercepts

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$\frac{-8}{a} + \frac{4}{a} = 1$$

$$\frac{-8+4}{a} = 1$$

$$-4 = a$$

$$\boxed{a = -4}$$

eqn

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$\frac{x}{-4} + \frac{y}{-4} = 1$$

$$\frac{x+y}{-4} = 1$$

$$x+y = -4$$

$$\boxed{x+y+4=0}$$

EXERCISE 5.4

1) Find the slope of the following straight lines

(i) $5y-3=0$

$\Rightarrow \boxed{0x+5y-3=0}$

Slope $= \frac{-\text{co}x}{\text{co}y} = -\frac{0}{5} = 0$

$\boxed{\text{slope}=0}$

(ii) $7x-\frac{3}{17}=0$

$\Rightarrow \boxed{7x+0y-\frac{3}{17}=0}$

$\therefore$ slope $= \frac{-\text{co}x}{\text{co}y} = \frac{-7}{0}$

$\boxed{\text{slope}=\text{n.d}}$

---

2) Find the slope of the line which is

(i) Parallel to $y=0.7x-11$

$\Rightarrow \boxed{0.7x-y-11=0}$

$\therefore$ slope $= -\frac{\text{co}x}{\text{co}y} = \frac{\cancel{-}0.7}{\cancel{-}1} = 0.7$

$\therefore \boxed{\text{slope}=0.7}$

(ii) Perpendicular to the line $x=-11$

$\Rightarrow x+11=0$

$\Rightarrow \boxed{x+0y+11=0}$

$\therefore$ Slope $= \frac{-\text{co}x}{\text{co}y} = \frac{-1}{0} = \text{n.d}$

$\therefore \boxed{\text{slope}=\text{n.d}}$

3) Check whether the given lines are Parallel or perpendicular?

i) $\frac{x}{3}+\frac{y}{4}+\frac{1}{7}=0$ and $\frac{2x}{3}+\frac{y}{2}+\frac{1}{10}=0$

$$\text{slope}=\frac{-\text{co}x}{\text{co}y}$$

$$=\frac{-\frac{1}{3}}{\frac{1}{4}}$$

$$=\frac{-1}{3}\times\frac{4}{1}$$

$$m_1=\frac{-4}{3}$$

$$\text{slope}=\frac{-\text{co}x}{\text{co}y}$$

$$=\frac{-\frac{2}{3}}{\frac{1}{2}}$$

$$=-\frac{2}{3}\times\frac{2}{1}$$

$$m_2=\frac{-4}{3}$$

$$\boxed{m_1=m_2}$$

$\Rightarrow$ It is parallel

(ii) $5x+23y+14=0$ and $23x-5y+9=0$

$$\text{slope}=\frac{-\text{co}x}{\text{co}y}$$

$$m_1=\frac{-5}{23}$$

$$\text{slope}=\frac{-\text{co}x}{\text{co}y}$$

$$=\frac{+23}{+5}$$

$$m_2=\frac{23}{5}$$

$$\boxed{m_1\times m_2=-1}$$

$$\frac{-\cancel{5}^{1}}{\cancel{23}}\times\frac{\cancel{23}}{\cancel{5}}=-1$$

$$-1=-1$$

$\Rightarrow$ $\boxed{\text{It is Perpendicular}}$

4) If the straight lines $12y = -(P+3)x + 12$, $12x - 7y = 16$ are perpendicular then find 'p'.

$$12y = -(P+3)x + 12$$

$$\boxed{(P+3)x + 12y - 12 = 0}$$

$$\text{Slope} = \frac{-\,\text{co}x}{\text{co}y}$$

$$\boxed{m_1 = \frac{-(P+3)}{12}}$$

$$12x - 7y = 16$$

$$\boxed{12x - 7y - 16 = 0}$$

$$\text{Slope} = \frac{-\,\text{co}x}{\text{co}y}$$

$$= \frac{\cancel{-}12}{\cancel{-}7}$$

$$\boxed{m_2 = \frac{12}{7}}$$

$$\boxed{\text{Perpendicular}}$$

$$\Downarrow$$

$$m_1 \times m_2 = -1$$

$$\frac{\cancel{-}(P+3)}{1\cancel{2}} \times \frac{1\cancel{2}}{7} = \cancel{-}1$$

$$\frac{P+3}{7} \rightarrow = 1$$

$$P + 3 = 7$$

$$P = 7 - 3$$

$$\boxed{P = 4}$$

---

5) Find the equation of a straight line passing through the point $P(-5, 2)$ and parallel to the line joining the points $Q(3, -2)$ and $R(-5, 4)$

$\boxed{\text{eqn?}}$ $\boxed{P(-5,2)}$

$Q\ (3,-2)\ R\ (-5,4)$
$x_1\ y_1 \quad x_2\ y_2$

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{4+2}{-5-3}$$

$$= \frac{\cancel{6}^{3}}{-\cancel{8}_{4}}$$

$$\boxed{m = \frac{3}{-4}}$$

$\boxed{\text{Parallel} \Rightarrow \text{slope}}$

$\boxed{m = \frac{3}{-4}}$ $P(-5,2)$
$x_1\ y_1$

eqn

$$y - y_1 = m\,(x - x_1)$$

$$y - 2 = \frac{3}{-4}\,(x+5)$$
$\nwarrow$

$$-4(y-2) = 3(x+5)$$

$$-4y + 8 = 3x + 15$$

$$3x + 4y + 15 - 8 = 0$$

$$\boxed{3x + 4y + 7 = 0}$$

---

6) Find the equation of a line passing through $(6,-2)$ and perpendicular to the line joining the points $(6,7)$ and $(2,-3)$

$\boxed{\text{eqn?}}$ $\boxed{(6,-2)}$

Perpendicular:

$(6,7)\ (2,-3)$
$x_1\ y_1 \quad x_2\ y_2$

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-3-7}{2-6}$$

$$= \frac{\cancel{+10}^{5}}{\cancel{+4}_{2}}$$

$$\boxed{\text{Slope} = \frac{5}{2}}$$

$$\perp^{r} \text{ slope} = -\frac{2}{5}$$

$\boxed{m = -\frac{2}{5}}$ $(6,-2)$
$x_1\ y_1$

eqn $y - y_1 = m\,(x - x_1)$

$$y + 2 = \frac{-2}{5}\,(x - 6)$$
$\nwarrow$

$$5(y+2) = -2\,(x-6)$$

$$5y + 10 = -2x + 12$$

$$2x + 5y + 10 - 12 = 0$$

$$\boxed{2x + 5y - 2 = 0}$$

7) A (-3, 0) B (10, -2) and C (12, 3) are the vertices of Δ ABC. Find the equation of the altitude through A and B.

eqn? Altitude through A and B

A (-3, 0) B (10, -2) C (12, 3)

(i) Altitude through A:

B (10, -2) C (12, 3)

x_1 y_1 x_2 y_2

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{3+2}{12-10}$$

$$\boxed{\text{Slope} = \frac{5}{2}}$$

$$\perp^r \text{ Slope} = \frac{-2}{5}$$

$\Downarrow$

$$\boxed{m = \frac{-2}{5}}$$ A (-3, 0)

x_1 y_1

eqn

$$y - y_1 = m(x - x_1)$$

$$y - 0 = -\frac{2}{5}(x + 3)$$

$$5y = -2x - 6$$

$$\boxed{2x + 5y + 6 = 0}$$

(ii) Altitude through B:

A (-3, 0) C (12, 3)

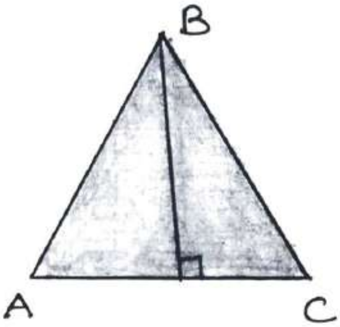
x_1 y_1 x_2 y_2

$$\text{Slope} = \frac{y_2-y_1}{x_2-x_1}$$

$$= \frac{3-0}{12+3}$$

$$= \frac{\cancel{3}^{1}}{\cancel{15}_{5}}$$

$$\boxed{\text{Slope} = \frac{1}{5}}$$



$$\perp^r \text{ slope} = \frac{-5}{1}$$

$$\Downarrow$$

$\boxed{m=-5}$ $B(10, -2)$
$\qquad\qquad\qquad\quad x_1\ \ y_1$

eqn

$$y-y_1 = m(x-x_1)$$

$$y+2 = -5(x-10)$$

$$y+2 = -5x+50$$

$$5x+y+2-50=0$$

$$\boxed{5x+y-48=0}$$

---

8) Find the equation of the perpendicular bisector of the line joining the Points A(-4,2) and B(6,-4)

(i) Perpendicular

$A(-4,2)\ \ B(6,-4)$
$\ \ x_1\ y_1 \qquad\ x_2\ y_2$

$$\text{Slope} = \frac{y_2-y_1}{x_2-x_1}$$

(ii) Bisector

$A(-4,2)\ \ B(6,-4)$
$\ \ x_1\ y_1 \qquad x_2\ y_2$

$$\text{Midpt} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$= \frac{-4-2}{6+4}$

$= \frac{-\cancel{6}^{3}}{\cancel{10}_{5}}$

$\boxed{\text{slope} = \frac{-3}{5}}$

$\perp^r$ slope $= \frac{5}{3}$

$\Downarrow$

$= \left(\frac{-4+6}{2}, \frac{2-4}{2}\right)$

$= \left(\frac{\cancel{2}^{1}}{\cancel{2}}, \frac{-\cancel{2}^{1}}{\cancel{2}}\right)$

$\boxed{= (1, -1)}$

$\boxed{m = \frac{5}{3}}$ $\quad (\underset{x_1}{1}, \underset{y_1}{-1})$

eqn

$$y - y_1 = m(x - x_1)$$

$$y + 1 = \frac{5}{3}(x - 1)$$

$$3(y+1) = 5(x-1)$$

$$3y + 3 = 5x - 5$$

$$5x - 3y - 5 - 3 = 0$$

$$\boxed{5x - 3y - 8 = 0}$$

---

9) Find the equation of a straight line through the intersection of lines $7x + 3y = 10$, $5x - 4y = 1$ and parallel to the line $13x + 5y + 12 = 0$

$\boxed{\text{eqn?}}$ (i) $\boxed{\text{Intersection}}$ $\Rightarrow$ [solve 2 eqns]

$7x + 3y = 10 \rightarrow$ ① $\times 4$

$5x - 4y = 1 \rightarrow$ ② $\times 3$

$$28x + \cancel{12y} = 40$$
$$\underline{15x - \cancel{12y} = 3}$$
$$43x = 43$$
$$x = \frac{\cancel{43}^{1}}{\cancel{43}}$$
$$\boxed{x = 1}$$

Put $x=1$ in ①

$$7x + 3y = 10$$
$$7(1) + 3y = 10$$
$$7 + 3y = 10$$
$$3y = 10 - 7$$
$$3y = 3$$
$$y = \frac{\cancel{3}^{1}}{\cancel{3}}$$
$$\boxed{y = 1}$$

$\Rightarrow$ $\boxed{\text{Intersection is } (1,1)}$

(ii) Parallel:

$$13x + 5y + 12 = 0$$
$$\text{Slope} = \frac{-\text{co } x}{\text{co } y}$$
$$= \frac{-13}{5}$$
$$\boxed{m = -\frac{13}{5}} \quad \underset{x_1,\ y_1}{(1,1)}$$

eqn

$$y - y_1 = m(x - x_1)$$
$$y - 1 = -\frac{13}{5}(x - 1)$$
$$\nwarrow$$
$$5(y-1) = -13(x-1)$$
$$5y - 5 = -13x + 13$$
$$13x + 5y - 5 - 13 = 0$$
$$\boxed{13x + 5y - 18 = 0}$$

10) Find the equation of a straight line through the intersection of lines $5x - 6y = 2$, $3x + 2y = 10$ and perpendicular to the line $4x - 7y + 13 = 0$

eqn?

(i) Intersection:

$$5x - 6y = 2 \rightarrow ①$$
$$3x + 2y = 10 \rightarrow ② \times 3$$

$$5x - \cancel{6y} = 2$$
$$9x + \cancel{6y} = 30$$

$$14x = 32$$

$$x = \frac{\cancel{32}\ 16}{\cancel{14}\ 7}$$

$$\boxed{x = \frac{16}{7}}$$

Put $x = \frac{16}{7}$ in ①

$$5x - 6y = 2$$

$$5\left(\frac{16}{7}\right) - 6y = 2$$

$$\frac{80}{7} - 6y = 2$$

$$-6y = 2 - \frac{80}{7}$$

$$-6y = \frac{14-80}{7}$$

$$\cancel{-}6y = \cancel{-}\frac{66}{7}$$

$$y = \frac{\cancel{66}\ 11}{7 \times \cancel{6}}$$

$$\boxed{y = \frac{11}{7}}$$

⇒ Intersection is

$$\left(\frac{16}{7}, \frac{11}{7}\right)$$

(ii) Perpendicular:

$$4x - 7y + 13 = 0$$

$$\text{slope} = -\frac{\text{co}x}{\text{co}y}$$

$$= \frac{\cancel{-}4}{\cancel{-}7}$$

$$\boxed{\text{Slope} = \frac{4}{7}}$$

$$\perp^r \text{slope} = \frac{-7}{4}$$

$$\Downarrow$$

$$\boxed{m = \frac{-7}{4}} \quad \left(\frac{16}{7}, \frac{11}{7}\right)$$

$$x_1 \quad y_1$$

eqn

$$y - y_1 = m(x - x_1)$$

$$y - \frac{11}{7} = -\frac{7}{4}\left(x - \frac{16}{7}\right)$$

$$\frac{7y-11}{\cancel{7}} = -\frac{7}{4}\left(\frac{7x-16}{\cancel{7}}\right)$$

$$4(7y-11) = -7(7x-16)$$

$$28y - 44 = -49x + 112$$

$$49x + 28y - 44 - 112 = 0$$

$$\boxed{49x + 28y - 156 = 0}$$

11) Find the equation of a straight line joining the point of Intersection of $3x+y+2=0$ and $x-2y-4=0$ to the point of intersection of $7x-3y=-12$ and $2y=x+3$.

eqn?

(i) Point of Intersection:

$3x+y\overrightarrow{+2=0}$

$x-2y\overrightarrow{-4=0}$

$3x+y=-2 \rightarrow ① \times 2$

$x-2y=4 \rightarrow ②$

$$\begin{aligned} 6x+\cancel{2y}&=-4 \\ x-\cancel{2y}&=4 \\ \hline 7x&=0 \end{aligned}$$

$x=\frac{0}{7}$

$x=0$

Put $x=0$ in ①

$3x+y=-2$

$3(0)+y=-2$

$0+y=-2$

$y=-2$

∴ Point of Intersection is $(0,-2)$

(ii) Point of Intersection!

$\boxed{7x-3y=-12} \rightarrow ①$

$2y=x+3$

$x-2y+3=0$

$\boxed{x-2y=-3} \rightarrow ②$

$7x-3y=-12 \rightarrow ①$

$x-2y=-3 \rightarrow ② \times 7$

$$\begin{aligned} \cancel{7x}-3y&=-12 \\ \cancel{7x}-14y&=-21 \\ (-)\quad (+)\quad &\quad (+) \\ \hline 11y&=9 \end{aligned}$$

$y=\frac{9}{11}$

Put $y=\frac{9}{11}$ in ①

$7x-3y=-12$

$7x-3\left(\frac{9}{11}\right)=-12$

$7x-\frac{27}{11}=-12$

$7x=-12+\frac{27}{11}$

$7x=\frac{-132+27}{11}$

$$7x = \frac{-105}{11}$$

$$x = \frac{-\cancel{105}^{\,15}}{11 \times \cancel{7}}$$

$$\boxed{x = \frac{-15}{11}}$$

Point of Intersection is $\left(\frac{-15}{11}, \frac{9}{11}\right)$

Now, we have

$$\underset{x_1\ \ y_1}{(0,-2)} \text{ and } \underset{x_2\ \ \ y_2}{\left(\frac{-15}{11}, \frac{9}{11}\right)}$$

eqn

$$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$$

$$\frac{y+2}{\frac{9}{11}+2} = \frac{x-0}{\frac{-15}{11}-0}$$

$$\frac{y+2}{\frac{9+22}{\cancel{11}}} = \frac{x}{\frac{-15}{\cancel{11}}}$$

$$\frac{y+2}{31} = \frac{x}{-15}$$

$$-15(y+2) = 31x$$

$$-15y - 30 = 31x$$

$$\boxed{31x + 15y + 30 = 0}$$

12) Find the equation of a straight line through the point of Intersection of the lines $8x+3y=18$, $4x+5y=9$ and bisecting the line segment joining the Points $(5,-4)$ and $(-7,6)$

(i) Point of Intersection:

$$8x+3y=18 \rightarrow ① \times 4$$

$$4x+5y=9 \rightarrow ② \times 8$$

$$\cancel{32x}+12y=72$$

$$\cancel{32x}+40y=72$$

$$(-) \quad (-) \quad (-)$$

$$-28y=0$$

$$y=\frac{0}{-28}$$

$$\boxed{y=0}$$

Put $y=0$ in ①

$$8x+3y=18$$

$$8x+3(0)=18$$

$$8x=18$$

$$x=\frac{\cancel{18}^{9}}{\cancel{8}_{4}}$$

$$\boxed{x=\frac{9}{4}}$$

⇒ Point of Intersection is $\left(\frac{9}{4}, 0\right)$

(ii) Bisecting:

$$(5,-4) \quad (-7,6)$$

$$x_1 \; y_1 \quad x_2 \; y_2$$

$$M.P = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$= \left(\frac{5-7}{2}, \frac{-4+6}{2}\right)$$

$$= \left(\frac{-\cancel{2}^{1}}{\cancel{2}}, \frac{\cancel{2}^{1}}{\cancel{2}}\right)$$

$$\boxed{M.P=(-1,1)}$$

eqn $\left(\frac{9}{4}, 0\right)$ and $(-1, 1)$

$x_1 \ y_1$ $\qquad$ $x_2 \ y_2$

$$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$$

$$\frac{y-0}{1-0} = \frac{x - \frac{9}{\nwarrow 4}}{-1 - \frac{9}{\nwarrow 4}}$$

$$\frac{y-0}{1-0} = \frac{\frac{4x-9}{\cancel{4}}}{\frac{-4-9}{\cancel{4}}}$$

$$\frac{y}{1} = \frac{4x-9}{-13}$$

$$-13y = 4x-9$$

$$\boxed{4x+13y-9=0}$$

CHAPTER-6

TRIGONOMETRY

RECIPROCAL RATIOS

1) $\sin\theta = \dfrac{1}{\operatorname{cosec}\theta}$

2) $\cos\theta = \dfrac{1}{\sec\theta}$

3) $\tan\theta = \dfrac{1}{\cot\theta}$

4) $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$

5) $\sec\theta = \dfrac{1}{\cos\theta}$

6) $\cot\theta = \dfrac{1}{\tan\theta}$

Identity	Equal forms
$\sin^2\theta + \cos^2\theta = 1$	$\sin^2\theta = 1 - \cos^2\theta$ (or) $\cos^2\theta = 1 - \sin^2\theta$
$1 + \tan^2\theta = \sec^2\theta$	$\tan^2\theta = \sec^2\theta - 1$ (or) $\sec^2\theta - \tan^2\theta = 1$
$1 + \cot^2\theta = \operatorname{cosec}^2\theta$	$\cot^2\theta = \operatorname{cosec}^2\theta - 1$ (or) $\operatorname{cosec}^2\theta - \cot^2\theta = 1$

ALGEBRAIC IDENTITIES

$(a+b)^2 = a^2 + b^2 + 2ab$

$(a-b)^2 = a^2 + b^2 - 2ab$

$a^2 - b^2 = (a+b)(a-b)$

$(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

$\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

EXERCISE 6.1

1) Prove the following Identities:

(i) $\cot\theta + \tan\theta = \sec\theta \operatorname{cosec}\theta$

L.H.S

$$\cot\theta + \tan\theta$$

$$= \frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta}$$

$$= \frac{\cos^2\theta + \sin^2\theta}{\sin\theta\cos\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta} \qquad [\because \sin^2\theta + \cos^2\theta = 1]$$

$$= \frac{1}{\cos\theta\sin\theta}$$

$$= \underset{\downarrow}{\frac{1}{\cos\theta}} \times \underset{\downarrow}{\frac{1}{\sin\theta}}$$

$$= \sec\theta \operatorname{Cosec}\theta$$

$$= \boxed{R.H.S}$$

(ii) $\tan^4\theta + \tan^2\theta = \sec^4\theta - \sec^2\theta$

L.H.S

$$\tan^4\theta + \tan^2\theta$$

$$= \tan^2\theta\,(\tan^2\theta + 1)$$

$$= \tan^2\theta\,(\sec^2\theta)$$

$$= (\sec^2\theta - 1)\sec^2\theta \qquad [\because \tan^2\theta = \sec^2\theta - 1]$$

$$= \sec^4\theta - \sec^2\theta$$

$$= \boxed{R.H.S}$$

---

2) Prove the following identities:

(i) $\dfrac{1-\tan^2\theta}{\cot^2\theta - 1} = \tan^2\theta$

$\boxed{L.H.S}$

$$\frac{1-\tan^2\theta}{\cot^2\theta - 1}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$= \frac{1 - \dfrac{\sin^2\theta}{\cos^2\theta}}{\dfrac{\cos^2\theta}{\sin^2\theta} - 1}$$

$$= \frac{\dfrac{\cos^2\theta - \sin^2\theta}{\cos^2\theta}}{\dfrac{\cos^2\theta - \sin^2\theta}{\sin^2\theta}}$$

$$= \frac{\cancel{\cos^2\theta - \sin^2\theta}}{\cos^2\theta} \times \frac{\sin^2\theta}{\cancel{\cos^2\theta - \sin^2\theta}}$$

$$= \frac{\sin^2\theta}{\cos^2\theta}$$

$$= \tan^2\theta$$

$$= \boxed{R.H.S}$$

(ii) $\dfrac{\cos\theta}{1+\sin\theta} = \sec\theta - \tan\theta$

L.H.S

$$\frac{\cos\theta}{1+\sin\theta} \times \frac{1-\sin\theta}{1-\sin\theta}$$

[Multiply by the conjugate]

$$= \frac{\cos\theta\,[1-\sin\theta]}{(1)^2-(\sin\theta)^2}$$

$$= \frac{\cos\theta\,(1-\sin\theta)}{1-\sin^2\theta}$$

$$= \frac{\cancel{\cos\theta}\,(1-\sin\theta)}{\cos^{\cancel{2}}\theta}$$

$$= \frac{1-\sin\theta}{\cos\theta}$$

$$= \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}$$

$$= \sec\theta - \tan\theta$$

= R.H.S

- $(a-b)(a+b) = a^2-b^2$
- $1-\sin^2\theta = \cos^2\theta$
- $\dfrac{1}{\cos\theta} = \sec\theta$
- $\dfrac{\sin\theta}{\cos\theta} = \tan\theta$

---

3) Prove the following identities:

(i) $\sqrt{\dfrac{1+\sin\theta}{1-\sin\theta}} = \sec\theta + \tan\theta$

L.H.S

$$\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} = \sqrt{\frac{(1+\sin\theta)}{(1-\sin\theta)} \times \frac{(1+\sin\theta)}{(1+\sin\theta)}}$$

[multiply by the conjugate]

$$= \sqrt{\frac{(1+\sin\theta)^2}{(1)^2-(\sin\theta)^2}}$$

$$(a-b)(a+b) = a^2-b^2$$

$$1-\sin^2\theta = \cos^2\theta$$

$$\frac{1}{\cos\theta} = \sec\theta$$

$$\frac{\sin\theta}{\cos\theta} = \tan\theta$$

$$= \sqrt{\frac{(1+\sin\theta)^2}{1^2-\sin^2\theta}}$$

$$= \cancel{\sqrt{\vphantom{1}}}\frac{(1+\sin\theta)^{\cancel{2}}}{\cos^{\cancel{2}}\theta}$$

$$= \frac{1+\sin\theta}{\cos\theta}$$

$$= \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}$$

$$= \sec\theta + \tan\theta$$

$$= \boxed{R\cdot H\cdot S}$$

(ii) $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} + \sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = 2\sec\theta$

$\boxed{L\cdot H\cdot S}$

$$\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} + \sqrt{\frac{1-\sin\theta}{1+\sin\theta}}$$

$$= \sqrt{\frac{(1+\sin\theta)}{(1-\sin\theta)} \times \frac{(1+\sin\theta)}{(1+\sin\theta)}} + \sqrt{\frac{(1-\sin\theta)}{(1+\sin\theta)} \times \frac{(1-\sin\theta)}{(1-\sin\theta)}}$$

$$= \sqrt{\frac{(1+\sin\theta)^2}{1^2-\sin^2\theta}} + \sqrt{\frac{(1-\sin\theta)^2}{1^2-\sin^2\theta}}$$

$$= \cancel{\sqrt{\vphantom{1}}}\frac{(1+\sin\theta)^{\cancel{2}}}{\cos^{\cancel{2}}\theta} + \cancel{\sqrt{\vphantom{1}}}\frac{(1-\sin\theta)^{\cancel{2}}}{\cos^{\cancel{2}}\theta}$$

$$= \frac{1+\sin\theta}{\cos\theta} + \frac{1-\sin\theta}{\cos\theta}$$

$$= \frac{1+\cancel{\sin\theta}+1-\cancel{\sin\theta}}{\cos\theta}$$

$$= \frac{2}{\cos\theta}$$

$$= 2 \times \frac{1}{\cos\theta}$$

$$= 2\sec\theta$$

= R.H.S

---

4) Prove the following identities:

(i) $\sec^6\theta = \tan^6\theta + 3\tan^2\theta\sec^2\theta + 1$

L.H.S

$\sec^6\theta$

$$(a+b)^3 = a^3+b^3+3ab(a+b)$$

$$= (\sec^2\theta)^3$$

$$\downarrow$$

$$= (\underset{ⓐ}{1}+\underset{ⓑ}{\tan^2\theta})^3$$

$$= (1)^3 + (\tan^2\theta)^3 + 3(1)(\tan^2\theta)(1+\tan^2\theta)$$

$$= 1 + \tan^6\theta + 3\tan^2\theta(\sec^2\theta)$$

$$= \tan^6\theta + 3\tan^2\theta\sec^2\theta + 1$$

= R.H.S

(ii) $(\sin\theta+\sec\theta)^2 + (\cos\theta+\operatorname{cosec}\theta)^2 = 1+(\sec\theta+\operatorname{cosec}\theta)^2$

L.H.S

$$(\underset{ⓐ}{\sin\theta} + \underset{ⓑ}{\sec\theta})^2 + (\underset{ⓐ}{\cos\theta} + \underset{ⓑ}{\mathrm{cosec}\,\theta})^2$$

$$= \sin^2\theta + \sec^2\theta + 2\sin\theta\sec\theta + \cos^2\theta + \mathrm{cosec}^2\theta + 2\cos\theta\,\mathrm{cosec}\,\theta$$

$$= \sin^2\theta + \cos^2\theta + \sec^2\theta + 2\sin\theta\left(\frac{1}{\cos\theta}\right) + \mathrm{cosec}^2\theta + 2\cos\theta\left(\frac{1}{\sin\theta}\right)$$

$$= 1 + \left[2\frac{\sin\theta}{\cos\theta} + 2\frac{\cos\theta}{\sin\theta}\right] + \sec^2\theta + \mathrm{cosec}^2\theta$$

$$= 1 + \left[\frac{2\sin^2\theta + 2\cos^2\theta}{\cos\theta\sin\theta}\right] + \sec^2\theta + \mathrm{cosec}^2\theta$$

$$= 1 + \frac{2(\sin^2\theta + \cos^2\theta)}{\cos\theta\sin\theta} + \sec^2\theta + \mathrm{cosec}^2\theta$$

$$= 1 + 2\left[\frac{1}{\cos\theta} \times \frac{1}{\sin\theta}\right] + \sec^2\theta + \mathrm{cosec}^2\theta$$

$$= 1 + \left[\sec^2\theta + \mathrm{cosec}^2\theta + 2(\sec\theta)(\mathrm{cosec}\,\theta)\right]$$

$$= 1 + (\sec\theta + \mathrm{cosec}\,\theta)^2$$

$= \boxed{\text{R.H.S}}$

---

5) Prove the following identities.

(i) $\sec^4\theta\,(1 - \sin^4\theta) - 2\tan^2\theta = 1$

$\boxed{\text{L.H.S}}$

$$\sec^4\theta\,(1 - \sin^4\theta) - 2\tan^2\theta$$

$$= \frac{1}{\cos^4\theta}\left[1 - (\sin^2\theta)^2\right] - 2\left[\frac{\sin^2\theta}{\cos^2\theta}\right]$$

$$= \frac{1}{\cos^4\theta}\left[(1+\sin^2\theta)\underset{\downarrow}{(1-\sin^2\theta)}\right] - 2\left[\frac{\sin^2\theta}{\cos^2\theta}\right]$$

$$= \frac{1}{\cos^{\cancel{4}\,2}\theta}(1+\sin^2\theta)(\cancel{\cos^2\theta}) - 2\frac{\sin^2\theta}{\cos^2\theta}$$

$$= \frac{1+\sin^2\theta}{\cos^2\theta} - \frac{2\sin^2\theta}{\cos^2\theta}$$

$$= \frac{1+\sin^2\theta - 2\sin^2\theta}{\cos^2\theta}$$

$$= \frac{1-\sin^2\theta}{\cos^2\theta}$$

$$= \frac{\cancel{\cos^2\theta}}{\cancel{\cos^2\theta}}$$

$$= 1$$

= R.H.S

(ii) $\dfrac{\cot\theta - \cos\theta}{\cot\theta + \cos\theta} = \dfrac{\operatorname{cosec}\theta - 1}{\operatorname{cosec}\theta + 1}$

L.H.S

$$\frac{\cot\theta - \cos\theta}{\cot\theta + \cos\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$= \frac{\frac{\cos\theta}{\sin\theta} - \cos\theta}{\frac{\cos\theta}{\sin\theta} + \cos\theta}$$

$$= \frac{\cancel{\cos\theta}\left[\frac{1}{\sin\theta} - 1\right]}{\cancel{\cos\theta}\left[\frac{1}{\sin\theta} + 1\right]} = \frac{\operatorname{cosec}\theta - 1}{\operatorname{cosec}\theta + 1}$$ = R.H.S

6) Prove the following identities:

(i) $$\frac{\sin A - \sin B}{\cos A + \cos B} + \frac{\cos A - \cos B}{\sin A + \sin B} = 0$$

L.H.S

$$\frac{\sin A - \sin B}{\cos A + \cos B} + \frac{\cos A - \cos B}{\sin A + \sin B}$$

$$= \frac{(\sin A - \sin B)(\sin A + \sin B) + (\cos A - \cos B)(\cos A + \cos B)}{(\cos A + \cos B)(\sin A + \sin B)}$$

$$= \frac{(\sin^2 A) - (\sin^2 B) + (\cos^2 A) - (\cos^2 B)}{(\cos A + \cos B)(\sin A + \sin B)}$$

$$= \frac{(\sin^2 A + \cos^2 A) - (\sin^2 B + \cos^2 B)}{(\cos A + \cos B)(\sin A + \sin B)}$$

$$= \frac{1 - 1}{(\cos A + \cos B)(\sin A + \sin B)}$$

$$= \frac{0}{(\cos A + \cos B)(\sin A + \sin B)}$$

$= 0$

$=$ R.H.S

(ii) $$\frac{\sin^3 A + \cos^3 A}{\sin A + \cos A} + \frac{\sin^3 A - \cos^3 A}{\sin A - \cos A} = 2$$

L.H.S

$$\frac{\sin^3 A + \cos^3 A}{\sin A + \cos A} + \frac{\sin^3 A - \cos^3 A}{\sin A - \cos A}$$

$$\boxed{\begin{aligned} a^3+b^3 &= (a+b)(a^2-ab+b^2) \\ a^3-b^3 &= (a-b)(a^2+ab+b^2) \end{aligned}}$$

$$= \frac{(\cancel{\sin A+\cos A})(\overleftrightarrow{\sin^2 A+\cos^2 A}-\sin A\cos A)}{(\cancel{\sin A+\cos A})} + \frac{(\cancel{\sin A-\cos A})(\overleftrightarrow{\sin^2 A+\cos^2 A}+\sin A\cos A)}{(\cancel{\sin A-\cos A})}$$

$$= 1-\cancel{\sin A\cos A}+1+\cancel{\sin A\cos A}$$

$$= 1+1$$

$$= 2 = \boxed{R.H.S}$$

---

7) (i) If $\sin\theta+\cos\theta=\sqrt{3}$, then prove that $\tan\theta+\cot\theta=1$

Given: $\sin\theta+\cos\theta=\sqrt{3}$

[squaring on both sides,]

$$(\sin\theta+\cos\theta)^2=(\cancel{\sqrt{3}})^{\cancel{2}}$$

ⓐ . ⓑ

$$\underleftrightarrow{\sin^2\theta+\cos^2\theta}+2\sin\theta\cos\theta=3$$

$$1+2\sin\theta\cos\theta=3$$

$$2\sin\theta\cos\theta=3-1$$

$$\cancel{2}\sin\theta\cos\theta=\cancel{2}^{1}$$

$$\boxed{\sin\theta\cos\theta=1} \rightarrow ①$$

To prove:

$$\tan\theta + \cot\theta = 1$$

L.H.S

$$\tan\theta + \cot\theta$$

$$= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}$$

$$= \frac{1}{\sin\theta\cos\theta}$$

$$= \frac{1}{1} \text{ (by ①)}$$

$$= 1 = \text{R.H.S}$$

(ii) If $\sqrt{3}\sin\theta - \cos\theta = 0$, then show that

$$\tan 3\theta = \frac{3\tan\theta - \tan^2\theta}{1 - 3\tan^2\theta}$$

Given

$$\sqrt{3}\sin\theta - \cos\theta = 0$$

$$\sqrt{3}\sin\theta = \cos\theta$$

$$\frac{\sin\theta}{\cos\theta} = \frac{1}{\sqrt{3}}$$

$$\tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$\tan\theta = \frac{1}{\sqrt{3}}$$

$$\tan\theta = \tan 30^\circ$$

$$\Rightarrow \boxed{\theta = 30^\circ}$$

show that

$$\tan 3\theta = \frac{3\tan\theta - \tan^2\theta}{1-3\tan^2\theta}$$

L.H.S

$$\tan 3\theta$$

$$= \tan 3(30^\circ) = \tan 90^\circ = \boxed{n.d} \checkmark$$

R.H.S

$$\frac{3\tan\theta - \tan^2\theta}{1-3\tan^2\theta}$$

$$= \frac{3\tan 30^\circ - \tan^2 30^\circ}{1-3\tan^2 30^\circ}$$

$$= \frac{3\left(\frac{1}{\sqrt{3}}\right) - \left(\frac{1}{\sqrt{3}}\right)^2}{1-3\left(\frac{1}{\sqrt{3}}\right)^2}$$

$$= \frac{\frac{3}{\sqrt{3}} - \frac{1}{3}}{1-\cancel{3}\left(\frac{1}{\cancel{3}}\right)}$$

$$= \frac{\frac{3}{\sqrt{3}} - \frac{1}{3}}{1-1}$$

$$= \frac{\frac{3}{\sqrt{3}} - \frac{1}{3}}{0}$$

$$= \boxed{n.d} \checkmark$$

$$\Rightarrow \boxed{L.H.S = R.H.S}$$

Hence Proved.

8) i) If $\frac{\cos\alpha}{\cos\beta} = m$ and $\frac{\cos\alpha}{\sin\beta} = n$, then prove that $(m^2+n^2)\cos^2\beta = n^2$.

Given: $\boxed{\frac{\cos\alpha}{\cos\beta} = m}$ $\boxed{\frac{\cos\alpha}{\sin\beta} = n}$

To prove $(m^2+n^2)\cos^2\beta = n^2$

L.H.S $(m^2+n^2)\cos^2\beta$

$$= \left[\frac{\cos^2\alpha}{\cos^2\beta} + \frac{\cos^2\alpha}{\sin^2\beta}\right]\cos^2\beta$$

$$= \left[\frac{\cos^2\alpha\sin^2\beta + \cos^2\alpha\cos^2\beta}{\cancel{\cos^2\beta}\sin^2\beta}\right]\cancel{\cos^2\beta}$$

$$= \frac{\cos^2\alpha\left[\sin^2\beta + \cos^2\beta\right]}{\sin^2\beta}$$

$$= \frac{\cos^2\alpha\,(1)}{\sin^2\beta}$$

$$= \frac{\cos^2\alpha}{\sin^2\beta}$$

$= n^2$

= R.H.S

(ii) If $\cot\theta + \tan\theta = x$ and $\sec\theta - \cos\theta = y$ then prove that $(x^2y)^{2/3} - (xy^2)^{2/3} = 1$

Given

$\Rightarrow$ $\cot\theta + \tan\theta = x$

$$\frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta} = x$$

$$\frac{\cos^2\theta + \sin^2\theta}{\sin\theta\cos\theta} = x$$

$$\boxed{\frac{1}{\sin\theta\cos\theta} = x}$$

$\Rightarrow$ $\sec\theta - \cos\theta = y$

$$\frac{1}{\cos\theta} - \cos\theta = y$$

$$\frac{1-\cos^2\theta}{\cos\theta} = y$$

$$\boxed{\frac{\sin^2\theta}{\cos\theta} = y}$$

Prove that $(x^2y)^{2/3} - (xy^2)^{2/3} = 1$

L.H.S

$$(x^2y)^{2/3} - (xy^2)^{2/3}$$

$$= \left[\frac{1}{\cancel{\sin^2\theta}\cos^2\theta} \times \frac{\cancel{\sin^2\theta}}{\cos\theta}\right]^{2/3} - \left[\frac{1}{\cancel{\sin\theta}\cos\theta} \times \frac{\sin^{\overset{3}{\cancel{4}}}\theta}{\cos^2\theta}\right]^{2/3}$$

$$= \left[\frac{1}{\cos^3\theta}\right]^{2/3} - \left[\frac{\sin^3\theta}{\cos^3\theta}\right]^{2/3}$$

$$= \frac{1}{\cos^{\cancel{3}\times\frac{2}{\cancel{3}}}\theta} - \frac{\sin^{\cancel{3}\times\frac{2}{\cancel{3}}}\theta}{\cos^{\cancel{3}\times\frac{2}{\cancel{3}}}\theta}$$

$$= \frac{1}{\cos^2\theta} - \frac{\sin^2\theta}{\cos^2\theta}$$

$$= \frac{1-\sin^2\theta}{\cos^2\theta}$$

$$= \frac{\cancel{\cos^2\theta}}{\cancel{\cos^2\theta}}$$

$$= 1 = \text{R.H.S}$$

---

9) (i) If $\sin\theta + \cos\theta = p$ and $\sec\theta + \operatorname{cosec}\theta = q$ then prove that $q(p^2-1) = 2p$

Given: $\sin\theta + \cos\theta = p$

$$\sec\theta + \operatorname{cosec}\theta = q$$

$$\frac{1}{\cos\theta} + \frac{1}{\sin\theta} = q$$

$$\boxed{\frac{\sin\theta + \cos\theta}{\cos\theta\sin\theta} = q}$$

To prove:

$$q(p^2-1) = 2p$$

L.H.S $\quad q(p^2-1)$

$$\left[\frac{\sin\theta + \cos\theta}{\cos\theta\sin\theta}\right]\left[(\underset{ⓐ}{\sin\theta} + \underset{ⓑ}{\cos\theta})^2 - 1\right]$$

$$= \left[\frac{\sin\theta+\cos\theta}{\cos\theta\,\sin\theta}\right]\left[\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta-1\right]$$

$$= \left[\frac{\sin\theta+\cos\theta}{\cos\theta\,\sin\theta}\right]\left(\cancel{1}+2\sin\theta\cos\theta-\cancel{1}\right)$$

$$= \left[\frac{\sin\theta+\cos\theta}{\cancel{\cos\theta}\,\cancel{\sin\theta}}\right]\left(2\,\cancel{\sin\theta}\,\cancel{\cos\theta}\right)$$

$$= 2(\sin\theta+\cos\theta)$$

$$= 2p$$

$$= \boxed{R.H.S}$$

(ii) If $\sin\theta(1+\sin^2\theta)=\cos^2\theta$, then prove that $\cos^6\theta-4\cos^4\theta+8\cos^2\theta=4$

Given

$$\boxed{\sin\theta\,(1+\sin^2\theta)=\cos^2\theta}$$

$$\downarrow$$

$$\sin\theta\,(1+\underline{1-\cos^2\theta})=\cos^2\theta$$

$$\boxed{\sin\theta\,(2-\cos^2\theta)=\cos^2\theta.}$$

Squaring on both sides,

$$\sin^2\theta\,(2-\cos^2\theta)^2=\cos^4\theta$$

$$\sin^2\theta\left[(2)^2+(\cos^2\theta)^2-2(2)(\cos^2\theta)\right]=\cos^4\theta$$

$$\downarrow$$

$$(1-\cos^2\theta)\left[4+\cos^4\theta-4\cos^2\theta\right]=\cos^4\theta$$

$$4+\cancel{\cos^4\theta}-4\cos^2\theta-4\cos^2\theta-\cos^6\theta+4\cos^4\theta=\cancel{\cos^4\theta}$$

✓ ✓

$$4 - 8\cos^2\theta - \cos^6\theta + 4\cos^4\theta = 0$$

$$4 = 8\cos^2\theta + \cos^6\theta - 4\cos^4\theta .$$

$$\Rightarrow \boxed{\cos^6\theta - 4\cos^4\theta + 8\cos^2\theta = 4}$$

Hence Proved.

10) If $\frac{\cos\theta}{1+\sin\theta} = \frac{1}{a}$, then prove that

$\frac{a^2-1}{a^2+1} = \sin\theta .$

Given $$\frac{\cos\theta}{1+\sin\theta} = \frac{1}{a}$$

squaring on both sides

$$\frac{\cos^2\theta}{(1+\sin\theta)^2} = \frac{1}{a^2}$$

$$\frac{1-\sin^2\theta}{(1+\sin\theta)^2} = \frac{1}{a^2}$$

$$\frac{1^2-\sin^2\theta}{(1+\sin\theta)(1+\sin\theta)} = \frac{1}{a^2}$$

$$\frac{\cancel{(1+\sin\theta)}(1-\sin\theta)}{\cancel{(1+\sin\theta)}(1+\sin\theta)} = \frac{1}{a^2}$$

$$\frac{1-\sin\theta}{1+\sin\theta} = \frac{1}{a^2}$$

$$\boxed{\frac{1+\sin\theta}{1-\sin\theta} = a^2} \text{ [Reciprocal]}$$

$\boxed{\text{To Prove}}$ $\frac{a^2-1}{a^2+1} = \sin\theta$

L.H.S ⇒ $\frac{a^2-1}{a^2+1}$

$$= \frac{\left(\frac{1+\sin\theta}{1-\sin\theta}\right)-1}{\left(\frac{1+\sin\theta}{1-\sin\theta}\right)+1}$$

$$= \frac{\frac{1+\sin\theta-1+\sin\theta}{1-\sin\theta}}{\frac{1+\sin\theta+1-\sin\theta}{1-\sin\theta}}$$

$$= \frac{(\cancel{1}+\sin\theta-\cancel{1}+\sin\theta)}{\cancel{(1-\sin\theta)}} \times \frac{\cancel{(1-\sin\theta)}}{(1+\cancel{\sin\theta}+1-\cancel{\sin\theta})}$$

$$= \frac{\cancel{2}\sin\theta}{\cancel{2}}$$

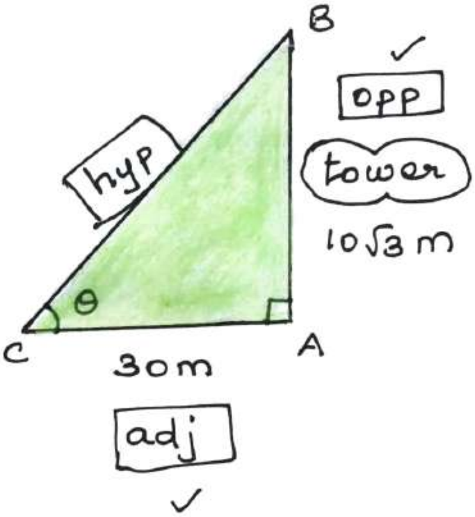
$$= \sin\theta$$

$$= \boxed{\text{R.H.S}}$$

EXERCISE 6.2

1) Find the angle of elevation of the top of a tower from a point on the ground, which is 30m away from the foot of a tower of height $10\sqrt{3}$ m.

$$\tan\theta = \frac{\text{opp}}{\text{adj}}$$



$$\tan\theta = \frac{1\cancel{0}\sqrt{3}}{3\cancel{0}}$$

$$\tan\theta = \frac{\cancel{\sqrt{3}}}{\cancel{\sqrt{3}}\times\sqrt{3}}$$

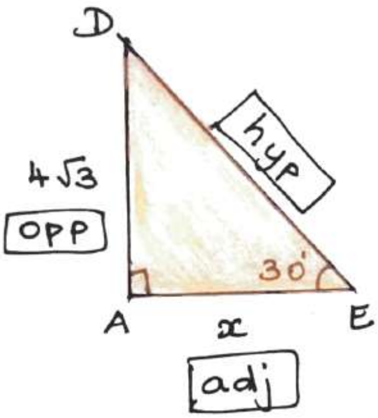
$$\tan\theta = \frac{1}{\sqrt{3}}$$

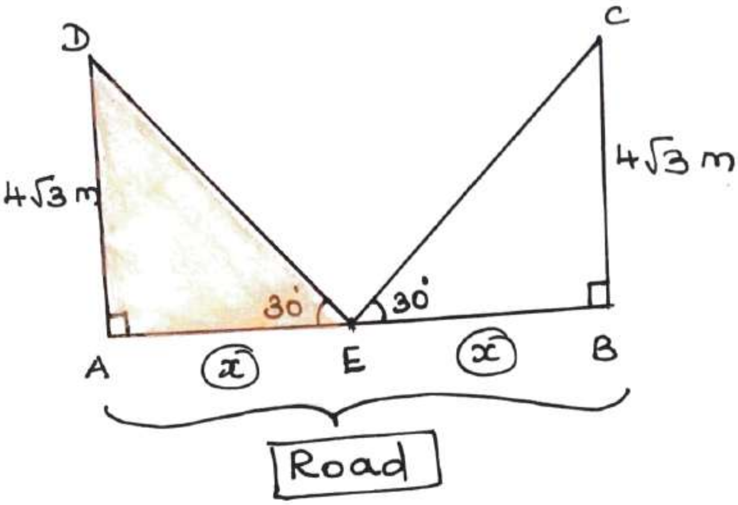
$$\tan\theta = \tan 30^\circ$$

$$\Rightarrow \boxed{\theta = 30^\circ}$$

---

2) A road is flanked on either side by continuous rows of houses of height $4\sqrt{3}$ m with no space in between them. A pedestrian is standing on the median of the road facing a row house. The angle of elevation from the pedestrian to the top of the house is 30°. Find the width of the road.





$$\tan\theta = \frac{opp}{adj}$$

$$\tan 30^\circ = \frac{4\sqrt{3}}{x}$$

$$\frac{1}{\sqrt{3}} = \frac{4\sqrt{3}}{x}$$

$$x = 4\sqrt{3} \times \sqrt{3}$$

$$x = 4 \times 3$$

$$\boxed{x = 12\,m}$$

$\Rightarrow$ Width of the Road

$$= x + x$$

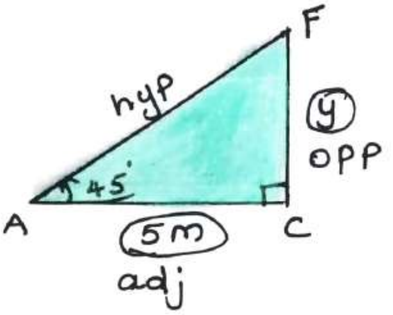
$$= 12 + 12$$

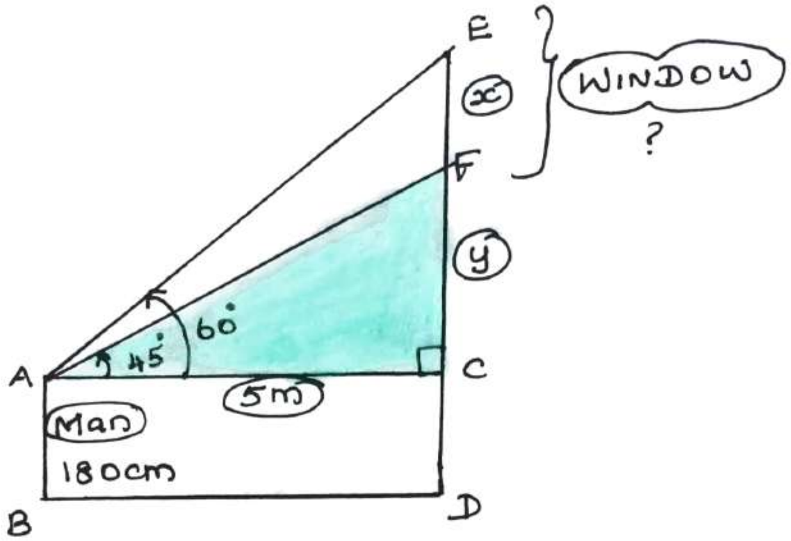
$$= 24\,m$$

---

3) To a man standing outside his house, the angles of elevations of the top and bottom of a window are 60° and 45° respectively. If the height of the

man is 180 cm and if he is 5m away from the wall, what is the height of the window? ($\sqrt{3} = 1.732$)





$$\tan\theta = \frac{opp}{adj}$$

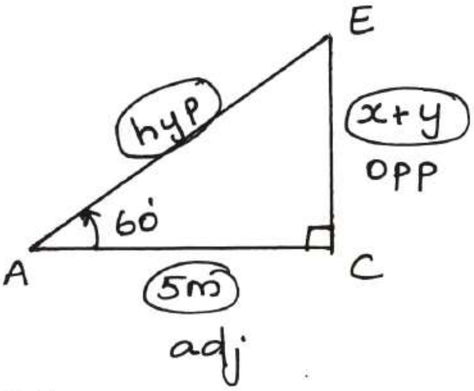
$$\tan 45^\circ = \frac{y}{5}$$

$$1 = \frac{y}{5}$$

$$5 = y$$

$$\boxed{y = 5} \rightarrow ①$$

$$\tan\theta = \frac{opp}{adj}$$



$$\tan 60^\circ = \frac{x+y}{5}$$

$$\sqrt{3} = \frac{x+y}{5}$$

$$5\sqrt{3} = x+y \quad (\text{by } ①)$$

$$5\sqrt{3} = x+5$$

$$5\sqrt{3} - 5 = x$$

$$5(\sqrt{3}-1) = x$$

$$5(1.732-1) = x$$

$$5(0.732) = x$$

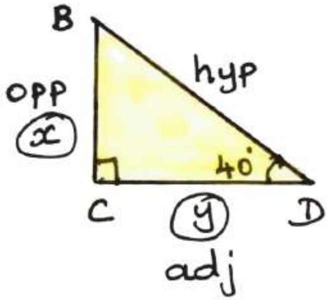
$$3.660 = x$$

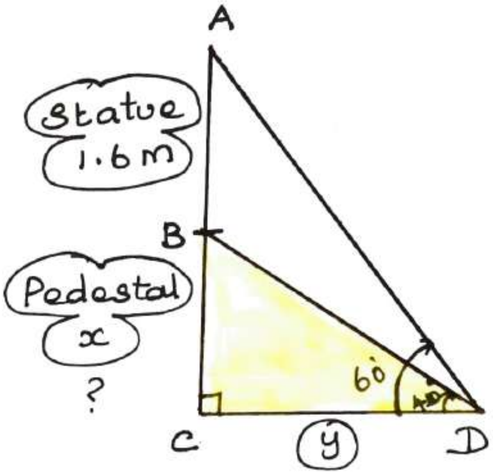
$$\boxed{x = 3.66\text{ m}}$$

$\Rightarrow$ Height of a window = 3.66m

---

4) A statue 1.6m tall stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60° and from the same point the angle of elevation of the top of the pedestal is 40°. Find the height of the pedestal. $(\tan 40° = 0.8391)$, $(\sqrt{3} = 1.732)$





$$\tan\theta = \frac{\text{opp}}{\text{adj}}$$

$$\tan 40° = \frac{x}{y}$$

$$0.8391 = \frac{x}{y}$$

$$\boxed{y = \frac{x}{0.8391}} \rightarrow ①$$

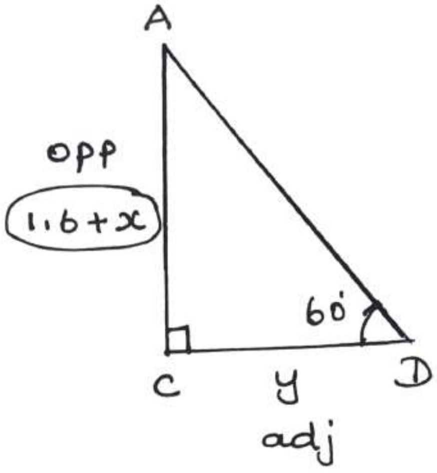
$$\tan\theta = \frac{opp}{adj}$$

$$\tan 60^\circ = \frac{1.6+x}{y}$$

$$\sqrt{3} = \frac{1.6+x}{y}$$

$$y = \frac{1.6+x}{\sqrt{3}}$$

$$\boxed{y = \frac{1.6+x}{1.732}} \rightarrow ②$$



$$\begin{array}{r} 8391 \times 16 \\ \hline 50346 \\ 8391 \\ \hline 134256 \end{array}$$

$$\begin{array}{r} 1.7320 \\ 0.8391 \\ \hline 0.8929 \end{array} \ (-)$$

from ① & ②

$$\frac{x}{0.8391} = \frac{1.6+x}{1.732}$$

$$1.732x = (1.6+x)(0.8391)$$

$$1.732x = 1.34256 + 0.8391x$$

$$1.732x - 0.8391x = 1.34256$$

$$0.8929x = 1.34256$$

$$x = \frac{1.34256}{0.8929}$$

$$x = \frac{1.34256}{0.8929} \times \frac{10000}{10000}$$

$$x = \frac{13425.6}{8929}$$

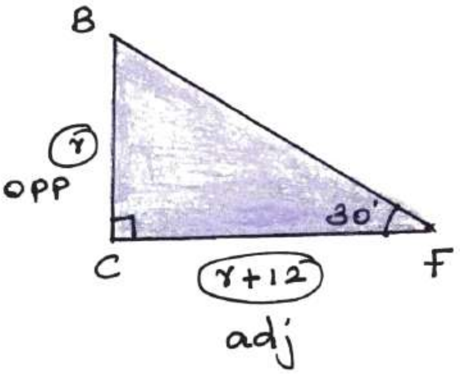
$$\begin{array}{r} 1.503 \\ 8929 \overline{)13425.6} \\ 8929 \\ \hline 44966 \\ 44645 \\ \hline 32100 \\ 26777 \\ \hline 5323 \end{array}$$

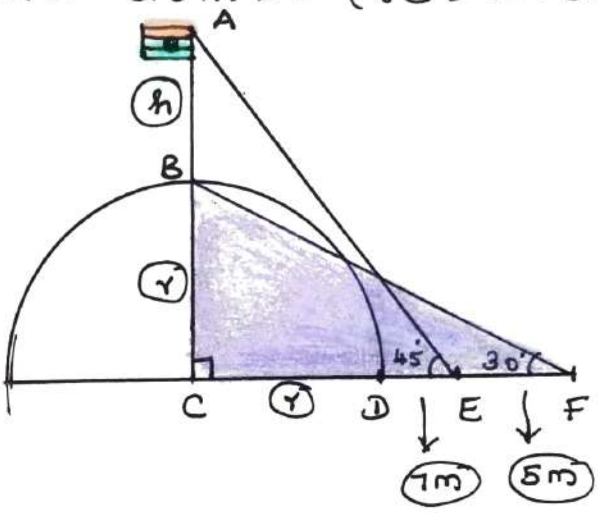
$$\boxed{x = 1.503\,m}$$

⇒ Height of the pedestal = 1.5 m

5) A flag pole 'h' metres is on the top of the hemispherical dome of radius 'r' metres. A man is standing 7m away from the dome. Seeing the top of the pole at an angle 45° and moving 5m away from the dome and seeing the bottom of the pole at an angle 30°. Find (i) the height of the pole.

(ii) radius of the dome. ($\sqrt{3} = 1.732$)





$$\tan\theta = \frac{opp}{adj}$$

$$\tan 30° = \frac{r}{r+12}$$

$$\frac{1}{\sqrt{3}} = \frac{r}{r+12}$$

$$r+12 = \sqrt{3}\,r$$

$$r+12 = 1.732r$$

$$12 = 1.732r - r$$

$$12 = 0.732r$$

$$0.732r = 12$$

$$r = \frac{12}{0.732}$$

```
       16.39
732 ) 12000
      732↓
      4680
      4392
       2880
       2196
        6840
        6588
         252
```

$$r = \frac{12}{0.732} \times \frac{1000}{1000}$$

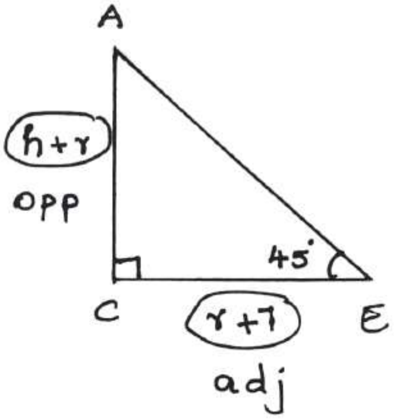
$$r = \frac{12000}{732}$$

$$\boxed{r = 16.39\ m}$$

⇒ Radius of the domb = 16.39m

$$\tan\theta = \frac{opp}{adj}$$

$$\tan 45^\circ = \frac{h+r}{r+7}$$



$$1 = \frac{h+r}{r+7}$$

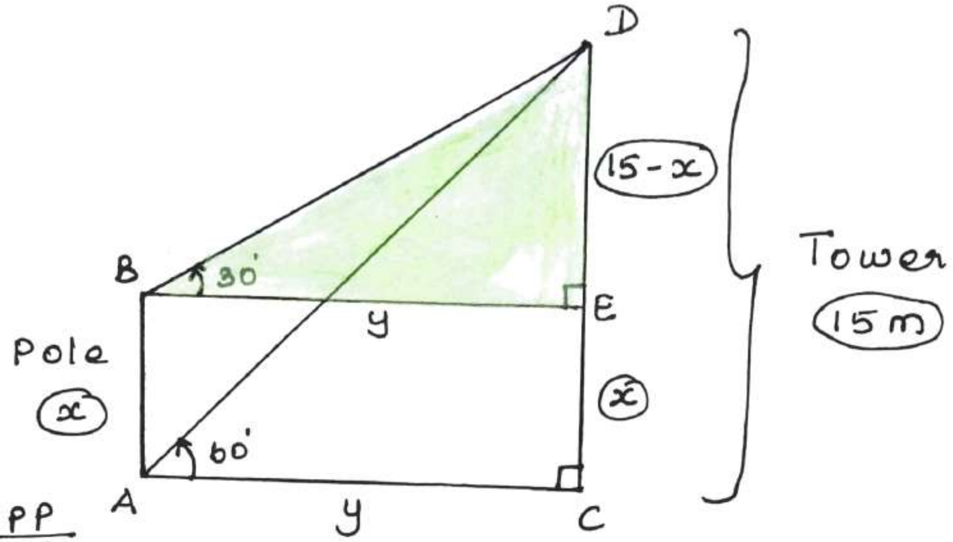
$$\cancel{r}+7 = h+\cancel{r}$$

$$7 = h$$

$$\boxed{h = 7\ m}$$

⇒ Height of the pole = 7m

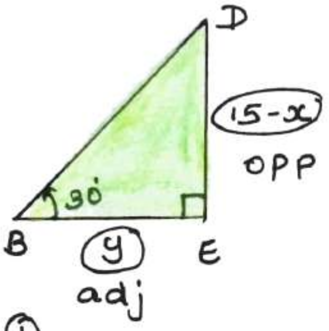
6) The top of a 15m high tower makes an angle of elevation of 60° with the bottom of an electronic pole and the angle of elevation of 30° with the top of the pole. What is the height of the electric pole?



$\tan\theta = \frac{opp}{adj}$

$\tan 30^\circ = \frac{15-x}{y}$

$\frac{1}{\sqrt{3}} = \frac{15-x}{y}$

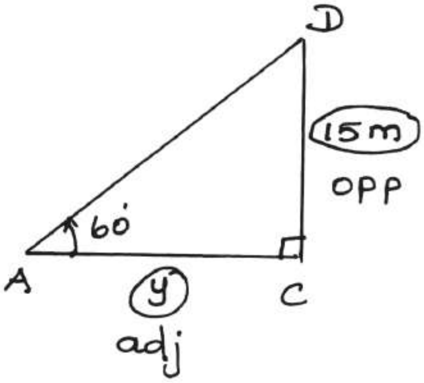


$\boxed{y = (15-x)\sqrt{3}} \rightarrow ①$

$\tan\theta = \frac{opp}{adj}$

$\tan 60^\circ = \frac{15}{y}$

$\sqrt{3} = \frac{15}{y}$



$\boxed{y = 15/\sqrt{3}} \rightarrow ②$

From ① & ②

$(15-x)\sqrt{3} = \frac{15}{\sqrt{3}}$

$(15-x)3 = 15$

$45 - 3x = 15$

$45 - 15 = 3x$

$30 = 3x$

$3x = 30$

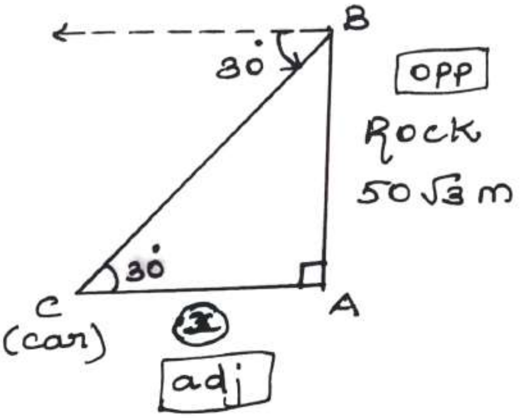
$x = \frac{\cancel{30}^{10}}{\cancel{3}}$

$\boxed{x = 10\text{ m}}$

Height of electric pole = 10 m.

EXERCISE 6.3

1) From the top of a rock $50\sqrt{3}$ m high, the angle of depression of a car on the ground is observed to be 30°. Find the distance of the car from the rock.



$$\tan\theta = \frac{\text{opp}}{\text{adj}}$$

$$\tan 30° = \frac{50\sqrt{3}}{x}$$

$$\frac{1}{\sqrt{3}} = \frac{50\sqrt{3}}{x}$$

$$x = 50\underbrace{\sqrt{3}\times 3}$$

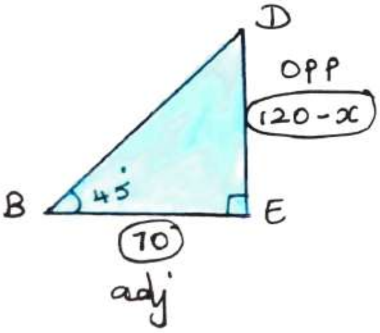
$$x = 50 \times 3$$

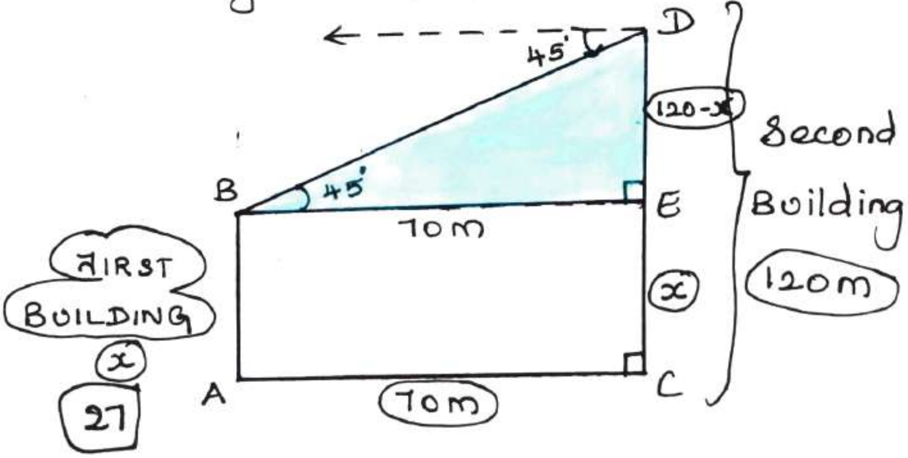
$$\boxed{x = 150\text{ m}}$$

⇒ Distance of car from the rock is 150m

---

2) The horizontal distance between two buildings is 70m. The angle of depression of the top of the first building when sees from the top of a second building is 45°. If the height of the second building is 120m, find the height of the first building.





$$\tan\theta = \frac{opp}{adj}$$

$$\tan 45^\circ = \frac{120-x}{70}$$

$$1 = \frac{120-x}{70}$$

$$70 = 120 - x$$

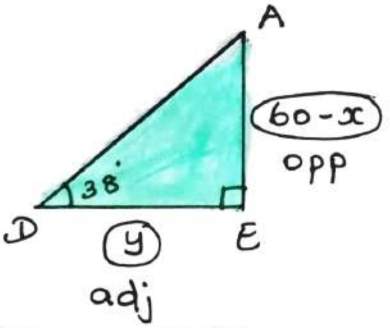
$$x = 120 - 70$$

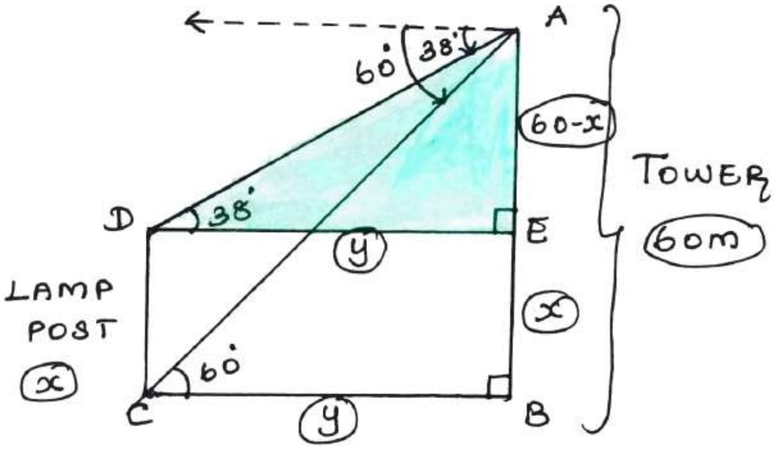
$$\boxed{x = 50m}$$

$\Rightarrow$ Height of first building = 50m

---

3) From the top of the tower 60m high the angles of depression of the top and bottom of a vertical lamp post are observed to be 38° and 60° respectively. Find the height of the lamp post. ($\tan 38^\circ = 0.7813$, $\sqrt{3} = 1.732$)





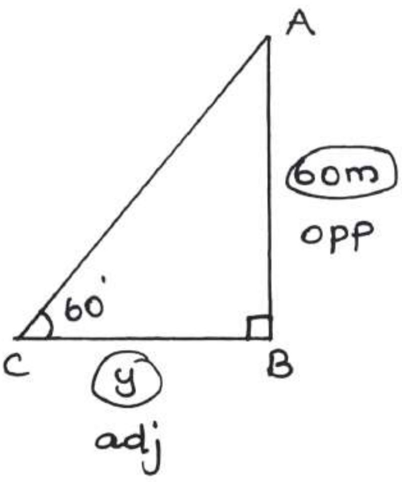
$$\tan\theta = \frac{opp}{adj}$$

$$\tan 38^\circ = \frac{60-x}{y}$$

$$0.7813 = \frac{60-x}{y}$$

$$\boxed{y = \frac{60-x}{0.7813}} \rightarrow ①$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}}$$



$$\tan 60^\circ = \frac{60}{y}$$

$$\sqrt{3} = \frac{60}{y}$$

$$\boxed{y = \frac{60}{\sqrt{3}}} \rightarrow ②$$

$$\begin{array}{r} \scriptstyle 9\,13\;\;8\,11\,10 \\ \cancel{1}\cancel{0}\cancel{3}.\cancel{9}\cancel{2}\cancel{0} \\ 46.878 \\ \hline 57.042 \\ \hline \end{array} \; (-)$$

From ① and ②

$$\frac{60-x}{0.7813} = \frac{60}{\sqrt{3}}$$

$$\sqrt{3}(60-x) = 60 \times 0.7813$$

$$60\sqrt{3} - \sqrt{3}x = 46.8780$$

$$60(1.732) - \sqrt{3}x = 46.8780$$

$$103.920 - \sqrt{3}x = 46.8780$$

$$103.920 - 46.878 = \sqrt{3}x$$

$$57.042 = \sqrt{3}x$$

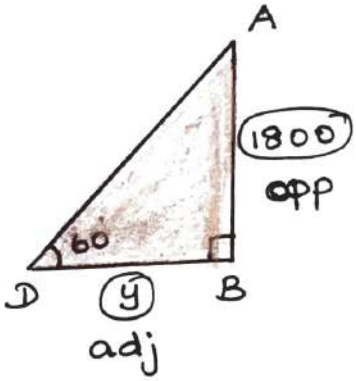
$$\sqrt{3}x = 57.042$$

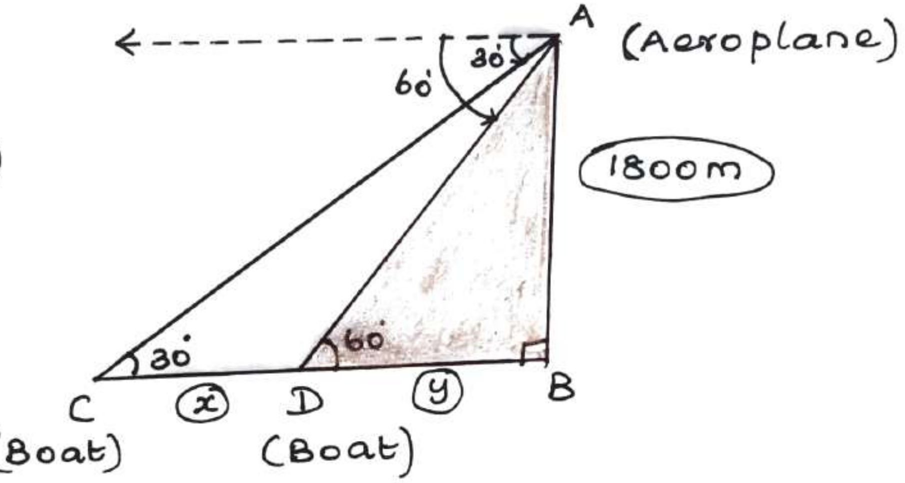
$$\begin{array}{r} 32.93 \\ 1732\overline{)57042} \\ 5196\downarrow \\ \hline 5082 \\ 3464 \\ \hline 16180 \\ 15588 \\ \hline 5920 \\ 5196 \\ \hline 724 \\ \hline \end{array}$$

$$x = \frac{57.042}{1.732} = 32.93\,\text{m}$$

⇒ Height of the Lamppost = 32.93 m

4) An aeroplane at an altitude of 1800 m finds that two boats are sailing towards it in the same direction. The angles of depression of the boats as observed from the aeroplane are 60° and 30° respectively. Find the distance between the two boats. ($\sqrt{3} = 1.732$)





$$\tan\theta = \frac{opp}{adj}$$

$$\tan 60^\circ = \frac{1800}{y}$$

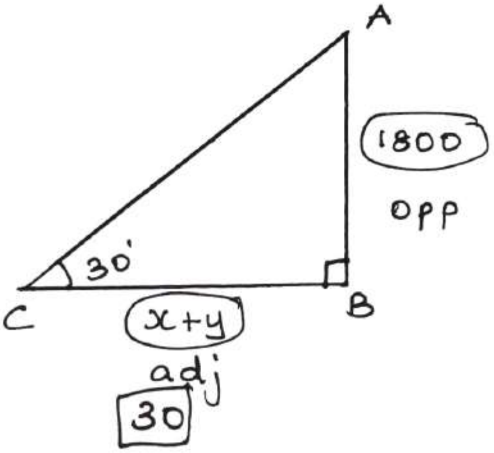
$$\sqrt{3} = \frac{1800}{y}$$

$$\boxed{y = \frac{1800}{\sqrt{3}}} \rightarrow ①$$

$$\tan\theta = \frac{opp}{adj}$$

$$\tan 30^\circ = \frac{1800}{x+y}$$

$$\frac{1}{\sqrt{3}} = \frac{1800}{x+y}$$



$$x + y = 1800\sqrt{3}$$

$$\boxed{y = 1800\sqrt{3} - x} \rightarrow ②$$

from ① and ②

$$\frac{1800}{\sqrt{3}} = 1800\sqrt{3} - x$$

$$x = 1800\sqrt{3} - \frac{1800}{\sqrt{3}}$$

$$x = \frac{(1800 \times 3) - 1800}{\sqrt{3}}$$

$$x = \frac{5400 - 1800}{1.732}$$

$$x = \frac{3600}{1.732}$$

$$x = \frac{3600}{1.732} \times \frac{1000}{1000}$$

$$x = \frac{3600000}{1732}$$

$$x = 2078.52\text{m}$$

$$\begin{array}{r}
2078.52 \\
1732\overline{)3600000} \\
3464 \\
\hline
13600 \\
12124 \\
\hline
14760 \\
13856 \\
\hline
9040 \\
8660 \\
\hline
3800 \\
3464 \\
\hline
336 \\
\hline
\end{array}$$

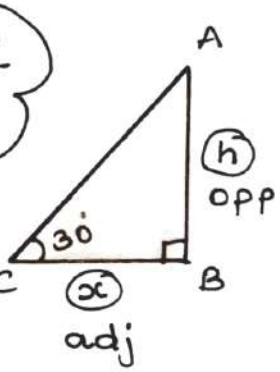
⇒ Distance between the two boats is 2078.52 m

---

5) From the top of a lighthouse, the angle of depression of two ships on the opposite sides of it are observed to be

30° and 60°. If the height of the lighthouse is 'h' metres and the line joining the ships passes through the foot of the lighthouse, show that the distance between the ships is $\frac{4h}{\sqrt{3}}$ m.

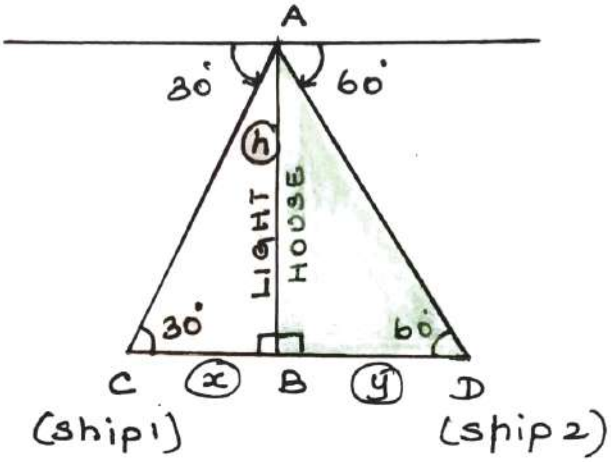
$\tan\theta = \frac{opp}{adj}$



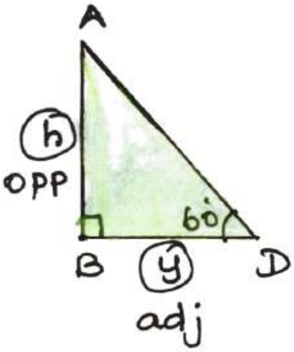
$\tan 30° = \frac{h}{x}$

$\frac{1}{\sqrt{3}} = \frac{h}{x}$

$\boxed{x = \sqrt{3}h} \rightarrow ①$



$\tan\theta = \frac{opp}{adj}$



$\tan 60° = \frac{h}{y}$

$\sqrt{3} = \frac{h}{y}$

$\boxed{y = \frac{h}{\sqrt{3}}}$

Show that $\frac{4h}{\sqrt{3}}$ m

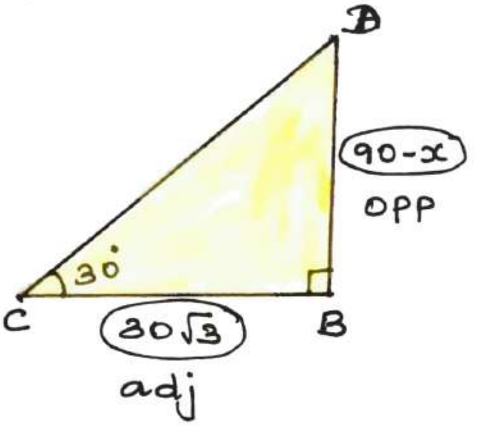
Distance between two ships

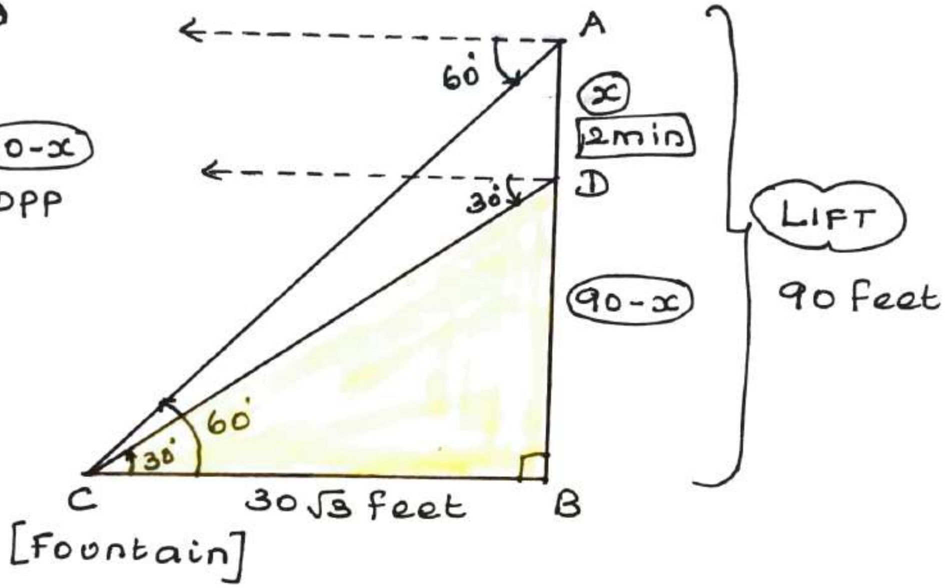
$= x + y$

$= \sqrt{3}h + \frac{h}{\sqrt{3}}$

$= \frac{3h + h}{\sqrt{3}} = \frac{4h}{\sqrt{3}}$ m Hence Proved.

6) A lift in a building of height 90 feet with transparent glass walls is descending from the top of the building. At the top of the building, the angle of depression to a fountain in the garden is 60°. Two minutes later, the angle of depression reduces to 30°. If the fountain is $30\sqrt{3}$ feet from the entrance of the lift, find the speed of the lift which is descending.





$\tan\theta = \frac{opp}{adj}$

$\tan 30° = \frac{90-x}{30\sqrt{3}}$

$\frac{1}{\cancel{\sqrt{3}}} = \frac{90-x}{30\cancel{\sqrt{3}}}$

$30 = 90 - x$

$x = 90 - 30$

$\boxed{x = 60 \text{ feet}}$

Time = 2min

Distance = Speed x Time

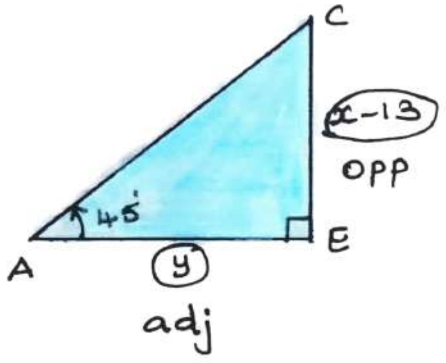
60 = Speed x 2

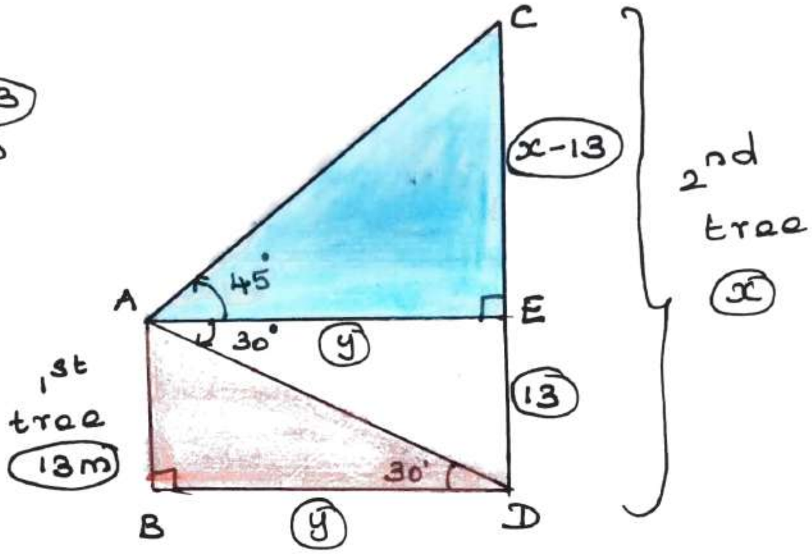
Speed = $\frac{\cancel{60}\ 30}{\cancel{2}}$

Speed = 30 feet/m

EXERCISE 6.4

1) From the top of a tree of height 13 m the angle of elevation and depression of the top and bottom of another tree are 45° and 30° respectively. Find the height of the second tree. ($\sqrt{3} = 1.732$)



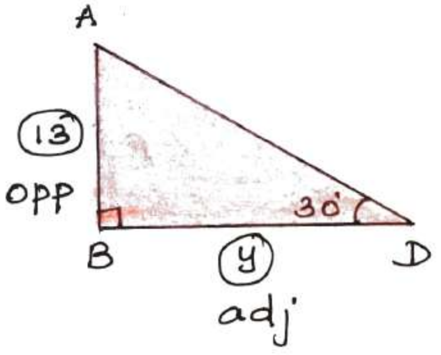


$$\tan\theta = \frac{opp}{adj}$$

$$\tan 45^\circ = \frac{x-13}{y}$$

$$1 = \frac{x-13}{y}$$

$$\boxed{y = x-13} \rightarrow ①$$



$$\tan\theta = \frac{opp}{adj}$$

$$\tan 30^\circ = \frac{13}{y}$$

$$\frac{1}{\sqrt{3}} = \frac{13}{y}$$

$$\boxed{y = 13\sqrt{3}} \rightarrow ②$$

From ① and ②

$x - 13 = 13\sqrt{3}$

$x = 13\sqrt{3} + 13$

$x = 13(\sqrt{3} + 1)$

$x = 13(1.732 + 1)$

$x = 13 \overset{\curvearrowright}{(2.732)}$

$x = 35.516$

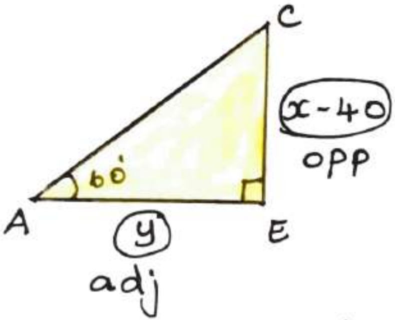
$\boxed{x = 35.52\text{m}}$

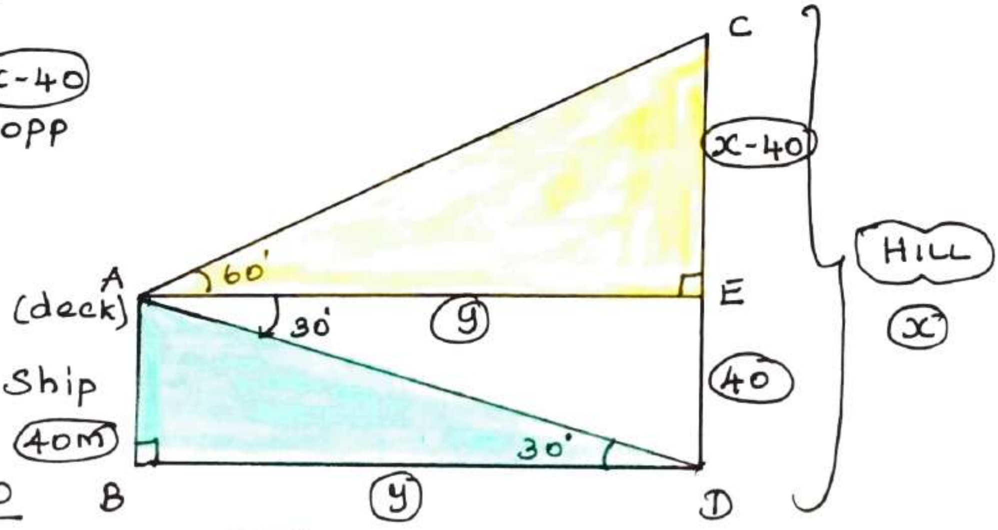
$$\boxed{\begin{array}{r} 2732 \times 13 \\ \hline 8196 \\ 2732 \\ \hline 35.516 \\ \hline \end{array}}$$

=> Height of second tree = 35.52 m

---

2) A man is standing on the deck of a ship, which is 40m above water level. He observes the angle of elevation of the top of a hill as 60° and the angle of depression of the base of the hill as 30°. Calculate the distance of the hill from the ship and the height of the hill. ($\sqrt{3} = 1.732$)





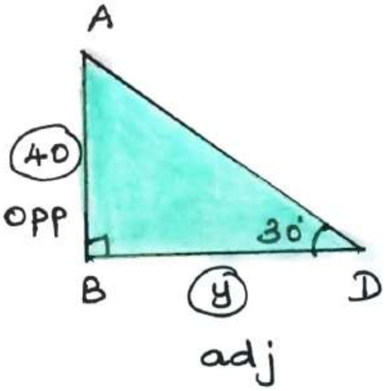
$\tan\theta = \dfrac{\text{opp}}{\text{adj}}$

$\tan 60° = \dfrac{x-40}{y}$

$$\sqrt{3} = \frac{x-40}{y}$$

$$\boxed{y = \frac{x-40}{\sqrt{3}}} \rightarrow ①$$

$$\tan\theta = \frac{opp}{adj}$$



$$\tan 30^\circ = \frac{40}{y}$$

$$\frac{1}{\sqrt{3}} = \frac{40}{y}$$

$$\boxed{y = 40\sqrt{3}} \rightarrow ②$$

From ① & ②

$$\frac{x-40}{\sqrt{3}} = 40\sqrt{3}$$

$$x-40 = 40\times 3$$

$$x-40 = 120$$

$$x = 120+40$$

$$\boxed{x = 160m}$$

Height of the hill = 160m

From ②

$$y = 40\sqrt{3}$$

$$y = 40\times 1.732$$

$$y = 69.280$$

$$\boxed{y = 69.28m}$$

⇒ Distance of the hill from the ship = 69.28m

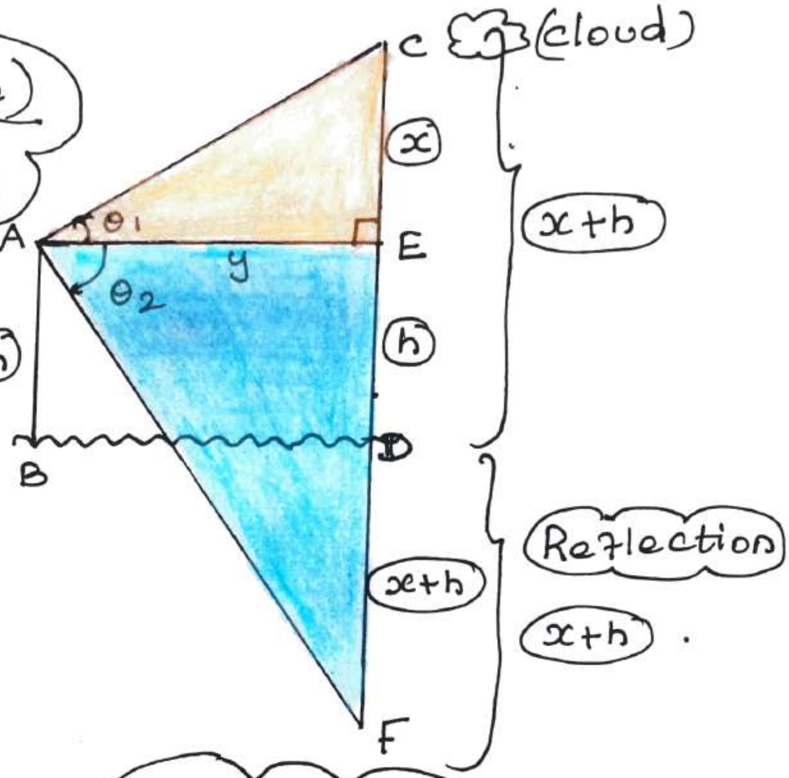
---

3) If the angle of elevation of a cloud from a point 'h' metres above a lake is

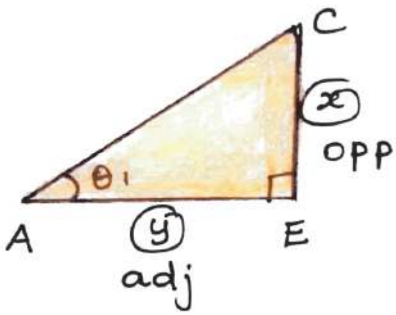
θ_1 and the angle of depression of its reflection in the lake is θ_2. Prove that the height that the cloud is located from the ground is $\frac{h(\tan\theta_1+\tan\theta_2)}{\tan\theta_2-\tan\theta_1}$

To prove

$$x+h = \frac{h(\tan\theta_1+\tan\theta_2)}{\tan\theta_2-\tan\theta_1}$$



$$EF = h+x+h$$
$$= x+2h$$

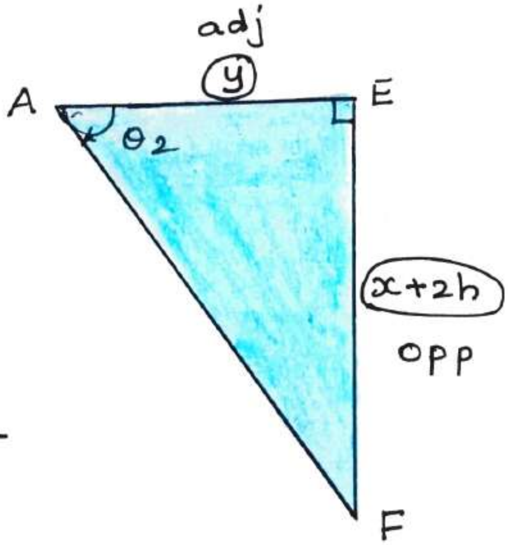


$$\tan\theta_1 = \frac{x}{y}$$

$$\boxed{y = \frac{x}{\tan\theta_1}} \rightarrow ①$$

$$\tan\theta_2 = \frac{x+2h}{y}$$

$$\boxed{y = \frac{x+2h}{\tan\theta_2}} \rightarrow ②$$



From ① and ②

$$\frac{x}{\tan\theta_1} = \frac{x+2h}{\tan\theta_2}$$

$$x \tan\theta_2 = x\tan\theta_1 + 2h\tan\theta_1$$

$$x\tan\theta_2 - x\tan\theta_1 = 2h\tan\theta_1$$

$$x(\tan\theta_2 - \tan\theta_1) = 2h\tan\theta_1$$

$$x = \frac{2h\tan\theta_1}{\tan\theta_2 - \tan\theta_1}$$

Adding 'h' on both sides

$$x+h = \frac{2h\tan\theta_1}{\tan\theta_2 - \tan\theta_1} + h \rightarrow$$

$$x+h = \frac{2h\tan\overleftrightarrow{\theta_1 + h\tan\theta_2} - h\tan\theta_1}{\tan\theta_2 - \tan\theta_1}$$

$$x+h = \frac{h\tan\theta_1 + h\tan\theta_2}{\tan\theta_2 - \tan\theta_1}$$

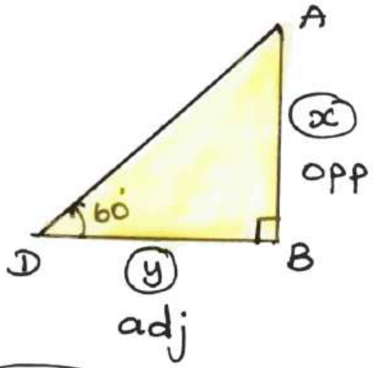
$$\boxed{x+h = \frac{h(\tan\theta_1 + \tan\theta_2)}{\tan\theta_2 - \tan\theta_1}}$$

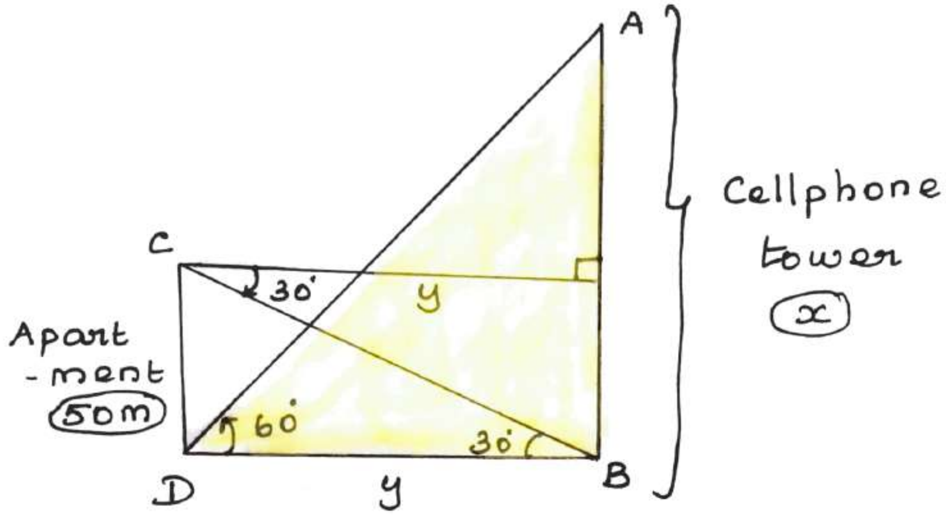
Hence Proved.

---

4) The angle of elevation of the top of a cell phone tower from the foot of a high apartment is 60° and the angle of depression of the foot of the tower from the top of the apartment is 30°. If the height of the apartment is 50m, find the height of the cellphone tower. According to radiations control norms, the minimum height of a cell phone

tower should be 120 m. State if the height of the above mentioned cell phone tower meets the radiation norms.





$$\tan\theta = \frac{opp}{adj}$$

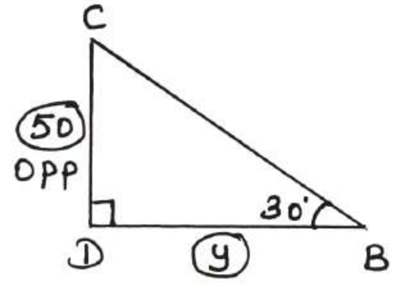
$$\tan 60^\circ = \frac{x}{y}$$

$$\sqrt{3} = \frac{x}{y}$$

$$\boxed{y = \frac{x}{\sqrt{3}}} \rightarrow ①$$

Height of cell phone tower should be 120m

$$\tan\theta = \frac{opp}{adj}$$



$$\tan 30^\circ = \frac{50}{y}$$

$$\frac{1}{\sqrt{3}} = \frac{50}{y}$$

$$\boxed{y = 50\sqrt{3}} \rightarrow ②$$

From ① & ②

$$\frac{x}{\sqrt{3}} = 50\sqrt{3} \rightarrow$$

$x = 50 \times 3$

$\boxed{x = 150\ \text{m}}$

$150\ \text{m} > 120\ \text{m}$

⇒ Yes, the height of cell phone tower meets the radiation norms.

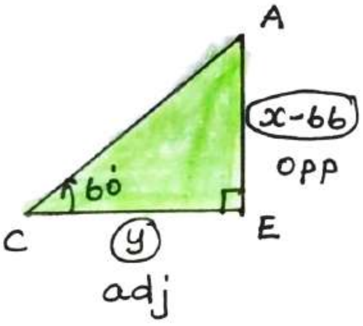
---

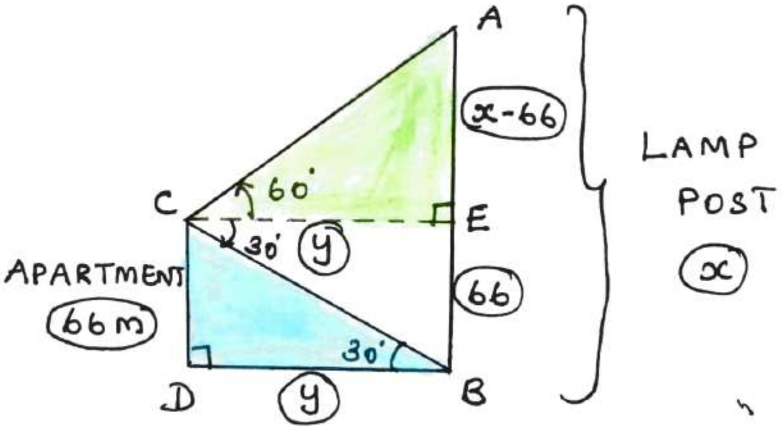
5) The angles of elevation and depression of the top and bottom of a Lamppost from the top of a 66 m high apartment are 60° and 30° respectively. Find

(i) The height of the lamp post.

(ii) The difference between height of the lamp post and the apartment.

(iii) The distance between the lamppost and the apartment. $(\sqrt{3} = 1.732)$





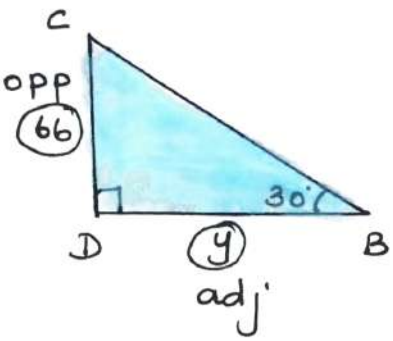
$\tan\theta = \frac{\text{opp}}{\text{adj}}$

$\tan 60° = \frac{x-66}{y}$

$\sqrt{3} = \frac{x-66}{y}$

$\boxed{y = \frac{x-66}{\sqrt{3}}} \rightarrow ①$

$$\boxed{\tan\theta = \frac{opp}{adj}}$$



$$\tan 30° = \frac{66}{y}$$

$$\frac{1}{\sqrt{3}} = \frac{66}{y}$$

$$\boxed{y = 66\sqrt{3}} \rightarrow ②$$

from ① & ②

$$\frac{x-66}{\sqrt{3}} = 66\sqrt{3} \rightarrow$$

$$x - 66 = 66 \times 3$$

$$\boxed{x - 66 = 198} \longrightarrow ③$$

$$x = 198 + 66$$

$$\boxed{x = 264\,m}$$

(i) Height of the Lamp post = $\boxed{264\,m}$

(ii) Difference between the height of Lamppost and Apartment

$$\Rightarrow x - 66 \Rightarrow \boxed{198m} \text{ (by 3)}$$

(iii) Distance between the Lamp post and the Apartment

$$\Rightarrow y = 66\sqrt{3} \text{ (by 2)}$$

$$y = 66 \times 1.732$$

$$y = 114.312$$

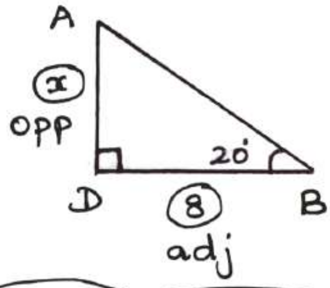
$$\boxed{y = 114.31\,m}$$

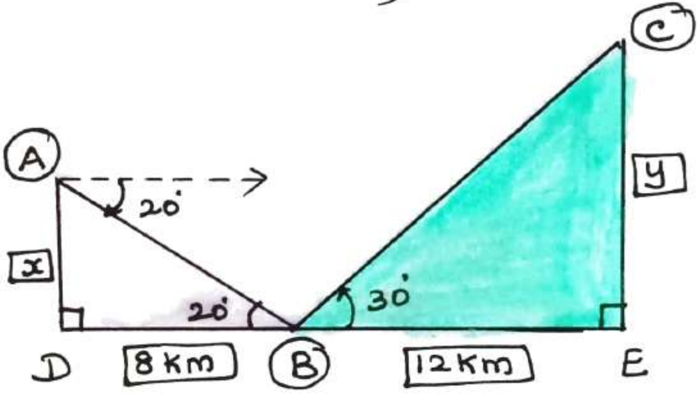
6) Three villagers A, B and C can see each other using telescope across a valley. The horizontal distance between A and B is 8 km and the horizontal distance between B and C is 12 km. The angle of depression of B from A is 20° and the angle of elevation of C from B is 30°. Calculate

(i) the vertical height between A and B.

(ii) the vertical height between B and C.

$(\tan 20^\circ = 0.3640, \sqrt{3} = 1.732)$





$$\tan\theta = \frac{opp}{adj}$$

$$\tan 20^\circ = \frac{x}{8}$$

$$0.3640 = \frac{x}{8}$$

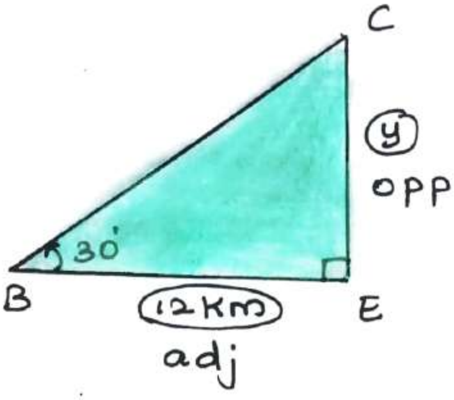
$$0.3640 \times 8 = x$$

$$2.9120 = x$$

$$\boxed{x = 2.91 \text{ Km}}$$

∴ (i) Vertical height between A and B = 2.91 km

$$\tan\theta = \frac{opp}{adj}$$



$$\tan 30° = \frac{y}{12}$$

$$\frac{1}{\sqrt{3}} = \frac{y}{12}$$

$$\frac{12}{\sqrt{3}} = y$$

$$y = \frac{12}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$y = \frac{\overset{4}{\cancel{12}} \times \sqrt{3}}{\cancel{3}}$$

$$y = 4 \times 1.732$$

$$y = 6.928$$

$$\boxed{y = 6.93 \text{ km}}$$

=> (ii) Vertical height between B and C is 6.93 km

Chapter - 7

Mensuration

SHAPE	CURVED SURFACE AREA [C.S.A]	TOTAL SURFACE AREA [T.S.A]
CYLINDER	$2\pi rh$ sq units	$2\pi r(h+r)$ sq units
HOLLOW CYLINDER	$2\pi h(R+r)$ sq units	$2\pi(R+r)(R-r+h)$ sq units
CONE	πrl sq units $l = \sqrt{h^2+r^2}$	$\pi r(l+r)$ sq units
FRUSTUM	$\pi l(R+r)$ sq units	$\pi l(R+r)+\pi(R^2+r^2)$ sq units
SPHERE	$4\pi r^2$ sq units	$4\pi r^2$ sq. units.
HEMISPHERE	$2\pi r^2$ sq units	$3\pi r^2$ sq units.
HOLLOW HEMISPHERE	$2\pi(R^2+r^2)$ sq units	$\pi(3R^2+r^2)$ sq. units

Exercise-7.1

① The radius and height of a cylinder are in the ratio 5:7 and its curved surface area is 5500 sq.cm. Find its radius and height.

Cylinder

Sol:-

$$r : h = 5 : 7$$

$$r = 5x$$

$$h = 7x$$

$$CSA = 5500 \text{ sq cm}$$

$$2\pi rh = 5500$$

$$2 \times \frac{22}{\cancel{7}} \times (\cancel{7}x) \times (5x) = 5500$$

$$220x^2 = 5500$$

$$x^2 = \frac{\overset{\cancel{50}\ 25}{\cancel{550}\cancel{0}}}{\underset{\cancel{2}\ 1}{\cancel{22}\cancel{0}}}$$

$$x^2 = 25$$

$$x^2 = 5^2$$

$$\boxed{x = 5}$$

$r = 5x = 5 \times 5 = 25$ cm.

$h = 7x = 7 \times 5 = 35$ cm.

② A Solid iron cylinder has total Surface area of 1848 sq m. Its Curved Surface area is five-sixth of its total Surface area. Find the radius and height of the iron cylinder.

Sol:- Cylinder

$$TSA = 1848 \text{ sq m.} \longrightarrow ①$$

$$CSA = \text{five-sixth of } TSA$$

$$2\pi rh = \frac{5}{\cancel{6}_{1}} \times \overset{308}{\cancel{1848}}$$

$$\begin{array}{r} 308 \\ \times\ 5 \\ \hline 1540 \end{array}$$

$$2\pi rh = 1540 \longrightarrow ②$$

From ①

$$TSA = 1848$$

$$2\pi r(h+r) = 1848$$

$$2\pi rh + 2\pi r^2 = 1848 \longrightarrow ③$$

Put ② in ③

$$1540 + 2\pi r^2 = 1848$$

$$\begin{array}{r} 1848 \\ -\ 1540 \\ \hline 308 \end{array}$$

$$2\pi r^2 = 1848 - 1540$$

$$2 \times \frac{22}{7} \times r^2 = 308$$

$$\left[\pi = \frac{22}{7}\right]$$

$$r^2 = \frac{\overset{\overset{7}{\cancel{77}}}{\cancel{\overset{154}{\cancel{308}}}} \times 7}{\underset{1}{\cancel{2}} \times \underset{\underset{1}{\cancel{11}}}{\cancel{22}}}$$

$$r^2 = 7^2$$

$$\boxed{r = 7\text{m}}$$

Put $r = 7$ in ②

$$2 \times \frac{22}{\cancel{7}} \times \cancel{7} \times h = 1540$$

$$h = \frac{\overset{\overset{\overset{35}{\cancel{385}}}{\cancel{770}}}{\cancel{1540}}}{\underset{1}{\cancel{2}} \times \underset{\underset{1}{\cancel{11}}}{\cancel{22}}}$$

$$\boxed{h = 35\text{m}}$$

$\therefore$ radius = 7m

height = 35m.

---

③ The external radius and the length of a hollow wooden log are 16cm and 13cm respectively. If its thickness is 4cm. then find its T.S.A.

Sol :-

Hollow Cylinder

external radius = $R = 16\text{cm}$

Length of log = $h = 13\text{cm}$

Thickness = 4cm

$$R - r = 4$$

$$16 - r = 4$$

$$16 - 4 = r$$

$$\boxed{r = 12\text{cm}}$$

$$TSA = 2\pi (R+r)(R-r+h)$$

$$= 2\times\frac{22}{7}(16+12)(16-12+13)$$

$$= 2\times\frac{22}{\cancel{7}_{1}}\times\overset{4}{\cancel{28}}\times 17$$

$$\boxed{TSA = 2992\ \text{cm}^2}$$

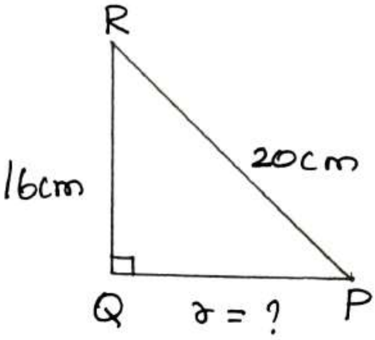
$$\begin{array}{r} 17 \\ \times 4 \\ \hline 68 \\ \times 44 \\ \hline 272 \\ 272 \\ \hline 2992 \\ \hline \end{array}$$

---

④ A right angled triangle PQR where $\angle Q = 90^\circ$ is rotated about QR and PQ. If QR = 16cm and PR = 20cm, Compare the Curved Surface areas of the right Circular Cones so formed by the triangle

Sol: -

Cone



$h = 16cm$

$l = 20cm$

$r = ?$

$h^2 + r^2 = l^2$

$16^2 + r^2 = 20^2$

$256 + r^2 = 400$

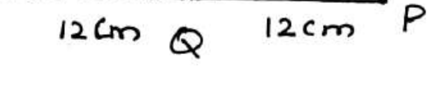
$r^2 = 400 - 256$

$r^2 = 144$

$r^2 = 12^2$

$\boxed{r = 12cm}$

Triangle rotated about QR.



$r = 12cm$

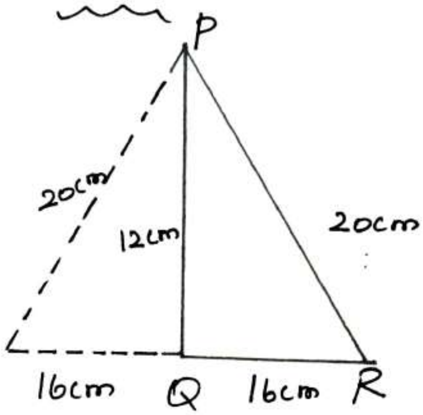
$l = 20cm$

$CSA = \pi r l$

$= \pi \times 12 \times 20$

$\boxed{CSA = 240\pi \ cm^2}$

Triangle rotated about PQ



$r = 16cm$

$l = 20cm$

$CSA = \pi r l$

$= \pi \times 16 \times 20$

$$\boxed{CSA = 320\pi cm^2}$$

$\therefore$ CSA of Cone rotated about PQ is Larger.

---

⑤ 4 person live in a conical tent whose slant height is 19m. If each person require $22m^2$ of the floor area then find the height of the tent.

Sol:-

Cone

Slant height = $l = 19m$

$\left.\begin{array}{l}\text{Area required}\\ \text{for 1 person}\end{array}\right\} = 22m^2$

$\therefore \left.\begin{array}{l}\text{Area required}\\ \text{for 4 person}\end{array}\right\} = 22 \times 4$

$= 88m^2$

The base of the Conical Tent is in the form of Circle

ie, Area of Circle = Area required by 4 person

$$\pi r^2 = 88$$

$$\frac{22}{7} \times r^2 = 88$$

$$r^2 = \cancel{88}^{4} \times \frac{7}{\cancel{22}}$$

$$\boxed{r^2 = 28}$$

To find height of tent.

$$h^2 + r^2 = l^2$$

$$h^2 + 28 = 19^2$$

$$h^2 + 28 = 361$$

$$h^2 = 361 - 28$$

$$h^2 = 333$$

$$h \doteq \sqrt{333}$$

$$h \doteq 18.25 \text{ m}$$

∴ height of Tent = 18.25m

$$\begin{array}{r} 19 \\ \times\ 19 \\ \hline 171 \\ 19 \\ \hline 361 \end{array}$$

$$\begin{array}{r} 361 \\ -\ 28 \\ \hline 333 \end{array}$$

18.248

1 | 333
1

28 | 233
224

362 | 900
724

3644 | 17600
- 14576

36488 | 302400
291904

(6) A girl wishes to prepare birthday Caps in the form of right Circular Cones for her birthday party, using a sheet of paper whose area is 5720 cm^2, how many caps can be made with a radius 5 cm and height 12 cm.

Sol :-

Cone

Total area of Sheet = 5720 cm^2

Area of Cone = CSA of cone

Area of cone = $\pi r l$

Cone

$r = 5cm$

$h = 12cm$

$l = ?$

$$l = \sqrt{h^2 + r^2}$$

$$= \sqrt{12^2 + 5^2}$$

$$= \sqrt{144 + 25}$$

$$= \sqrt{169}$$

$$l = 13 \text{ cm}$$

$$\text{Area of cone} = \pi r l$$

$$= \frac{22}{7} \times 5 \times 13$$

$$\therefore \text{No of caps} = \frac{\text{Area of Sheet}}{\text{Area of cone}}$$

$$= \frac{5720}{\frac{22}{7} \times 5 \times 13}$$

$$= \frac{\overset{\overset{\overset{4}{\cancel{52}}}{\cancel{260}}}{\overset{\cancel{2860}}{\cancel{5720}}} \times 7}{\underset{\underset{1}{\cancel{11}}}{\cancel{22}} \times \underset{1}{\cancel{5}} \times \underset{1}{\cancel{13}}}$$

$$= 4 \times 7$$

$$\boxed{\text{No of caps} = 28}$$

---

⑦ The ratio of the radii of two right circular cones of same height is 1:3. Find the ratio of their curved surface area when the height of each cone is 3 times the radius of the smaller cone.

Sol :- Cone

Two right circular cone $= r_1 : r_2 = 1:3$

$$\text{ie, } r_1 = 1x$$
$$r_2 = 3x$$

Given :- $h_1 = h_2$

Height = 3 times the radius of Smaller Cone

$$h = 3r_1$$
$$h_1 = 3(x)$$

$$\Rightarrow \boxed{h_1 = h_2 = 3x}$$

To find :- Ratio of the Curved Surface area

CSA 1

$r_1 = x$

$h_1 = 3x$

$l_1 = \sqrt{h_1^2 + r_1^2}$

$= \sqrt{(3x)^2 + x^2}$

$= \sqrt{9x^2 + x^2} = \sqrt{10x^2}$

$\boxed{l_1 = \sqrt{10}\, x}$

CSA 2

$r_2 = 3x$

$h_1 = 3x$

$l_2 = \sqrt{h_2^2 + r_2^2}$

$= \sqrt{(3x)^2 + (3x)^2}$

$= \sqrt{9x^2 + 9x^2}$

$= \sqrt{18x^2}$

$\boxed{l_2 = \sqrt{18}\, x}$

$$\frac{CSA_1}{CSA_2} = \frac{\cancel{\pi} r_1 l_1}{\cancel{\pi} r_2 l_2}$$

$$= \frac{\cancel{x} \cdot \sqrt{10}\,\cancel{x}}{3\cancel{x} \cdot \sqrt{18}\,\cancel{x}}$$

$\sqrt{10} = \sqrt{5} \times \sqrt{2}$

$\sqrt{18} = \sqrt{3 \times 3 \times 2}$

$= 3\sqrt{2}$

$$= \frac{\sqrt{5} \times \cancel{\sqrt{2}}}{3 \times 3 \times \cancel{\sqrt{2}}}$$

$$= \frac{\sqrt{5}}{9}$$

$\therefore CSA_1 : CSA_2 = \sqrt{5} : 9$

---

⑧ The radius of a sphere increases by 25%. Find the percentage increase in i/s surface area.

Sol :

Sphere

Surface area $= 4\pi r^2$

radius = increased by 25%

New radius $= r + 25\%$

$$= r + \frac{\cancel{25}^{1} r}{\cancel{100}_{4}}$$

$$= r + \frac{r}{4}$$

$$= \frac{4r + r}{4}$$

New radius $= \frac{5r}{4}$

∴ New Surface area with New radius $\Big\} = 4\pi\left(\frac{5r}{4}\right)^2$

$$= 4\pi \times \frac{25r^2}{16}$$

Percentage increase in its Surface area $\Big\} = \frac{\text{New S.A} - \text{Old S.A}}{\text{Old S.A}} \times 100$

$$= \frac{4\pi \times \frac{25r^2}{16} - 4\pi r^2}{4\pi r^2} \times 100$$

$$= \frac{\cancel{4\pi r^2}\left(\frac{25}{16} - 1\right)}{\cancel{4\pi r^2}} \times 100$$

$$= \left(\frac{25 - 16}{16}\right) \times 100$$

$$= \frac{9}{16} \times 100$$

$$\begin{array}{r} 0.5625 \\ 16\overline{)\,90} \\ -80 \\ \hline 100 \\ -96 \\ \hline 40 \\ -32 \\ \hline 80 \\ -80 \\ \hline 0 \end{array}$$

$$= 0.5625 \times 100$$

$$= 56.25\,\%$$

---

⑨ The internal and external diameters of a hollow hemispherical Vessel are 20cm and 28cm respectively. Find the cost to paint the Vessel all over at ₹0.14 per cm^2.

Sol :-

Hollow hemisphere

Internal diameter $= d = 20cm$

$$\therefore r = \frac{20}{2} = 10cm$$

External diameter $= D = 28\,cm$

$$R = \frac{28}{2} = 14\,cm.$$

$$TSA = \pi(3R^2 + r^2)$$

$$= \frac{22}{7}\left(3(14)^2 + 10^2\right)$$

$$\begin{array}{r} 14 \\ \times 14 \\ \hline 56 \\ 14 \\ \hline 196 \end{array}$$

$$= \frac{22}{7}\left(3(196)+100\right)$$

$$= \frac{22}{7}\left(588+100\right)$$

$$= \frac{22}{7} \times 688$$

$$= \frac{15136}{7}$$

$$\boxed{TSA = 2162.28\ cm^2}$$

$$\begin{array}{r} 196 \\ \times\ 3 \\ \hline 588 \end{array}$$

$$\begin{array}{r} 688 \\ \times\ 22 \\ \hline 1376 \\ 1376 \\ \hline 15136 \end{array}$$

$$\begin{array}{r} 2162.28 \\ 7\overline{)15136} \\ 14 \\ \hline 11 \\ 7 \\ \hline 43 \\ 42 \\ \hline 16 \\ 14 \\ \hline 20 \\ 14 \\ \hline 60 \\ 56 \\ \hline 40 \end{array}$$

Cost of painting 1 cm^2 = ₹0.14

∴ cost of painting 2162.28 cm^2 =

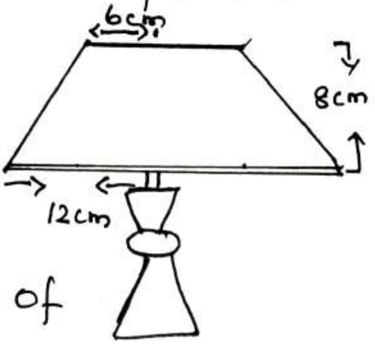
0.14×2162.28

= ₹ 302.72

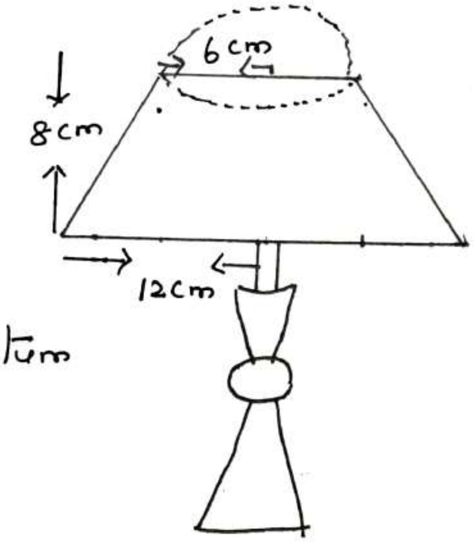
$$\begin{array}{r} \overset{2}{2}162.\overset{3}{2}8 \\ \times 0.14 \\ \hline 864912 \\ 216228 \\ \hline 302.7192 \end{array}$$

---

⑩ The frustum shaped outer portion of the table lamp has to be painted including the top part. Find the total cost of painting the lamp if the cost of painting 1sq.cm is ₹2.



Sol Frustum



$R = 12$ cm

$r = 6$ cm

$h = 8$ cm

$l = \sqrt{h^2 + (R-r)^2} \Rightarrow$ Frustum

$l = \sqrt{8^2 + (12-6)^2}$

$= \sqrt{64 + (6)^2}$

$= \sqrt{64 + 36}$

$= \sqrt{100}$

$l = 10$ cm

CSA of Frustum + Area of top circular region

$\Rightarrow \pi l (R+r) + \pi r^2$

$\pi [10(12+6) + 6^2]$

$\pi [(10 \times 18) + 36]$

$\pi [180 + 36]$

$\pi [216]$

$= \frac{22}{7} \times 216$

$= \frac{4752}{7}$

$= 678.85 \text{ cm}^2$

Cost of painting 1 cm^2 = ₹ 2

∴ cost of painting 678.85 cm^2 =

$= 678.85 \times 2$

$=$ ₹1357.70

$$\begin{array}{r} 216 \\ \times\ 22 \\ \hline 432 \\ 432 \\ \hline 4752 \end{array}$$

$$\begin{array}{r} 678.85 \\ 7\,\overline{)\,4752} \\ 42 \\ \hline 55 \\ 49 \\ \hline 62 \\ 56 \\ \hline 60 \\ 56 \\ \hline 40 \\ 35 \\ \hline 0 \end{array}$$

$$\begin{array}{r} 678.85 \\ \times\ 2 \\ \hline 1357.70 \end{array}$$

Volume

SHAPE	VOLUME [cubic units]
Cylinder	$\pi r^2 h$
Hollow cylinder	$\pi h(R^2 - r^2)$
Cone	$\frac{1}{3}\pi r^2 h$
Frustum	$\frac{\pi h}{3}(R^2 + Rr + r^2)$
Sphere	$\frac{4}{3}\pi r^3$
Hemisphere	$\frac{2}{3}\pi r^3$
Hollow Hemisphere	$\frac{2}{3}\pi(R^3 - r^3)$

Exercise - 7.2

① A 14 m deep well with inner diameter 10 m is dug and the earth taken out is evenly spread all round the well to form an embankment of width 5 m. Find the height of the embankment.

Sol :-

Cylinder

$h = 14m$

$d = 10m \Rightarrow r = \frac{10}{2} = 5m$

Vol of cylinder $= \pi r^2 h$

$$= \frac{22}{\cancel{7}_{1}} \times 5 \times 5 \times \overset{2}{\cancel{14}}$$

$$\begin{array}{r} 44 \\ \times 25 \\ \hline 220 \\ 88 \\ \hline 1100 \\ \hline \end{array}$$

$$\boxed{\text{Vol. of Cylinder} = 1100 m^3}$$

Hollow cylinder ⇒ Embankment

$r = 5m$

Width ⇒ $w = 5m$

$R = r + w$

$[W = R - r$
$\therefore R = w + r]$

$= 5 + 5$

$R = 10\,m$

$h = ?$

$\because$ Volume of Hollow cylinder $= \pi h(R^2 - r^2)$

$= \frac{22}{7} \times h(10^2 - 5^2)$

$= \frac{22}{7} \times h(100 - 25)$

Vol of Embankment $= \frac{22}{7} \times h(75)$

$\therefore$ Vol of Embankment $=$ Vol of cylinder

$\frac{22}{7} \times h(75) = 1100$

$h = \frac{\cancel{1100}^{\cancel{550}\ \cancel{50}\ \cancel{10}\ 2} \times 7}{\cancel{22}_{\cancel{11}\ 1} \times \cancel{75}_{\cancel{15}\ 3}}$

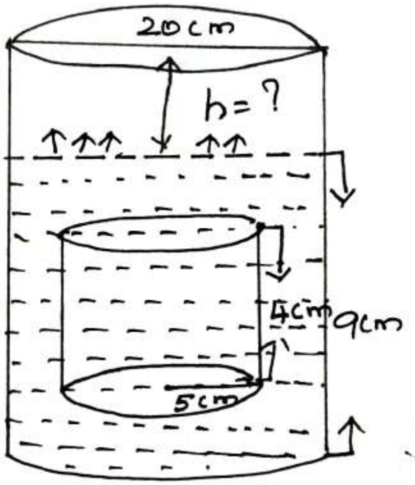
$h = \frac{2 \times 7}{3}$

$h = \frac{14}{3}$

$$\boxed{h = 4.7\,m}$$

$$\begin{array}{r} 4.66 \\ 3\overline{)14} \\ 12 \\ \hline 20 \\ 18 \\ \hline 20 \end{array}$$

② A cylindrical glass with diameter 20cm has water to a height of 9cm. A small cylindrical metal of radius 5cm and height 4cm is immersed it completely. Calculate the raise of the water in the glass ?



Sol:-

cylindrical glass	A small cylinder
$d = 20cm$	$r = 5cm$
$r = \frac{20}{2} = 10cm$	$h = 4cm$
$h = ?$ [raise of Water]	$V = \pi r^2 h$
$V = \pi r^2 h$	$= \pi \times 5 \times 5 \times 4$
$= \pi \times 10 \times 10 \times h$	

∴ Vol of cylindrical glass = Vol of small cylinder

$$\cancel{\pi} \times 10 \times 10 \times h = \cancel{\pi} \times 5 \times 5 \times 4$$

$$h = \frac{\overset{1}{\cancel{5}} \times \overset{1}{\cancel{5}} \times \cancel{4}\,\overset{1}{\cancel{2}}}{\underset{\cancel{2}_{1}}{\cancel{10}} \times \underset{\cancel{2}_{1}}{\cancel{10}}}$$

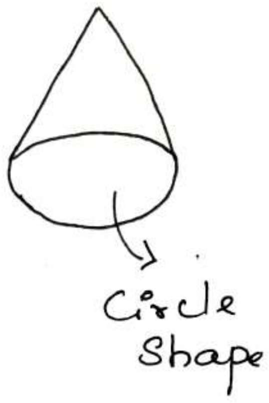
$$\boxed{h = 1cm}$$

Raise of Water level in the glass = 1cm

③ If the Circumference of a Conical woooden piece is 484 cm. then find its Volume when its height is 105 cm.

Sol :-

Cone



Circumference = 484 cm

$2\pi r = 484$

$2 \times \frac{22}{7} \times r = 484$

$r = \frac{\overset{11}{\cancel{\overset{121}{\cancel{\overset{242}{\cancel{484}}}}}} \times 7}{\underset{1}{\cancel{2}} \times \underset{1}{\cancel{\underset{11}{\cancel{22}}}}}$

$r = 11 \times 7$

$r = 77\ cm$

$h = 105\ cm$ (Given)

$\therefore$ Volume $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{\underset{1}{\cancel{3}}} \times \frac{22}{\underset{1}{\cancel{7}}} \times 77 \times 77 \times \overset{\overset{5}{\cancel{35}}}{\cancel{105}}$

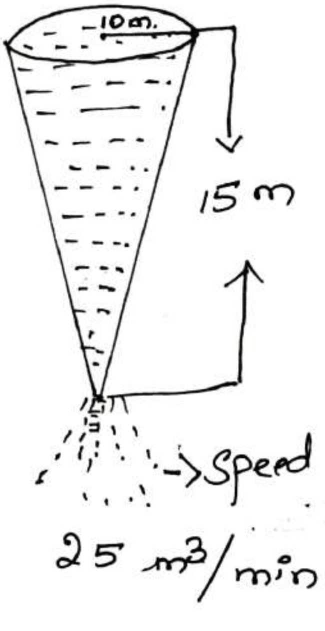
$\boxed{\text{Volume} = 652190\ cm^3}$

$$\begin{array}{r} \overset{4}{77} \\ \times 77 \\ \hline 539 \\ 539 \\ \hline 5929 \\ \times 22 \\ \hline 11858 \\ 11858 \\ \hline 130438 \\ \times 5 \\ \hline 652190 \end{array}$$

④ A Conical Container is fully filled with petrol. The radius is 10m and the height is 15m. If the container can release the petrol through its bottom at the rate of 25 cu. meter per minute, in how many minutes the Container will be emptied. Round off your answer to the nearest minute.

Sol:

Cone



$r = 10m$

$h = 15m$

$V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{\cancel{3}_{1}} \times \frac{22}{7} \times 10 \times 10 \times \overset{5}{\cancel{15}}$

$V = \frac{22}{7} \times 10 \times 10 \times 5$

$\therefore$ Vol of Container $= \frac{22}{7} \times 10 \times 10 \times 5 \ m^3 \longrightarrow ①$

Speed of Petrol $= 25 m^3/min$

$\therefore$ Speed = Vol of Petrol

∴ Vol of petrol × T = Vol of container

$$25 \times T = \frac{22}{7} \times 10 \times 10 \times 5$$

$$T = \frac{22}{7} \times \frac{10 \times \overset{2}{\cancel{10}} \times \overset{1}{\cancel{5}}}{\underset{1}{\cancel{\underset{5}{\cancel{25}}}}}$$

$$= \frac{440}{7}$$

$$7\overline{)440} = 62.8$$
42
20
14
60
56
4

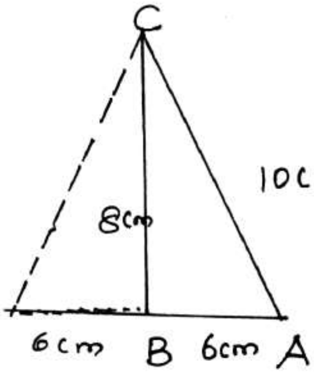
∴ $\boxed{\text{Time} = 63 \text{ min}}$

---

⑤ A right angled triangle whose sides are 6cm, 8cm and 10cm is revolved about the sides containing the right angle in two ways. Find the difference in volumes of the two solids so formed.

Sol Cone

Revolving about BC fixed.



$r = 6\text{cm}$

$h = 8\text{cm}$

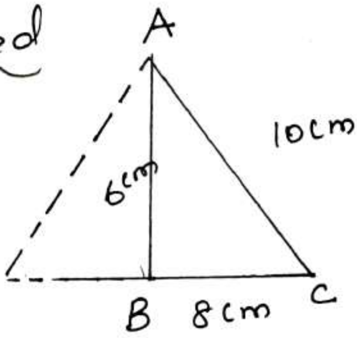
$$V = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{\underset{1}{\cancel{3}}} \times \frac{22}{7} \times 6 \times \overset{2}{\cancel{6}} \times 8$$

$$\boxed{V = \frac{2112}{7}\ cm^3}$$

$$\begin{array}{r} 132 \\ \times 16 \\ \hline 792 \\ 132 \\ \hline 2112 \end{array}$$

Revolving about AB fixed



$r = 8cm$

$h = 6cm$

$$V = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{\underset{1}{\cancel{3}}} \times \frac{22}{7} \times 8 \times 8 \times \overset{2}{\cancel{6}}$$

$$\boxed{V = \frac{2816}{7}\ cm^3}$$

$$\begin{array}{r} 176 \\ \times 16 \\ \hline 1056 \\ 176 \\ \hline 2816 \end{array}$$

$\therefore$ Difference in Volumes $= \frac{2816}{7} - \frac{2112}{7}$

$$= \frac{704}{7}$$

$$= 100.5\ cm^3$$

$$\begin{array}{r} 2816 \\ -2112 \\ \hline 704 \end{array}$$

$$\begin{array}{r} 100.5 \\ 7 \overline{)\ 704} \\ 700 \\ \hline 40 \\ 35 \end{array}$$

⑥ The Volumes of two Cones of Same base radius are $3600 cm^3$ and $5040 cm^3$. Find the ratio of heights.

Sol

Cone

Vol of Cone 1 $= \pi r_1^2 h_1 = 3600 cm^3$

Vol of Cone 2 $= \pi r_2^2 h_2 = 5040 cm^3$

Given :- $\boxed{r_1 = r_2 = r}$

To find :- $\boxed{h_1 : h_2}$

$$\frac{\text{Vol } 1}{\text{Vol } 2} = \frac{360\cancel{0}}{504\cancel{0}}$$

$$\frac{\cancel{\frac{1}{3}\pi r} h_1}{\cancel{\frac{1}{3}\pi r} h_2} = \frac{360}{504}$$

$$\frac{h_1}{h_2} = \frac{\cancel{360}\ \cancel{90}\ \cancel{30}\ \cancel{15}\ 5}{\cancel{504}\ \cancel{126}\ \cancel{42}\ \cancel{14}\ 7}$$

$$\therefore \boxed{h_1 : h_2 = 5 : 7}$$

⑦ If the ratio of radii of two Spheres is 4 : 7, find the ratio of their Volumes.

Sol :-

Sphere

$r_1 = 4$

$r_2 = 7$

$$\therefore \frac{V_1}{V_2} = \frac{\cancel{\frac{4}{3}\pi} r_1^3}{\cancel{\frac{4}{3}\pi} r_2^3}$$

$$= \frac{(4)^3}{(7)^3}$$

$$\frac{V_1}{V_2} = \frac{64}{343}$$

$$\therefore \boxed{V_1 : V_2 = 64 : 343}$$

---

⑧ A Solid sphere and a Solid hemisphere have equal total Surface area. prove that the ratio of their Volume is $3\sqrt{3} : 4$.

Sol :-

Sol:.

To prove :- $\dfrac{\text{Volume of sphere}}{\text{Volume of Hemisphere}} = \dfrac{3\sqrt{3}}{4}$

ie, $\boxed{\text{Vol of Sphere : Vol of Hemisphere} = 3\sqrt{3} : 4}$

Given:-

TSA of Sphere = TSA of Hemisphere

$$4\cancel{\pi} r_1^2 = 3\cancel{\pi} r_2^2$$

$$4 r_1^2 = 3 r_2^2$$

$$\frac{r_1^2}{r_2^2} = \frac{3}{4}$$

$$\frac{r_1}{r_2} = \sqrt{\frac{3}{4}}$$

$$\frac{r_1}{r_2} = \frac{\sqrt{3}}{2}$$

$\therefore$ $\boxed{r_1 = \sqrt{3}x}$ $\boxed{r_2 = 2x}$

$$\frac{\text{Vol of Sphere}}{\text{Vol of Hemisphere}} = \frac{\frac{4}{\cancel{3}}\cancel{\pi} r_1^3}{\frac{2}{\cancel{3}}\cancel{\pi} r_2^3}$$

$$= \frac{\cancel{4}^{2}(\sqrt{3}x)^3}{\cancel{2}_{1}(2x)^3}$$

$\sqrt{3}\times\sqrt{3}\times\sqrt{3}$
$= 3\sqrt{3}$

$$= \frac{\cancel{2} \times 3\sqrt{3}\cancel{x^3}}{\cancel{8}_{4}\cancel{x^3}}$$

$$\frac{\text{Vol of Sphere}}{\text{Vol of Hemisphere}} = \frac{3\sqrt{3}}{4}$$

$$\boxed{\text{Vol of Sphere : Vol of Hemisphere} = 3\sqrt{3} : 4}$$

---

⑨ The Outer and the inner Surface areas of a Spherical copper Shell are $576\pi\ cm^2$ and $324\pi\ cm^2$ respectively. Find the Volume of the material required to make the shell.

Sol

Sphere

Outer Surface area $= 576\pi$

$$4\cancel{\pi}R^2 = 576\cancel{\pi}$$

$$R^2 = \frac{\cancel{576}^{144}}{\cancel{4}}$$

$$R^2 = 12^2$$

$$\boxed{R = 12\ cm}$$

Inner Surface area $= 324\pi$

$$4\cancel{\pi}r^2 = 324\cancel{\pi}$$

$$r^2 = \frac{\cancel{324}\,81}{\cancel{4}}$$

$$r^2 = 9^2$$

$$\boxed{r = 9\ cm}$$

Volume of Hollow Sphere $= \frac{4}{3}\pi(R^3 - r^3)$

$$= \frac{4}{3}\pi(12^3 - 9^3)$$

$$= \frac{4}{3}\pi(1728 - 729)$$

$$= \frac{4}{\cancel{3}_{1}}\pi(\overset{3\,3\,3}{\cancel{999}})$$

$$= 1332\pi \text{ cm}^3$$

$12 \times 12 \times 12$

144×12

288
144

1728

$9^3 = 9 \times 9 \times 9$
$= 81 \times 9$
$= 729$

1728
729
999

---

⑩ A Container open at the top is in the form of a frustum of a cone of height 16cm and with radii of its lower and upper ends are 8cm and 20 cm respectively. Find the cost of milk which can completely fill a Container at the rate of ₹40 per litre.

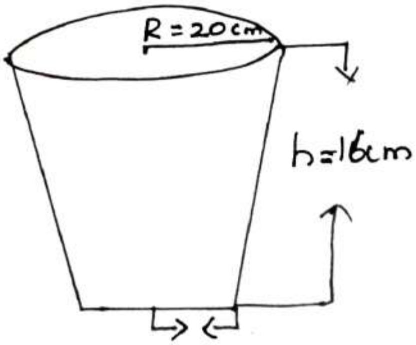
Sol:-

Frustum

height = 16cm

lower end radius = 8cm = r

Upper end radius = 20cm = R



Volume $= \frac{\pi}{3} h \left(R^2 + r^2 + Rr\right)$

$= \frac{22}{7} \times \frac{1}{3} \times 16 \left(20^2 + 8^2 + 20(8)\right)$

$= \frac{22}{7} \times \frac{1}{3} \times 16 \left(400 + 64 + 160\right)$

$= \frac{22}{7} \times \frac{1}{\overset{}{\underset{1}{\cancel{3}}}} \times 16 \left(\overset{208}{\cancel{624}}\right)$

$= \frac{73216}{7}$

$= 10459.42 \text{ cm}^3$

$\therefore$ Volume $= 10459 \text{ cm}^3$

$\therefore$ Total Quantity of Milk $= \frac{10459}{1000}$

$= 10.459\ l$

$$
\begin{array}{r}
22 \\
\times\ 16 \\
\hline
132 \\
22\ \ \\
\hline
352 \\
\times\ 208 \\
\hline
2816 \\
000\ \ \\
704\ \ \ \ \\
\hline
73216
\end{array}
$$

$$
\begin{array}{r|l}
 & 10459.42 \\
7 & 73216 \\
 & -7 \\
\hline
 & 32 \\
 & 28 \\
\hline
 & 41 \\
 & 35 \\
\hline
 & 66 \\
 & 63 \\
\hline
 & 30 \\
 & 28 \\
\hline
 & 20
\end{array}
$$

Cost of milk per litre = ₹40

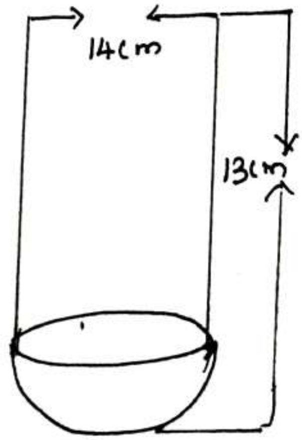
$\therefore$ cost of 10.459 l of milk $= 10.459 \times 40$

$= ₹\ 418.360$

$\Rightarrow ₹\ 418.36$

Exercise-7.3

① A Vessel is in the form of a hemispherical bowl mounted by a hollow cylinder. The diameter is 14cm and the height of the Vessel is 13cm. Find the capacity of the Vessel.



Sol:

Hemisphere

$D = 14\text{ cm}$

$r = \frac{14}{2}$

$r = 7\text{cm}$

$V = \frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \frac{22}{\cancel{7}} \times 7 \times 7 \times \cancel{7}$

$\boxed{V = \frac{2156}{3}\text{ cm}^3}$

cylinder

$r = 7\text{cm}$

$h = 13 - 7 = 6\text{cm}$

$V = \pi r^2 h$

$= \frac{22}{\cancel{7}} \times 7 \times \cancel{7} \times 6$

$\boxed{V = 924\text{ cm}^3}$

$$\begin{array}{r} 49 \\ \times 44 \\ \hline 196 \\ 196 \\ \hline 2156 \end{array}$$

$$\begin{array}{r} 22 \\ \times 42 \\ \hline 44 \\ 88 \\ \hline 924 \end{array}$$

Capacity = Volume = Vol of Hemisphere + Vol of cylinder

$$= \frac{2156}{3} + 924$$

$$\begin{array}{r} 924 \\ \times 3 \\ \hline 2772 \end{array}$$

$$= \frac{2156 + 2772}{3}$$

$$\begin{array}{r} 2772 \\ +2156 \\ \hline 4928 \end{array}$$

$$= \frac{4928}{3}$$

$$= 1642.666...$$

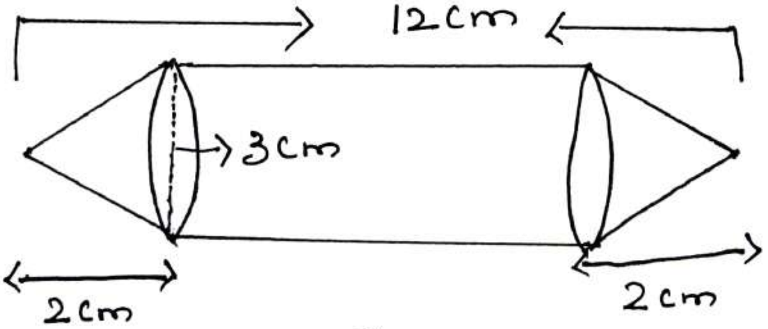
$$\begin{array}{r} 1642.66 \\ 3\overline{)4928} \\ 3 \\ \hline 19 \\ 18 \\ \hline 12 \\ 12 \\ \hline 8 \\ 6 \\ \hline 20 \\ 18 \\ \hline 20 \end{array}$$

$$\therefore \boxed{\text{Volume} = 1642.67\ cm^3}$$

---

② Nathan, an engineering student was asked to make a model shaped like cylinder with two cones attached at its two ends. The diameter of the model is 3cm and its length is 12cm. If each cone has a height of 2cm. find the Volume of the model. that Nathan made.

Sol:-



cylinder

$d = 3\text{cm}$

$r = \frac{3}{2}\ \text{cm}$

$h = 12 - 2(2)$

$= 12 - 4$

$h = 8\text{cm}$

$V = \pi r^2 h$

$= \frac{22}{7} \times \frac{3}{\underset{1}{\cancel{2}}} \times \frac{3}{\cancel{2}} \times \overset{\overset{2}{\cancel{4}}}{\cancel{8}}$

$V = \frac{396}{7}\ \text{cm}^3$

Cone

$r = \frac{3}{2}\ \text{cm}$

$h = 2\text{cm}$

$V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{\cancel{3}} \times \frac{\overset{11}{\cancel{22}}}{7} \times \frac{3}{\underset{1}{\cancel{2}}} \times \frac{\cancel{3}}{\cancel{2}} \times \overset{1}{\cancel{2}}$

$V = \frac{33}{7}\ \text{cm}^3$

$$\begin{array}{r} 44 \\ \times\ 9 \\ \hline 396 \end{array}$$

$\therefore$ Volume of model = Vol of cylinder + Vol of 2 cones.

$= \frac{396}{7} + 2\left(\frac{33}{7}\right)$

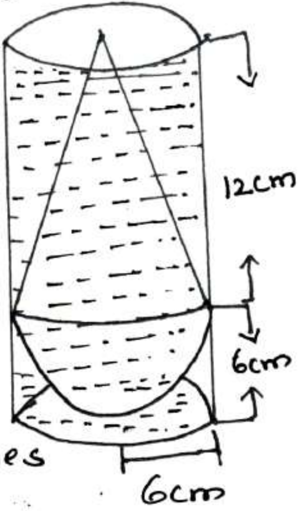
$= \frac{396}{7} + \frac{66}{7}$

$= \frac{462}{7}$

$$\begin{array}{r} 66 \\ 7\,\overline{)462} \\ -42 \\ \hline 42 \\ -42 \\ \hline 0 \end{array}$$

Volume of model $= 66\ \text{cm}^3$

④ A Solid consisting of a right Circular cone of height 12cm and radius 6cm standing On a hemisphere of radius 6cm is placed upright in a right circular cylinder full of water Such that it touches the bottom. Find the Volume of the water displaced Out of cylinder, if the radius of the cylinder is 6cm and the height is 18cm.



Sol :-

Cone

$r = 6cm$

$h = 12cm$

$Vol = \frac{1}{3}\pi r^2 h$

$= \frac{1}{\cancel{3}_1}\times\frac{22}{7}\times6\times6\times\overset{4}{\cancel{12}}$

$Vol = \frac{3168}{7} cm^3$

Hemisphere

$r = 6cm$

$V = \frac{2}{3}\pi r^3$

$= \frac{2}{\cancel{3}_1}\times\frac{22}{7}\times6\times6\times\overset{2}{\cancel{6}}$

$= \frac{44\times72}{7}$

$Vol = \frac{3168}{7} cm^3$

$$\begin{array}{r} 22 \\ \times 36 \\ \hline 132 \\ 66 \\ \hline 792 \\ \times 4 \\ \hline 3168 \end{array}$$

$$\begin{array}{r} 44 \\ \times 72 \\ \hline 88 \\ 308 \\ \hline 3168 \end{array}$$

Vol of water discharged = Vol of Cone + Vol of Hemisphere

$$= \frac{3168}{7} + \frac{3168}{7}$$

$$= \frac{6336}{7}$$

Vol of water discharged = 905.14 cm^3

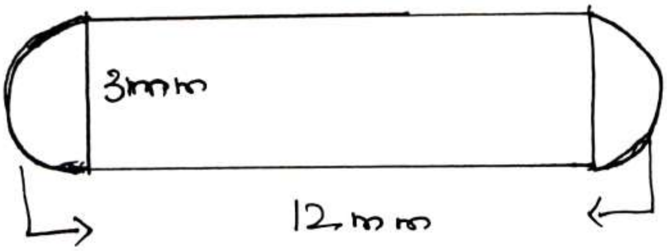
```
  3168
+ 3168
  6336

    905.14
 7) 6336
  - 63↓↓
    036
     35
      10
       7
      30
```

---

⑤ A capsule is in the shape of a cylinder with two hemisphere stuck to each of its ends. If the length of the entire capsule is 12mm and the diameter of the capsule is 3mm, how much medicine it can hold?

Sol



Cylinder

$d = 3\,mm$

$r = \frac{3}{2}\,mm$

$h = 12 - \overset{1}{\cancel{2}}(3/\cancel{2})$

$h = 12 - 3$

$h = 9\,mm$

$V = \pi r^2 h$

$= \frac{\overset{11}{\cancel{22}}}{7} \times \frac{3}{2} \times \frac{3}{\underset{1}{\cancel{2}}} \times 9$

$V = \frac{891}{14}\,mm^3$

Hemisphere.

$r = \frac{3}{2}\,mm$

$V = \frac{2}{3}\pi r^3$

$= \frac{\overset{1}{\cancel{2}}}{\cancel{3}} \times \frac{\overset{11}{\cancel{22}}}{7} \times \frac{3}{2} \times \frac{3}{\underset{1}{\cancel{2}}} \times \frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{2}}}$

$= \frac{11 \times 9}{14}$

$V = \frac{99}{14}\,mm^3$

11
$\times 81$
$\overset{1\;1}{88}$
891

891
198
1089

77.78
$14)\overline{1089}$
98
109
98
110
98
120
112

$\therefore$ Quantity of medicine = Total Volume of Capsule

Total volume of capsule = Vol of cylinder + 2 Vol of Hemisphere

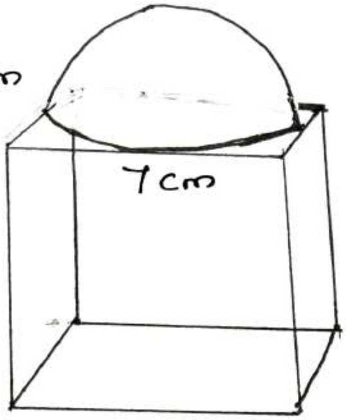
$= \frac{891}{14} + 2\left(\frac{99}{14}\right)$

$= \frac{891}{14} + \frac{198}{14}$

$= \frac{1089}{14}$

$= 77.78\,mm^3$

⑥ As shown in figure a cubical block of side 7cm is surmounted by a hemisphere. Find the surface area of the solid.



Sol:-

Hemisphere

$r = \frac{7}{2}$ cm

$CSA = 2\pi r^2$

$= \overset{1}{\cancel{2}} \times \frac{\overset{11}{\cancel{22}}}{\underset{1}{\cancel{7}}} \times \frac{7}{\underset{1}{\cancel{2}}} \times \frac{\overset{1}{\cancel{7}}}{\underset{1}{\cancel{2}}}$

$CSA = 77\ cm^2$

Cube

$a = 7$ cm

$TSA = 6a^2$

$= 6(7)^2$

$= 6(49)$

$TSA = 294\ cm^2$

Base of Hemisphere.

$r = \frac{7}{2}$ cm

$Area = \pi r^2$

$= \frac{\overset{11}{\cancel{22}}}{\underset{1}{\cancel{7}}} \times \frac{7}{2} \times \frac{\overset{1}{\cancel{7}}}{\underset{1}{\cancel{2}}}$

$Area = \frac{77}{2}\ cm^2$

$$\left.\begin{array}{l}\text{Surface area}\\ \text{of the solid}\end{array}\right\} = \begin{array}{c}\text{CSA of}\\ \text{H.S}\end{array} + \begin{array}{c}\text{TSA of}\\ \text{Cube}\end{array} - \begin{array}{c}\text{Base area}\\ \text{of}\\ \text{H.S}\end{array}$$

$$= 77 + 294 - \frac{77}{2}$$

$$= 371 - \frac{77}{2}$$

$$= \frac{742 - 77}{2}$$

$$\begin{array}{r} 742 \\ -77 \\ \hline 665 \end{array}$$

$$= \frac{665}{2}$$

$$\begin{array}{r} 332.5 \\ 2\overline{)665} \\ -6 \\ \hline 06 \\ 6 \\ \hline 05 \\ 4 \\ \hline 10 \\ 10 \\ \hline 0 \end{array}$$

Surface area of Solid $= 332.5 \text{ cm}^2$

---

⑦ A right circular cylinder just enclose a sphere of radius r units Calculate

(i) the Surface area of the Sphere.

(ii) the Curved Surface area of the cylinder

(iii) the ratio of the area obtained in (i) and (ii)

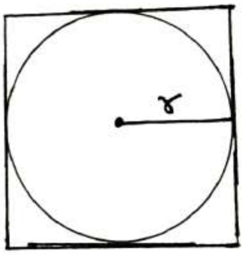
Sol :- cylinder

radius = r

height = r+r
= 2r

Sphere

radius = r



(i) Surface area of Sphere $= 4\pi r^2$ sq. units

(ii) Curved Surface area $= 2\pi rh = 2\pi r(2r)$
$= 4\pi r^2$ units.

(iii) Ratio of CSA of Sphere to CSA of cylinder $= \frac{\overset{1}{\cancel{4\pi r^2}}}{\underset{1}{\cancel{4\pi r^2}}} = 1:1$

Exercise - 7.4

① An aluminium sphere of radius 12 cm is melted to make a cylinder of radius 8 cm. Find the height of the cylinder.

Sol

Sphere	cylinder
$r = 12\text{ cm}$	$r = 8\text{ cm}$
	$h = ?$
$V = \frac{4}{3}\pi r^3$	$V = \pi r^2 h$
$= \frac{4}{\cancel{3}} \times \pi \times 12 \times 12 \times \overset{4}{\cancel{12}}$	$= \pi \times 8 \times 8 \times h$
$= 4\pi \times 12 \times 12 \times 4$	

$\therefore$ Vol of cylinder = Vol of sphere

$$\cancel{\pi} \times 8 \times 8 \times h = 4\cancel{\pi} \times 12 \times 12 \times 4$$

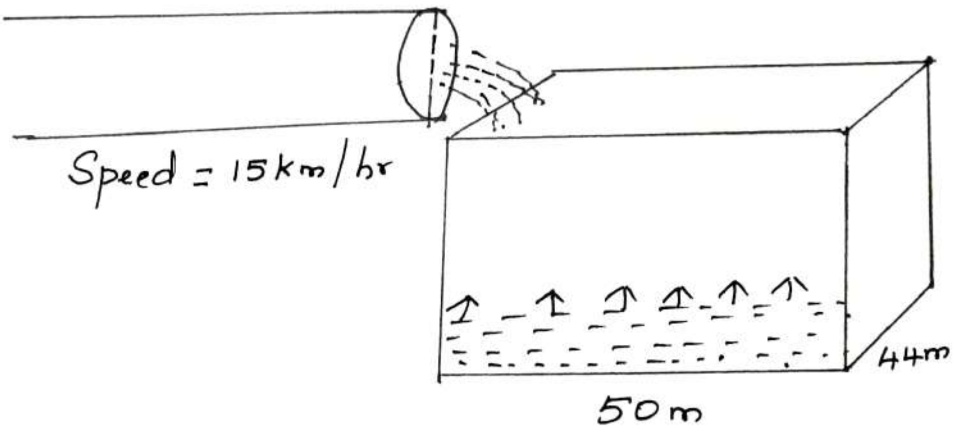
$$h = \frac{\cancel{4} \times 12 \times \overset{3}{\cancel{\overset{6}{\cancel{12}}}} \times \cancel{4}}{\underset{1}{\cancel{\underset{2}{\cancel{8}}}} \times \underset{1}{\cancel{\underset{2}{\cancel{8}}}}}$$

$$h = 12 \times 3$$

$$h = 36\text{ cm}$$

② Water is flowing at the rate of 15 km per hour through a pipe of diameter 14 cm into a Rectangular tank which is 50 m long and 44m wide. Find the time in which the level of water in the tanks will rise by 21cm.

Sol:-



Cylinder [pipe]

$d = 14\text{ cm}$

$r = \frac{14}{2} = 7\text{ cm} = \frac{7}{100}\text{ m}$

$\text{Speed} = 15\text{ km/hr}$

$= 15 \times 1000 = 15000\text{ m}$

$\text{Time} = ?$

$h \Rightarrow \text{Distance} = \text{speed} \times T$

$h = 15000 \times T$

$V = \pi r^2 h$

$= \frac{\overset{11}{\cancel{22}}}{\underset{1}{\cancel{7}}} \times \frac{\cancel{7}}{\underset{\underset{1}{\cancel{2}}}{\cancel{100}}} \times \frac{7}{\cancel{100}} \times \overset{3}{\cancel{15000}} \times T$

$V = 11 \times 7 \times 3 \times T$

Cuboid [Rectangular tank]

$l = 50\text{ m}$

$b = 44\text{ m}$

$h = 21\text{ cm} = \frac{21}{100}\text{ m}$

$V = l \times b \times h$

$= \overset{1}{\cancel{50}} \times \overset{22}{\cancel{44}} \times \frac{21}{\underset{\underset{1}{\cancel{2}}}{\cancel{100}}}$

$V = 22 \times 21$

Vol of cylinder = Vol of cuboid

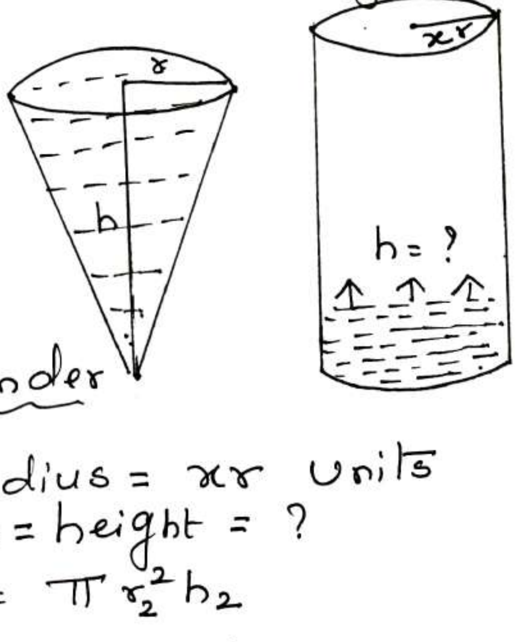
$$11\times 7\times 3\times T = 22\times 21$$

$$T = \frac{\cancel{22}^{2}\times \cancel{21}^{3}}{\cancel{11}_{1}\times \cancel{7}_{1}\times \cancel{3}_{1}}$$

$$\boxed{T = 2\text{ hrs.}}$$

③ A conical flask is full of water. The flask has base radius r units and height h units, the water is poured into a cylindrical flask of base radius xr units Find the height of water in the cylindrical flask.

Sol:-



Cone

radius = r units

h_1 = height = h. units

$$V = \frac{1}{3}\pi r_1^2 h_1$$

$$V = \frac{1}{3}\pi r^2 h$$

Cylinder

radius = xr units

h_2 = height = ?

$$V = \pi r_2^2 h_2$$

$$= \pi\times(xr)^2\times h_2$$

$$V = \pi x^2 r^2\times h_2$$

∴ Vol of cylinder = Vol of cone.

$$\cancel{\pi} x^2 \cancel{r^2} h_2 = \frac{1}{3}\cancel{\pi}\cancel{r^2} h$$

$$h_2 = \frac{1h}{3x^2}$$

∴ height of water level = $\boxed{h_2 = \frac{h}{3x^2}}$

---

④ A solid right circular cone of diameter 14cm and height 8cm is melted to form a hollow sphere. If the external diameter of the sphere is 10cm, find the internal diameter.

Sol :-

Cone

$d = 14\text{cm}$

$r = \frac{14}{2} = 7\text{cm}$

$h = 8\text{cm}$

$$V = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3} \times \pi \times 7 \times 7 \times 8$$

$$V = \frac{392}{3}\pi \text{ cm}^3$$

Hollow Sphere

External Diameter = 10cm

$D = 10\text{cm}$

$\therefore R = \frac{10}{2} = 5\text{cm}$

Internal Diameter = $d = ?$

$$V = \frac{4}{3}\pi (R^3 - r^3)$$

$$= \frac{4}{3}\pi (5^3 - r^3)$$

$$= \frac{4}{3}\pi (125 - r^3)$$

∴ Vol of Hollow Sphere = Vol of cone

$$\frac{4}{\cancel{3}}\cancel{\pi}(125 - r^3) = \frac{392}{\cancel{3}}\cancel{\pi}$$

$$125 - r^3 = \frac{\cancel{392}^{\,98}}{\cancel{4}_{\,1}}$$

$$\begin{array}{r} 125 \\ -\ 98 \\ \hline 7 \end{array}$$

$$125 - r^3 = 98$$

$$125 - 98 = r^3$$

$$27 = r^3$$

$$r^3 = 3^3$$

$$\therefore \boxed{r = 3\text{ cm}}$$

∴ Internal diameter $= d = 2r$

$$d = 2(3)$$

$$\boxed{d = 6\text{cm}}$$

---

⑤ Seenu's house has an Overhead tank in the Shape of cylinder. This is filled by Pumping water from a Sump (Underground tank) which is in the Shape of a cuboid. The Sump has dimensions 2m × 1.5m × 1m. The Overhead tank has its radius of 60 cm and height 105 cm. Find the Volume of the water left in the sump after the Overhead tank has been completely filled with water from the Sump which has been full, Initially.

Sol:-

Cuboid

[Sump]

$l = 2\,m$

$b = 1.5\,m$

$h = 1\,m$

$V = l \times b \times h$

$= 2 \times 1.5 \times 1$

$V = 3\,m^3$

Cylinder

[Overhead tank]

$r = 60\,cm = \frac{6\cancel{0}}{10\cancel{0}} = \frac{\overset{3}{\cancel{6}}}{\underset{5}{\cancel{10}}} = \frac{3}{5}\,m$

$h = 105\,cm = \frac{\overset{21}{\cancel{105}}}{\underset{20}{\cancel{100}}} = \frac{21}{20}\,m$

$V = \pi r^2 h$

$= \frac{\overset{11}{\cancel{22}}}{\underset{1}{\cancel{7}}} \times \frac{3}{5} \times \frac{3}{5} \times \frac{\overset{3}{\cancel{21}}}{\underset{10}{\cancel{20}}}$

$= 11 \times \frac{27}{250}$

$= \frac{297}{250}$

$V = 1.188\,m^3$

27×11

27

27

297

$250 \overline{)297}$ = 1.188

250

470

250

2200

2000

2000

2000

0

Remaining water in Sump = Vol of Cuboid − Vol of cylinder

$= 3 - 1.188$

$= 1.812 \ m^3 = 1.812 \times 1000$

$= 1812$ litres.

$$\begin{array}{r} 3.000 \\ 1.188 \\ \hline 1.812 \\ \hline \end{array}$$

---

⑥ The internal and external diameter of a hollow hemispherical shell are 6cm and 10cm respectively. If it is melted and recast into a solid cylinder of diameter 14cm, then find the height of the cylinder.

Sol:-

Hollow hemisphere

Internal diameter

$d = 6cm$

$r = \frac{6}{2} = 3\ cm$

External diameter

$D = 10cm$

$R = \frac{10}{2} = 5cm$

$$V = \frac{2}{3}\pi (R^3 - r^3)$$

$$= \frac{2}{3} \times \frac{22}{7} (5^3 - 3^3)$$

$$= \frac{2}{3} \times \frac{22}{7} (125 - 27)$$

cylinder

$d = 14cm$

$r = \frac{14}{2} = 7\ cm$

$h = ?$

$$V = \pi r^2 h$$

$$= \frac{22}{\cancel{7}} \times 7 \times \cancel{7} \times h$$

$$V = 154 h$$

$\therefore$ Vol of cylinder = $\therefore$ Vol of Hollow cylinder

$$154\,h = \frac{2}{3} \times \frac{22}{7}(125-27)$$

$$\begin{array}{r} 125 \\ -\,27 \\ \hline 98 \end{array}$$

$$h = \frac{\overset{1}{\cancel{2}} \times \overset{2}{\cancel{22}} \times \overset{14}{\cancel{98}}}{\underset{\underset{\cancel{7}_{1}}{\cancel{77}}}{\cancel{154}}}$$

$$\boxed{h = 28\,\text{cm}}$$

---

⑦ A Solid Sphere of radius 6cm is melted into a hollow cylinder of uniform thickness. If the external radius of the base of the cylinder is 5 cm and its height is 32cm, then find the thickness of the cylinder.

Sol :-

Sphere

$r = 6$ cm

$$V = \frac{4}{3}\pi r^3$$

$$= \frac{4}{3}\pi\, 6^3$$

$$= \frac{4}{\underset{1}{\cancel{3}}}\pi \times 6 \times 6 \times \overset{2}{\cancel{6}}$$

$$V = 4\pi \times 6 \times 6 \times 2$$

Hollow cylinder

External radius = R = 5 cm

h = 32cm

Internal radius = r = ?

Thickness = R − r

$$V = \pi h (R^2 - r^2)$$

$$= \pi \times 32 (5^2 - r^2)$$

$$= \pi \times 32 (25 - r^2)$$

Vol of Hollow cylinder = Vol of Sphere

$$\cancel{\pi}\times 32(25-r^2) = 4\cancel{\pi}\times 6\times 6\times 2$$

$$25-r^2 = \frac{\overset{1}{\cancel{4}}\times\overset{3}{\cancel{6}}\times\overset{3}{\cancel{6}}\times\overset{1}{\cancel{2}}}{\cancel{32}\ \cancel{8}\ \cancel{4}\ \cancel{2}}$$

$$25-r^2 = 9$$

$$25-9 = r^2$$

$$16 = r^2$$

$$r^2 = 4^2$$

$$\boxed{r = 4\text{ cm}}$$

$\therefore$ Thickness $= R - r$

$= 5 - 4$

$\boxed{\text{Thickness} = 1\text{cm}}$

---

⑧ A hemispherical bowl is filled to the brim with juice. The juice is poured into a cylindrical Vessel whose radius is 50% more than its height. If the diameter is same for both the bowl and the cylinder then find the Percentage of juice that can be transfered from the bowl into the cylindrical Vessel.

Sol :-

Hemisphere

$\left.\begin{matrix}\text{Diameter} \\ \text{of} \\ \text{H.S}\end{matrix}\right\} = \begin{matrix}\text{Diameter} \\ \text{of} \\ \text{Cylinder}\end{matrix}$

$\Rightarrow$ Radius $= r$

$\therefore V = \frac{2}{3}\pi r^3$ cubic units

cylinder

$r =$ 50% more than Height

$$\therefore r = \frac{\overset{1}{\cancel{50}}h}{\underset{2}{\cancel{100}}} + h$$

$$= \frac{h}{2} + h$$

$$= \frac{h+2h}{2}$$

$$r = \frac{3h}{2}$$

$$\boxed{\frac{2r}{3} = h}$$

$\therefore$ Radius $= r$

$$V = \pi r^2 h$$

$$= \pi (r^2)\left(\frac{2r}{3}\right)$$

$$V = \frac{2}{3}\pi r^3 \text{ cubic units}$$

$\therefore$ Vol of Hemisphere = Vol of cylinder

$\therefore$ 100% of juice can be transfered from the bowl into the cylindrical Vessel.

Chapter-8

Statistics And Probability

Arithmetic Mean($\bar{x}$)

Ungrouped data

① Direct Method

$$\bar{x} = \frac{\Sigma x}{n}$$

Grouped data

① $\bar{x} = \frac{\Sigma fx}{\Sigma f}$

② Assumed Mean Method

$$\bar{x} = A + \frac{\Sigma fd}{\Sigma f} \qquad \boxed{d = x - A}$$

③ Step Deviation Method

$$\bar{x} = A + \frac{\Sigma fd}{\Sigma f} \times c \qquad \boxed{d = \frac{x - A}{c}}$$

2. Range $= L - S$

L = large Value
S = Small Value

3. Coefficient of range $= \frac{L-S}{L+S}$

4. Standard Deviation (σ)

1. Direct Method

$$\sigma = \sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2}$$

2. Mean Method

$$\sigma = \sqrt{\frac{\Sigma d^2}{n}} \qquad \boxed{d = x - \bar{x}}$$

3. Assumed Mean Method.

$$\sigma = \sqrt{\frac{\Sigma d^2}{n} - \left(\frac{\Sigma d}{n}\right)^2} \qquad \boxed{d = x - A}$$

4. Step deviation Method

$$\sigma = \sqrt{\frac{\Sigma d^2}{n} - \left(\frac{\Sigma d}{n}\right)^2} \times c \qquad \boxed{d = \frac{x-A}{c}}$$

5. Variance (σ^2)

Exercise - 8.1

1. Find the range and coefficient of range of the following data

(i) 63, 89, 98, 125, 79, 108, 117, 68

Sol:- Largest Value = L = 125

Smallest Value = S = 63

$$\text{Range} = L - S$$

$$= 125 - 63$$

$$\boxed{\text{Range} = 62}$$

$$\text{Coefficient of Range} = \frac{L-S}{L+S}$$

$$= \frac{125-63}{125+63}$$

$$= \frac{62}{188}$$

$$= 0.33$$

```
   0.329
188)620
    564
    560
    376
    1840
    1692
```

(ii) 43.5, 13.6, 18.9, 38.4, 61.4, 29.8

Sol

$L = 61.4$

$S = 13.6$

$$\begin{array}{r} 61.4 \\ -\ 13.6 \\ \hline 47.8 \end{array}$$

Range $= L - S$

$= 61.4 - 13.6$

$$\boxed{\text{Range} = 47.8}$$

$$\begin{array}{r} 61.4 \\ +\ 13.6 \\ \hline 75.0 \end{array}$$

Coefficient of Range $= \dfrac{L-S}{L+S}$

$$\begin{array}{r|l} & 0.637 \\ 750 & 4780 \\ & -4500 \\ \hline & 2800 \\ & 2250 \\ \hline & 5500 \end{array}$$

$$= \frac{61.4 - 13.6}{61.4 + 13.6}$$

$$= \frac{47.8}{75} \times \frac{10}{10}$$

$$= \frac{478}{750}$$

$$= 0.64$$

---

② If the range and the Smallest Value of a Set of data are 36.8 and 13.4 respectively; then find the largest Value.

Sol:-

Range = 36.8

Smallest Value = 13.4

∴ Largest Value = ?

$$L - S = R$$

$$L - 13.4 = 36.8$$

$$L = 36.8 + 13.4$$

$$\boxed{L = 50.2}$$

$$\begin{array}{r} 36.8 \\ +13.4 \\ \hline 50.2 \\ \hline \end{array}$$

---

③ Calculate the range of the following data.

INCOME	400-450	450-500	500-550	550-600	600-650
NO OF WORKERS	8	12	30	21	6

Sol:-

$L = 650$

$S = 400$

$$\text{Range} = L - S$$

$$= 650 - 400$$

$$\boxed{\text{Range} = 250}$$

$$\begin{array}{r} 650 \\ -400 \\ \hline 250 \\ \hline \end{array}$$

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④ A teacher asked the Students to Complete 60 pages of a record note book. Eight

Students have completed only 32, 35, 37, 30, 33, 36, 35 and 37 pages. Find the Standard deviation of the pages completed by them.

Sol:-

$$\bar{x} = \frac{\Sigma x}{n}$$

$$= \frac{32+35+37+30+33+36+35+37}{8}$$

$$= \frac{275}{8}$$

$$\boxed{\bar{x} = 34.4}$$

$$\begin{array}{r} 34.375 \\ 8\overline{)275} \\ 24 \\ \hline 35 \\ 32 \\ \hline 30 \\ 24 \\ \hline 60 \\ 56 \\ \hline 40 \end{array}$$

x	$d = x - \bar{x}$	d^2
32	$32 - 34.4 = -2.4$	5.76
35	$35 - 34.4 = 0.6$	0.36
37	$37 - 34.4 = 2.6$	6.76
30	$30 - 34.4 = -4.4$	19.36
33	$33 - 34.4 = -1.4$	1.96
36	$36 - 34.4 = 1.6$	2.56
35	$35 - 34.4 = 0.6$	0.36
37	$37 - 34.4 = 2.6$	6.76
275		43.88

$$\begin{array}{r} -34.4 \\ 32.0 \\ \hline -2.4 \end{array} \qquad \begin{array}{r} 2.4 \\ \times 2.4 \\ \hline 96 \\ 48 \\ \hline 5.76 \end{array}$$

$$\begin{array}{r} 35.0 \\ -34.4 \\ \hline 0.6 \end{array} \qquad \begin{array}{r} 2.6 \\ \times 2.6 \\ \hline 156 \\ 52 \\ \hline 6.76 \end{array}$$

$$\begin{array}{r} 37.0 \\ -34.4 \\ \hline 2.6 \end{array}$$

$$\begin{array}{r} -34.4 \\ 30.0 \\ \hline 4.4 \end{array} \qquad \begin{array}{r} 4.4 \\ \times 4.4 \\ \hline 176 \\ 176 \\ \hline 19.36 \end{array}$$

$$\begin{array}{r} -34.4 \\ 33.0 \\ \hline 1.4 \end{array}$$

$$S.D = \sigma = \sqrt{\frac{d^2}{n}}$$

$$= \sqrt{\frac{43.88}{8}}$$

$$= \sqrt{5.485}$$

$$\boxed{\sigma = 2.34}$$

$$\begin{array}{r|l}
 & \overline{2.34} \\
2 & 5.48500 \\
 & 4 \\
\hline
43 & 148 \\
 & 129 \\
\hline
464 & 2150 \\
 & 1856
\end{array}$$

$$\begin{array}{r}
5.485 \\
8\,\overline{)\,43.88} \\
-40 \\
\hline
38 \\
-32 \\
\hline
68 \\
-64 \\
\hline
40 \\
-40 \\
\hline
0
\end{array}$$

---

⑤ Find the Variance and standard deviation of the wages of 9 workers given below: ₹310, ₹290, ₹320, ₹280, ₹300, ₹290, ₹320, ₹310, ₹280

Sol:-

$$\overline{X} = \frac{\Sigma x}{n}$$

$$= \frac{310+290+320+280+300+290+320+310+280}{9}$$

$$= \frac{2700}{9}$$

$$\boxed{\overline{X} = 300}$$

x	$d = x - \bar{x}$	d^2
310	$310 - 300 = 10$	100
290	$290 - 300 = -10$	100
320	$320 - 300 = 20$	400
280	$280 - 300 = -20$	400
300	$300 - 300 = 0$	0
290	$290 - 300 = -10$	100
320	$320 - 300 = 20$	400
310	$310 - 300 = 10$	100
280	$280 - 300 = -20$	400
2700		2000

$$\text{Variance} = \sigma^2 = \frac{\Sigma d^2}{n}$$

$$= \frac{2000}{9}$$

$$\boxed{\sigma^2 = 222.22}$$

$$\begin{array}{r} 222.22 \\ 9\overline{)2000} \\ -18 \\ \hline 20 \\ -18 \\ \hline 20 \\ -18 \\ \hline 20 \\ -18 \\ \hline 20 \\ -18 \end{array}$$

$$\text{Standard deviation} = \sigma = \sqrt{\frac{\Sigma d^2}{n}}$$

ie, $\sigma = \sqrt{222.22}$

$$\boxed{\sigma = 14.9}$$

14.9
1 | 222.22
-1
24 | 122
-96
289 | 2622
-2601
298 | 2100

---

⑥ A wall clock strikes the bell once at 1 o'clock, 2 times at 2 o'clock, 3 times at 3 o'clock and so on. How many times will it strike in a particular day. Find the standard deviation of the number of strikes the bell make a day.

Sol:-

Clock Strikes

One time – 1 o'clock

Two times – 2 o'clock

and so on

We know that in a day

1 o'clock, 2 o'clock, ... so on comes Two times.

∴ Required S.D = 2 (S.D of first n natural no)

$$\boxed{n = 12}$$

S.D of first n natural No. $\Big\}$ $\sigma = \sqrt{\frac{n^2-1}{12}}$

$$\sigma = \sqrt{\frac{12^2-1}{12}}$$

$$= \sqrt{\frac{144-1}{12}}$$

$$= \sqrt{\frac{143}{12}}$$

$$= \sqrt{11.92}$$

$$\boxed{\sigma = 3.45}$$

$$\begin{array}{r|l} & 11.916 \\ 12 & 143 \\ & 12 \\ & 23 \\ & 12 \\ & 110 \\ & 108 \\ & 20 \\ & 12 \\ & 80 \\ & 72 \end{array}$$

$$\begin{array}{r|l} & 3.45 \\ 3 & 11.92 \\ & -9 \\ 64 & 292 \\ & -256 \\ 685 & 3600 \\ & -3425 \end{array}$$

∴ Required S.D = 2 (S.D of first n Natural No.)

$$= 2(3.45)$$

$$\boxed{\sigma = 6.90}$$

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(7) Find the standard deviation of first 21 natural Numbers.

Sol: $\boxed{n = 21}$

S.D of first n Natural No ∴ $= \sqrt{\frac{n^2-1}{12}}$

$$\sigma = \sqrt{\frac{21^2-1}{12}}$$

$$= \sqrt{\frac{441-1}{12}}$$

$$= \sqrt{\frac{440}{12}}$$

$$= \sqrt{36.66}$$

$$\boxed{\sigma = 6.05}$$

21
21
21
42
441

$12 \overline{)440}$ 36.66
36
80
72
80
72
80
72

6.05
6 | 36.66
36
1205 | 06600
6025

---

⑧ If the standard deviation of a data is 4.5 and if each value of the data is decreased by 5 then find the new standard deviation.

Sol :- $\sigma = 4.5$

each value of data is decreased by 5

When a data is increased (or) decreased There will be no change in New S.D

∴ New S.D $\boxed{\sigma = 4.5}$

⑨ If the standard deviation of a data is 3.6 and each Value of data is divided by 3, then find the new Variance and new standard deviation.

Sol

$$\sigma = 3.6$$

The data of each Value is divided by 3

$$\therefore \text{New } \sigma = \frac{3.6}{3}$$

$$\boxed{\sigma = 1.2}$$

$$\text{New } \sigma^2 \text{ (Variance)} = (1.2)^2$$

$$\boxed{\sigma^2 = 1.44}$$

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⑩ The rainfall recorded in Various places of five districts in a week are given below. Find its standard deviation

Rainfall (in mm)	45	50	55	60	65	70
Number of places	5	13	4	9	5	4

A = Middle Value

x	f	$A = 60$ $d = x - A$	d^2	fd	fd^2
45	5	$45 - 60 = -15$	225	-75	1125
50	13	$50 - 60 = -10$	100	-130	1300
55	4	$55 - 60 = -5$	25	-20	100
$\overset{A}{\boxed{60}}$	9	$60 - 60 = 0$	0	0	0
65	5	$65 - 60 = 5$	25	25	125
70	4	$70 - 60 = 10$	100	40	400
TOTAL	40			-160	3050

$$\sigma = \sqrt{\frac{\Sigma fd^2}{\Sigma f} - \left(\frac{\Sigma fd}{\Sigma f}\right)^2}$$

$$= \sqrt{\frac{305\cancel{0}}{4\cancel{0}} - \left(\frac{-\overset{4}{1\cancel{60}}}{\underset{1}{\cancel{40}}}\right)^2}$$

$$= \sqrt{\frac{305}{4} - 16}$$

$$= \sqrt{\frac{305 - 64}{4}}$$

$$\begin{array}{r} -130 \\ -75 \\ -20 \\ \hline -225 \\ \hline \end{array} \qquad \begin{array}{r} -225 \\ 65 \\ \hline -160 \\ \hline \end{array}$$

$$= \sqrt{\frac{241}{4}}$$

$$= \sqrt{60.25}$$

$$\boxed{\sigma = 7.76}$$

$$\begin{array}{r} 305 \\ -\ 64 \\ \hline 241 \end{array}$$

$$\begin{array}{r} 60.25 \\ 4\,)\,241 \\ -\,240 \\ \hline 10 \\ -\,8 \\ \hline 20 \\ -\,20 \\ \hline 0 \end{array}$$

$$\begin{array}{r|l} & 7.76 \\ 7 & 60.25 \\ & 49 \\ \hline 147 & 1125 \\ & 1029 \\ \hline 1546 & 9600 \\ & 8276 \end{array}$$

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(11) In a study about Viral fever, the number of people affected in a town were noted as Find its standard deviation.

Age (in years)	0-10	10-20	20-30	30-40	40-50	50-60	60-70
No of people affected.	3	5	16	18	12	7	4

Sol :-

C.I	x	f	$A = 35$ $C = 10$ $d = \frac{x-A}{c}$	d^2	fd	fd^2
0-10	5	3	$\frac{5-35}{10} = \frac{-30}{10} = -3$	9	-9	27
10-20	15	5	$\frac{15-35}{10} = \frac{-20}{10} = -2$	4	-10	20
20-30	25	16	$\frac{25-35}{10} = \frac{-10}{10} = -1$	1	-16	16
30-40	$\overset{A}{\boxed{35}}$	18	$\frac{35-35}{10} = \frac{0}{10} = 0$	0	0	0
40-50	45	12	$\frac{45-35}{10} = \frac{10}{10} = 1$	1	12	12
50-60	55	7	$\frac{55-35}{10} = \frac{20}{10} = 2$	4	14	28
60-70	65	4	$\frac{65-35}{10} = \frac{30}{10} = 3$	9	12	36
	TOTAL	65			3	139

$$\sigma = \sqrt{\frac{\Sigma fd^2}{\Sigma f} - \left(\frac{\Sigma fd}{\Sigma f}\right)^2} \times c$$

$$= \sqrt{\frac{139}{65} - \left(\frac{3}{65}\right)^2} \times 10$$

$$= \sqrt{\frac{139}{65} - \frac{9}{4225}} \times 10$$

$$= \sqrt{\frac{139(65) - 9}{4225}} \times 10$$

$$= \sqrt{\frac{9035 - 9}{4225}} \times 10$$

$$= \sqrt{\frac{9026}{4225}} \times 10$$

$$= \sqrt{2.13} \times 10$$

$$= 1.45 \times 10$$

$$\therefore \boxed{\sigma = 14.5}$$

$$\begin{array}{r} 65 \\ \times 65 \\ \hline 325 \\ 390 \\ \hline 4225 \\ \hline \end{array}$$

$$\begin{array}{r} 139 \\ \times 65 \\ \hline 695 \\ 834 \\ \hline 9035 \\ \hline \end{array}$$

$$\begin{array}{r} 9035 \\ -\ 9 \\ \hline 9026 \\ \hline \end{array}$$

$$\begin{array}{r} 2.13 \\ 4225\overline{)9026} \\ 8450 \\ \hline 6760 \\ 4225 \\ \hline 15350 \\ 12675 \\ \hline \end{array}$$

$$\begin{array}{r|l} & 1.45 \\ \hline 1 & 2.13 \\ & 1 \\ \hline 24 & 113 \\ & 96 \\ \hline 285 & 1700 \\ & 1425 \end{array}$$

⑫ The measurements of the diameters (in cms) of the plates prepared in a factory are given below. Find its standard deviation.

Diameter (Cm)	21-24	25-28	29-32	33-36	37-40	41-44
Number of plates	15	18	20	16	8	7

Sol :-

C.I	x	f	$A = 30.5$ $c = 4$ $d = \frac{x-A}{c}$	d^2	fd	fd^2
20.5 - 24.5	22.5	15	$\frac{22.5-30.5}{4} = \frac{-8}{4} = -2$	4	-30	60
24.5 - 28.5	26.5	18	$\frac{26.5-30.5}{4} = \frac{-4}{4} = -1$	1	-18	18
28.5 - 32.5	$\overset{A}{\overset{\uparrow}{\boxed{30.5}}}$	20	$\frac{30.5-30.5}{4} = 0$	0	0	0
32.5 - 36.5	34.5	16	$\frac{34.5-30.5}{4} = \frac{4}{4} = 1$	1	16	16
36.5 - 40.5	38.5	8	$\frac{38.5-30.5}{4} = \frac{8}{4} = 2$	4	16	32
40.5 - 44.5	42.5	7	$\frac{42.5-30.5}{4} = \frac{12}{4} = 3$	9	21	63
Total		84			5	189

$$\sigma = \sqrt{\frac{\Sigma fd^2}{\Sigma f} - \left(\frac{\Sigma fd}{\Sigma f}\right)^2} \times c$$

$$= \sqrt{\frac{189}{84} - \left(\frac{5}{84}\right)^2} \times 4$$

$$= \sqrt{\frac{189}{84} - \frac{25}{7056}} \times 4$$

$$= \sqrt{\frac{189(84) - 25}{7056}} \times 4$$

$$= \sqrt{\frac{15876 - 25}{7056}} \times 4$$

$$= \sqrt{\frac{15851}{7056}} \times 4$$

$$= \sqrt{2.24} \times 4$$

$$= 1.5 \times 4$$

$$\boxed{\sigma = 6}$$

189
×84
756
1512
15876

15876
−25
15851

$$\begin{array}{r} 2.24 \\ 7056\overline{)15851} \\ -14112 \\ \hline 17390 \\ 14112 \\ \hline 32780 \\ 28224 \\ \hline \end{array}$$

$$\begin{array}{r|l} & 1.\overset{5}{4}9 \\ 1 & 2.2400 \\ & 1 \\ \hline 24 & 124 \\ & 96 \\ \hline 289 & 2800 \\ & 2601 \end{array}$$

$$\begin{array}{r} \overset{2}{1}.5 \times 4 \\ \hline 6.0 \end{array}$$

⑬ The time taken by 50 students to complete a 100 meter race are given below. Find its standard deviation.

Time Taken (sec)	8·5 - 9·5	9·5 - 10·5	10·5 - 11·5	11·5 - 12·5	12·5 - 13·5
Number of Students	6	8	17	10	9

Sol :-

C·I	x	f	$A = 11$ $d = x - A$	d^2	fd	fd^2.
8·5 - 9·5	9	6	$9 - 11 = -2$	4	−12	24
9·5 - 10·5	10	8	$10 - 11 = -1$	1	−8	8
10·5 - 11·5	$\overset{\overset{A}{\uparrow}}{\boxed{11}}$	17	$11 - 11 = 0$	0	0	0
11·5 - 12·5	12	10	$12 - 11 = 1$	1	10	10
12·5 - 13·5	13	9	$13 - 11 = 2$	4	18	36
Total		50			8	78.

$$\sigma = \sqrt{\frac{\Sigma fd^2}{\Sigma f} - \left(\frac{\Sigma fd}{\Sigma f}\right)^2}$$

$$= \sqrt{\frac{78}{50} - \left(\frac{8}{50}\right)^2}$$

$$= \sqrt{\frac{78}{50} - \frac{64}{2500}}$$

$$= \sqrt{\frac{78(50) - 64}{2500}}$$

$$= \sqrt{\frac{3900 - 64}{2500}}$$

$$= \sqrt{\frac{3836}{2500}}$$

$$= \sqrt{1.534}$$

$$\boxed{\sigma = 1.23}$$

$$\begin{array}{r} 78 \\ \times\ 50 \\ \hline 3900 \\ \hline \end{array}$$

$$\begin{array}{r} 3900 \\ -\ 64 \\ \hline 3836 \end{array}$$

$$\begin{array}{r} 1.534 \\ 2500 \overline{)\,3836} \\ -2500 \\ \hline 13360 \\ -12500 \\ \hline 8600 \\ -7500 \\ \hline 11000 \\ 10000 \\ \hline \end{array}$$

$$\begin{array}{r|l} & \ \ 1.23 \\ \hline 1 & 1.534 \\ & 1\ \downarrow\downarrow \\ \hline 22 & 0\ 53 \\ & \ \ 44 \\ & \ \ \ 900 \\ 243 & \ \ \ 729 \end{array}$$

⑭ For a group of 100 Candidates the mean and standard deviation of their marks were found to be 60 and 15 respectively. Later on it was found that the scores 45 and 72 were wrongly entered as

40 and 27. Find the correct mean and standard deviation.

Sol: - $\boxed{n = 100}$

Incorrect Values	Correct Values
Mean $= \bar{x} = 60$ S.D $= \sigma = 15$ Scores $= 40, 27$	Scores $= 45, 72$

$$\bar{x} = 60$$

$$\frac{\Sigma x}{n} = 60$$

$$\frac{\Sigma x}{100} = 60 \nearrow$$

$$\Sigma x = 60 \times 100$$

$$\Sigma x = 6000 \text{ [incorrect Total]}$$

$\therefore$ Correct Total $\Rightarrow$ $\Sigma x = 6000 - \underline{40 - 27} + \underline{45 + 72}$

$$\Sigma x = 6000 - 67 + 117$$

$$= 6117 - 67$$

Correct Total $\boxed{\Sigma x = 6050}$

∴ Correct Mean $\overline{x} = \frac{\Sigma x}{n}$ (correct Total)

$$\overline{x} = \frac{605\cancel{0}}{10\cancel{0}}$$

$$\boxed{\overline{x} = 60.5}$$

Incorrect S.D ⇒ $\sigma = \sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2}$

$$\sqrt{\frac{\Sigma x^2}{100} - (60)^2} = 15$$ [Incorrect Values]

Squaring on both sides.

$$\frac{\Sigma x^2}{100} - 3600 = 225$$

$$\frac{\Sigma x^2}{100} = 225 + 3600$$

$$\frac{\Sigma x^2}{100} = 3825$$

$$\Sigma x^2 = 3825 \times 100$$

$$\begin{array}{r} 27 \\ \times 27 \\ \hline 189 \\ 54 \\ \hline 729 \end{array} \qquad \begin{array}{r} 45 \\ \times 45 \\ \hline 225 \\ 180 \\ \hline 2025 \end{array} \qquad \begin{array}{r} 72 \\ \times 72 \\ \hline 144 \\ 504 \\ \hline 5184 \end{array}$$

Incorrect $\Sigma x^2 = 382500$

∴ Correct $\Sigma x^2 = 382500 - 40^2 - 27^2 + 45^2 + 72^2$

$$\Sigma x^2 = 382500 - \underline{1600 - 729} + \underline{2025 + 5184}$$

$$= 382500 - 2329 + 7209$$

$$= 389709 - 2329$$

$$\Sigma x^2 = 387380 \quad [\text{correct } x^2]$$

$$\therefore \text{Correct } \sigma = \sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2}$$

$$= \sqrt{\frac{387380}{100} - (60.5)^2}$$

$$= \sqrt{3873.8 - 3660.25}$$

$$= \sqrt{213.55}$$

$$\therefore \boxed{\sigma = 14.6}$$

$$\begin{array}{r} 60.5 \\ \times\ 60.5 \\ \hline 3025 \\ 000 \\ 3630 \\ \hline 3660.25 \end{array}$$

$$\begin{array}{r} 3873.8 \\ 3660.25 \\ \hline 213.55 \end{array}$$

$$\begin{array}{r|l} & 14.6 \\ 1 & 213.55 \\ & 1 \\ \hline 24 & 113 \\ & 96 \\ \hline 286 & 1755 \\ & 1716 \end{array}$$

$\therefore$ Correct Mean $\bar{x} = 60.5$

Correct S.D $\sigma = 14.6$

(15) The mean and Variance of Seven Observations are 8 and 16 respectively. If five of these are 2, 4, 10, 12 and 14. then find the remaining two Observation.

Sol: -

x	x^2
2	4
4	16
10	100
12	144
14	196
a	a^2
b	b^2
$\Sigma x = 42 + a + b$	$\Sigma x^2 = 460 + a^2 + b^2$

Given: $\boxed{\bar{X} = 8}$, $\boxed{\text{Variance } \sigma^2 = 16}$

$$\bar{X} = 8$$

$$\frac{\Sigma x}{n} = 8$$

$$\frac{42+a+b}{7} \xrightarrow{} 8$$

$$\begin{array}{r} 56 \\ -42 \\ \hline 14 \end{array}$$

$$42 + a + b = 56$$

$$a + b = 56 - 42$$

$$\boxed{a+b=14} \longrightarrow ①$$

$$\sigma^2 = 16$$

$$\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2 = 16$$

$$\frac{460+a^2+b^2}{7} - (8)^2 = 16$$

$$\frac{460+a^2+b^2}{7} - 64 = 16$$

$$\frac{460+a^2+b^2}{7} = 16+64$$

$$\frac{460+a^2+b^2}{7} = 80$$

$$460+a^2+b^2 = 80\times 7$$

$$460+a^2+b^2 = 560$$

$$a^2+b^2 = 560-460$$

$$a^2+b^2 = 100$$

We know, $(a+b)^2 = a^2+b^2+2ab$

$$\therefore (a+b)^2 - 2ab = a^2+b^2$$

$$\therefore a^2+b^2 = (a+b)^2 - 2ab$$

$\therefore (a+b)^2 - 2ab = 100$ Sub ①

$(14)^2 - 2ab = 100$

$196 - 2ab = 100$

$-2ab = 100 - 196$

$+2ab = \cancel{-}96$

$ab = \frac{\cancel{96}^{48}}{\cancel{2}_{1}}$

$\boxed{ab = 48}$

$a = \frac{48}{b} \longrightarrow$ ②

Put ② in ①

48

$-6 \; -8$

$\frac{48}{b} + b = 14$

$48 + b^2 = 14b$

$b^2 - 14b + 48 = 0$

$(b-6)(b-8) = 0$

$b - 6 = 0 \quad \| \quad b - 8 = 0$

$\boxed{b = 6} \quad \| \quad \boxed{b = 8}$

Put b Value in ②

$$\text{If } b=6 \Rightarrow a=\frac{48}{6} \Rightarrow 8$$

$$b=8 \Rightarrow a=\frac{48}{8}=6$$

$$\therefore \boxed{\begin{matrix} a=8 \\ b=6 \end{matrix}} \qquad \boxed{\begin{matrix} a=6 \\ b=8 \end{matrix}}$$

Coefficient of Variation

$$\boxed{C.V = \frac{\sigma}{\overline{x}} \times 100\%}$$

Exercise - 8.2

① The Standard deviation and mean of a data are 6.5 and 12.5 respectively. Find the coefficient of Variation.

Sol:-

$$\sigma = 6.5$$

$$\overline{x} = 12.5$$

$$C.V = \frac{\sigma}{\overline{x}} \times 100\%$$

$$= \frac{6.5}{12.5} \times 100$$

$$= \frac{6.5}{12.5} \times \frac{10}{10} \times 100$$

To remove decimal point multiply by 10

$$= \frac{\overset{13}{\cancel{65}}}{\underset{\underset{1}{\cancel{25}}}{\cancel{125}}} \times \overset{4}{\cancel{100}}$$

$C.V = 13 \times 4 \ \%$

$$\boxed{C.V = 52\%}$$

---

② The Standard deviation and Coefficient of Variation of a data are 1.2 and 25.6 respectively. Find the Value of mean.

Sol

$S.D = \sigma = 1.2$

$C.V = 25.6$

$\bar{x} = \text{mean} = ?$

$C.V = 25.6$

$$\frac{\sigma}{\bar{x}} \times 100 = 25.6$$

$$\frac{1.2}{\bar{x}} \times 100 \overset{\longleftrightarrow}{=} 25.6$$

$$\frac{1.2}{25.6} \times 100 = \bar{x}$$

Multiply by 10
Remove Decimal

$$\frac{1.2}{25.6} \times \frac{10}{10} \times 100 = \bar{x}$$

$$\frac{\overset{3}{\cancel{12}}}{\underset{\underset{16}{\cancel{64}}}{\cancel{256}}} \times \overset{25}{\cancel{100}} = \bar{x}$$

$$\bar{x} = \frac{3 \times 25}{16}$$

$$= \frac{75}{16}$$

$$\boxed{\bar{x} = 4.7}$$

$$\begin{array}{r} 4.\overline{68} \\ 16\overline{)\,75} \\ 64 \\ \hline 110 \\ 96 \\ \hline 140 \\ 128 \\ \hline 12 \end{array}$$

---

③ If the mean and Coefficient of Variation of a data are 15 and 48 respectively. then find the Value of Standard deviation

Sol:-

Mean = $\bar{x}$ = 15

Coefficient of Variation = 48

S.D σ = ?

$$C.V = 48$$

$$\frac{\sigma}{\bar{x}} \times 100 = 48$$

$$\frac{\sigma}{15} \times 100 = 48 \longrightarrow$$

$$\sigma = \frac{\overset{24}{\cancel{48}} \times \cancel{15}^{\,3}}{\underset{\underset{\cancel{20}\ 10}{}}{\cancel{100}}}$$

$$= \frac{24 \times 3}{10}$$

$$= \frac{72}{10}$$

$$\boxed{\sigma = 7.2}$$

---

④ If $n = 5$, $\bar{x} = 6$, $\Sigma x^2 = 765$ then Calculate the Coefficient of Variation

Sol:-

$n = 5$

$\bar{x} = 6$

$x^2 = 765$

$\sigma = ?$

$C.V = ?$

$$\sigma = \sqrt{\frac{\Sigma x^2}{n} - (\bar{x})^2}$$

$$= \sqrt{\frac{\cancel{765}^{\,153}}{\cancel{5}_{\,1}} - (6)^2}$$

$$= \sqrt{153 - 36}$$

$$= \sqrt{117}$$

153
-36
117

10.81
$1 \mid 117$
1
$208 \mid 0\,1700$
1664
$2161 \mid 3600$
2161

$$\boxed{\sigma = 10.81}$$

$$C.V = \frac{\sigma}{\bar{x}} \times 100$$

$$= \frac{10.81}{6} \times 100$$

$$= \frac{1081.0}{6}$$

$$\boxed{C.V = 180.16\%}$$

$$\begin{array}{r} 180.16 \\ 6\overline{)1081} \\ \underline{6} \\ 48 \\ \underline{48} \\ 10 \\ \underline{6} \\ 40 \\ 36 \end{array}$$

---

⑤ Find the Coefficient of Variation of

24, 26, 33, 37, 29, 31

$$\bar{x} = \frac{\Sigma x}{n}$$

Sol 24, 26, 33, 37, 29, 31

$$\bar{x} = \frac{24 + 26 + 33 + 37 + 29 + 31}{6}$$

$$= \frac{180}{6}$$

$$\boxed{\bar{x} = 30}$$

$$\sigma = \sqrt{\frac{d^2}{n}}$$

x	$\bar{x} = 30$ $d = x - \bar{x}$	d^2
24	$24 - 30 = -6$	36
26	$26 - 30 = -4$	16
33	$33 - 30 = 3$	9
37	$37 - 30 = 7$	49
29	$29 - 30 = -1$	1
31	$31 - 30 = 1$	1
		112

$$\therefore \quad \sigma = \sqrt{\frac{112}{6}}$$

$$= \sqrt{18{\cdot}66}$$

$$\boxed{\sigma = 4{\cdot}32}$$

$$C{\cdot}V = \frac{\sigma}{\bar{x}} \times 100$$

$$= \frac{\overset{1{\cdot}44}{\cancel{4{\cdot}32}}}{\underset{1}{\cancel{30}}} \times 10\cancel{0}$$

$$= 1{\cdot}44 \times 10$$

$$\boxed{C{\cdot}V = 14{\cdot}4\ \%}$$

```
   18·66
6)112
  6
  52
  48
   40
   36
    40
    36
```

```
        4·319 (5)
  4   | 18·66
      | 16
 83   |  266
      |  249
 861  |   1700
      |    861
 8629 |    83900
      |    79661
```

⑥ Time taken to complete a home work by 8 students in a day are given by 38, 40, 47, 44, 46, 43, 49, 53 (Time in minute) Find the coefficient of Variation.

Sol 38, 40, 47, 44, 46, 43, 49, 53

$$\bar{X} = \frac{\Sigma x}{n}$$

$$= \frac{38+40+47+44+46+43+49+53}{8}$$

$$= \frac{360^{45}}{\cancel{8}}$$

$$\boxed{\bar{X} = 45}$$

x	$\bar{x} = 45$ $d = x - \bar{x}$	d^2
38	$38-45 = -7$	49
40	$40-45 = -5$	25
47	$47-45 = 2$	4
44	$44-45 = -1$	1
46	$46-45 = 1$	1
43	$43-45 = -2$	4
49	$49-45 = 4$	16
53	$53-45 = 8$	64
Total		164

$$\sigma = \sqrt{\frac{\Sigma d^2}{n}}$$

$$= \sqrt{\frac{164}{8}}$$

$$= \sqrt{20.5}$$

$$\boxed{\sigma = 4.527}$$

20.5
8) 164
16
40
40
0

4.527
4 | 20.5
16
85 | 450
425
902 | 2500
1804
9047 | 69600
63329

$$C.V = \frac{\sigma}{\bar{x}} \times 100$$

$$= \frac{4.527}{45} \times 100$$

$$= \frac{452.7}{45}$$

$$\boxed{C.v = 10.06\ \%}$$

10.06
45) 452.7
- 450
270
- 270
0

---

(7) The total marks Scored by two Students Sathya and Vidhya in 5 Subjects are 460 and 480 with Standard deviation 4.6 and 2.4 respectively who is more consistent in performance?

Sol

More Consistent = less C.V Value

Sathya

$n = 5$

Total marks $= \Sigma x = 460$

$\sigma = 4.6$

$\bar{x} = \dfrac{\Sigma x}{n}$

$= \dfrac{\overset{92}{\cancel{460}}}{\underset{1}{\cancel{5}}}$

$\boxed{\bar{x} = 92}$

$C.V = \dfrac{\sigma}{\bar{x}} \times 100\ \%$

$= \dfrac{4.6}{92} \times 100$

$= \dfrac{\overset{5}{\cancel{460}}.0}{\cancel{92}}$

$\boxed{C.V = 5\%}$

Vidhya

$n = 5$

$\Sigma x = 480$

$\sigma = 2.4$

$\bar{x} = \dfrac{\Sigma x}{n}$

$= \dfrac{\overset{96}{\cancel{480}}}{\underset{1}{\cancel{5}}}$

$\boxed{\bar{x} = 96}$

$C.V = \dfrac{\sigma}{\bar{x}} \times 100\ \%$

$= \dfrac{2.4}{96} \times 100$

$= \dfrac{\overset{2.5}{\cancel{240}}.0}{\underset{1}{\cancel{96}}}$

$\boxed{C.V \doteq 2.5\%}$

$\therefore$ Vidhya is more Consistent

(8) The mean and standard deviation of marks obtained by 40 students of a class in three subjects Mathematics, Science and Social Science are given below. Which of the three subjects shows more consistent and which shows less consistent in Marks.

Subject	Mean	S.D
Mathematics	56	12
Science	65	14
Social Science	60	10

Sol:-

Maths

$\bar{x} = 56$

$\sigma = 12$

$CV = \frac{\sigma}{\bar{x}} \times 100$

$= \frac{\cancel{12}^{\cancel{6}\,3}}{\cancel{56}_{\cancel{28}\,\cancel{14}\,7}} \times \cancel{100}^{50}$

$= \frac{3 \times 50}{7}$

$= \frac{150}{7}$

$$\begin{array}{r} 21.4 \\ 7\overline{)150} \\ 14 \\ \hline 10 \\ 7 \\ \hline 30 \end{array}$$

$\boxed{C.V = 21.4}$

Science

$\bar{x} = 65$

$\sigma = 14$

$C.V = \frac{\sigma}{\bar{x}} \times 100$

$= \frac{14}{\cancel{65}_{13}} \times \cancel{100}^{20}$

$= \frac{14 \times 20}{13}$

$= \frac{280}{13}$

$$\begin{array}{r} 21.5 \\ 13\overline{)280} \\ 26 \\ \hline 20 \\ 13 \\ \hline 70 \\ 65 \\ \hline 5 \end{array}$$

$\boxed{C.V = 21.5}$

Social Science

$\bar{x} = 60$

$\sigma = 10$

$C.V = \frac{\sigma}{\bar{x}} \times 100$

$= \frac{\cancel{10}^{5}}{\cancel{60}_{3}} \times 10\cancel{0}$

$= \frac{50}{3}$

$$\begin{array}{r} 16.6 \\ 3\overline{)50} \\ 3 \\ \hline 20 \\ 18 \\ \hline 20 \end{array}$$

$\boxed{C.V = 16.6}$

∴ Social Science is more consistent

Science is less consistent.

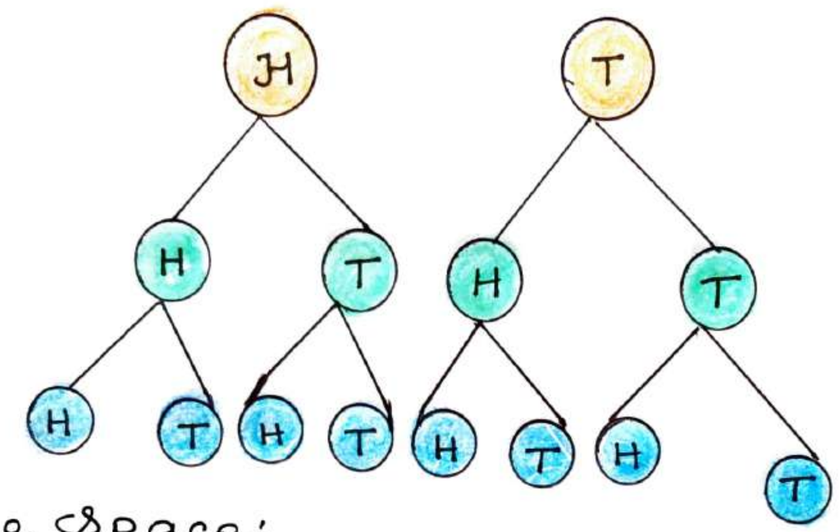
CHAPTER-8

PROBABILITY

EXERCISE 8.3

1) Write the sample space for tossing three coins using tree diagram.

Three Coins



Sample Space:

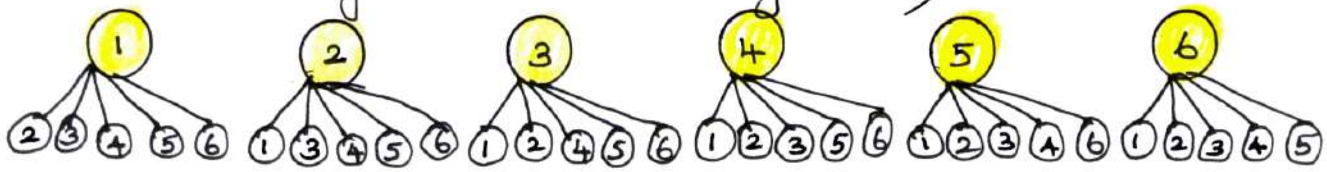
$$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$$

$n(S) = 8$

---

2) Write the sample space for selecting two balls from a bag containing 6 balls numbered 1 to 6

(using tree diagram)



SAMPLE SPACE:

$S = \{(1,2)\ (1,3)\ (1,4)\ (1,5)\ (1,6)$
$(2,1)\ (2,3)\ (2,4)\ (2,5)\ (2,6)$
$(3,1)\ (3,2)\ (3,4)\ (3,5)\ (3,6)$
$(4,1)\ (4,2)\ (4,3)\ (4,5)\ (4,6)$
$(5,1)\ (5,2)\ (5,3)\ (5,4)\ (5,6)$
$(6,1)\ (6,2)\ (6,3)\ (6,4)\ (6,5)\}$

$n(S) = 30$

---

3) If A is an event of a random experiment such that $P(A) : P(\bar{A}) = 17:15$ and $n(S) = 640$, then Find (i) $P(\bar{A})$ (ii) $n(A)$

GIVEN: $P(A) : P(\bar{A}) = 17:15$ $\boxed{n(S) = 640}$

$\Rightarrow P(A) = 17x$
$P(\bar{A}) = 15x$

We know, $P(A) + P(\bar{A}) = 1$

$$17x + 15x = 1$$

$$32x = 1$$

$$\boxed{x = \frac{1}{32}}$$

$\Rightarrow P(A) = 17x = 17\left(\frac{1}{32}\right) = \frac{17}{32}$

Find:

(i) $\boxed{P(\bar{A})} = 15x = 15\left(\frac{1}{32}\right) \boxed{= \frac{15}{32}}$

(ii) $n(A) = ?$

$$P(A) = \frac{n(A)}{n(S)}$$

$$\frac{17}{32} = \frac{n(A)}{640}$$

$$\frac{n(A)}{640} = \frac{17}{32}$$

$$n(A) = \frac{\overset{20}{\cancel{640}} \times 17}{\cancel{32}}$$

$$n(A) = 20 \times 17$$

$$n(A) = 340$$

---

4) A coin is tossed thrice. What is the probability of getting two consecutive tails?

A coin tossed thrice:

$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

$n(S) = 8$

Let A be the probability of getting two consecutive tails.

$A = \{HTT, TTH, TTT\}$

$n(A) = 3$

$$P(A) = \frac{n(A)}{n(S)} = \frac{3}{8}$$

5) At a fete, cards bearing numbers 1 to 1000, one number on one card are put in a box. Each player selects one card at random and that card is not replaced. If the selected card has a perfect square number greater than 500, the player wins a prize. What is the probability that

(i) the first player wins a prize.

(ii) the second player wins a prize, if the first has won?

Cards

$$S = \{1, 2, 3, 4, \cdots 1000\}$$

$$n(S) = 1000$$

Selected card has a perfect square numbers greater than 500

$22 \times 22 = 484$ ×

$28 \times 23 = 529$ ✓

$24 \times 24 = 576$ ✓

$25 \times 25 = 625$ ✓

$26 \times 26 = 676$ ✓

$27 \times 27 = 729$ ✓

$28 \times 28 = 784$

$29 \times 29 = 841$

$30 \times 30 = 900$

$31 \times 31 = 961$

$32 \times 32 = 1024$ ×

(i) Let A be the probability that the first player wins a prize.

$A = \{23^2, 24^2, 25^2, 26^2, 27^2, 28^2, 29^2, 30^2, 31^2\}$

$A = \{529, 576, 625, 676, 729, 784,$
$841, 900, 961\}$

$n(A) = 9$

$P(A) = \frac{n(A)}{n(S)} = \frac{9}{1000}$.

(ii) Let B be the probability that the second player wins a prize.

$n(B) = 8$

$P(B) = \frac{n(B)}{n(S)} = \frac{8}{999}$.

---

6) A bag contains 12 blue balls and x red balls. If one ball is drawn at random (i) What is the probability that it will be a Red ball?

(ii) If 8 more red balls are put in the bag, and if the probability of drawing a red ball will be twice that of the probability in (i), then find x.

Blue balls $= 12$

Red balls $= x$

$\therefore n(S) = 12 + x$

(i) Let A be the probability that it will be a Red ball.

$$n(A) = x$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{x}{12+x}$$

$$\Rightarrow \boxed{P(A) = \frac{x}{12+x}} \longrightarrow ①$$

(ii) 8 more Red balls are put

$$\Rightarrow \text{Red balls} = x+8$$

$$\therefore n(S) = 12 + x + 8$$

$$\boxed{n(S) = 20 + x}$$

Let B be the probability that it will be a New Red ball.

$$n(B) = x + 8$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{x+8}{20+x}$$

$$\Rightarrow \boxed{P(B) = \frac{x+8}{20+x}} \longrightarrow ②$$

Given:

Probability of Red ball is twice that of probability in (i)

$$\Rightarrow \frac{x+8}{20+x} = 2\left(\frac{x}{12+x}\right)$$

$$\frac{x+8}{20+x} = \frac{2x}{12+x}$$

$$(x+8)(12+x) = 2x(20+x)$$

$$12x + x^2 + 96 + 8x = 40x + 2x^2$$

$$\underbrace{2x^2 - x^2} + 40x \underbrace{-12x - 8x} - 96 = 0$$

$$x^2 + 40\underbrace{x - 20}x - 96 = 0$$

$$x^2 + 20x - 96 = 0$$

$$(x-4)\ (x+24) = 0$$

-96

$-4 \quad 24$

$x - 4 = 0$ ‖ $x + 24 = 0$

$\boxed{x = 4}$ ✓ ‖ $\boxed{x = -24}$

(omitting -ve value)

$$\therefore ① \Rightarrow P(A) = \frac{x}{12+x}$$

$$= \frac{4}{12+4}$$

$$= \frac{\cancel{4}^{1}}{\cancel{16}_{4}}$$

$$\Rightarrow \boxed{P(A) = \frac{1}{4}}$$

---

7) Two unbiased dice are rolled once. Find the probability of getting

(i) a doublet (equal numbers on both dice)

(ii) the product as a prime number.

(iii) the sum as a prime number.

(iv) the sum as 1.

TWO DICE:

$S = \{(1,1)\ (1,2)\ (1,3)\ (1,4)\ (1,5)\ (1,6)$

$(2,1)\ (2,2)\ (2,3)\ (2,4)\ (2,5)\ (2,6)$

$(3,1)\ (3,2)\ (3,3)\ (3,4)\ (3,5)\ (3,6)$

$(4,1)\ (4,2)\ (4,3)\ (4,4)\ (4,5)\ (4,6)$

$(5,1)\ (5,2)\ (5,3)\ (5,4)\ (5,5)\ (5,6)$

$(6,1)\ (6,2)\ (6,3)\ (6,4)\ (6,5)\ (6,6)\}$

$n(S) = 36$

(i) Let A be the probability of getting a doublet (equal numbers on both dice)

$A = \{(1,1)\ (2,2)\ (3,3)\ (4,4)\ (5,5)\ (6,6)\}$

$n(A) = 6$

$$P(A) = \frac{n(A)}{n(S)} = \frac{6}{36}$$

(ii) Let B be the probability of getting the product as a prime number.

$B = \{(1,2)\ (1,3)\ (1,5)\ (2,1)\ (3,1)\ (5,1)\}$

$n(B) = 6$

$$P(B) = \frac{n(B)}{n(S)} = \frac{6}{36}$$

(iii) Let C be the probability of getting the sum as a prime number.

$C = \{(1,1)\ (1,2)\ (1,4)\ (1,6)\ (2,1)\ (2,3)\ (2,5)$

$(3,2)\ (3,4)\ (4,1)\ (4,3)\ (5,2)\ (5,6)\ (6,1)\ (6,5)\}$

$n(C) = 15$

$$P(C) = \frac{n(C)}{n(S)} = \frac{15}{36}$$

(iv) Let D be the probability of getting the sum as 1

$n(D) = 0$

$$P(D) = \frac{n(D)}{n(S)} = \frac{0}{36} = 0.$$

---

8) Three fair Coins are tossed together. Find the probability of getting
(i) all heads (ii) atleast one tail
(iii) atmost one head (iv) atmost two tails.

Three Coins

S = {HHH, TTT, HHT, TTH, HTT, THH, HTH, THT}

$n(S) = 8$

(i) Let A be the probability of getting all heads.

A = {HHH}

$n(A) = 1$

$$P(A) = \frac{n(A)}{n(S)} = \frac{1}{8}$$

(ii) Let B be the probability of getting atleast one tail. ↑

B = {HHT, THH, HTH, TTH, HTT, THT, TTT}

$n(B) = 7$

$P(B) = \frac{n(B)}{n(S)} = \frac{7}{8}$

(iii) Let C be the probability of getting atmost one head. ↓

$C = \{TTH, HTT, THT, TTT\}$

$n(C) = 4$

$P(C) = \frac{n(C)}{n(S)} = \frac{4}{8}$

(iv) Let D be the probability of getting atmost two tails ↓

$D = \{HTT, THT, TTH, THH, HHT, HTH, HHH\}$

$n(D) = 7$

$P(D) = \frac{n(D)}{n(S)} = \frac{7}{8}$

---

9) A bag contains 5 red balls, 6 white balls, 7 green balls, 8 black balls. One ball is drawn at random from the bag. Find the probability that the ball drawn is (i) white (ii) black or red (iii) not white (iv) neither white nor black

Red balls = 5

White balls = 6

Green balls = 7

Black balls = 8

$n(S) = \underline{26}$

(i) Let A be the probability that the ball drawn is white.

$$n(A) = 6$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{6}{26}$$

(ii) $\boxed{\text{BLACK OR RED}}$

$$\boxed{A} \quad \cup \quad \boxed{B}$$

Let A be the probability that the ball drawn is black.

$$n(A) = 8$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{8}{26}$$

Let B be the probability that the ball drawn is Red.

$$n(B) = 5$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{5}{26}$$

$$\boxed{A \cap B = 0}$$

$$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{8}{26} + \frac{5}{26}$$

$$= \frac{8+5}{26}$$

$$\boxed{P(A \cup B) = \frac{13}{26}}$$

(iii) Not white:

Let A be the probability that the ball drawn is not white.

$$n(A) = 5+7+8 = 20$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{20}{26}$$

(iv) neither $\underset{Ⓐ}{\text{white}}$ nor $\underset{Ⓑ}{\text{black}}$:

Let A be the probability that the ball drawn is white.

$$n(A) = 6$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{6}{26}$$

Let B be the probability that the ball drawn is black.

$$n(B) = 8$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{8}{26}$$

$$A \cap B = 0$$

$$\therefore P(A \cup B) = P(A) + P(B)$$

$$= \frac{6}{26} + \frac{8}{26}$$

$$P(A \cup B) = \frac{14}{26}$$

Neither nor:

$$P(A \cup B)' = 1 - P(A \cup B)$$

$$= 1 - \frac{14}{26}$$

$$\nwarrow$$

$$= \frac{26-14}{26} = \frac{12}{26}$$

10) In a box there are 20 non-defective and some defective bulbs. If the Probability that a bulb selected at random from the box found to be defective is $\frac{3}{8}$ then, find the number of defective bulbs.

Non-defective bulbs = 20 $\quad [n(A)] = 20$

defective bulbs = ? $\quad [n(\bar{A})] = x$

$n(s) = 20 + x$

Probability of defective $= \frac{3}{8}$

$\Rightarrow \boxed{P(\bar{A}) = \frac{3}{8}}$

We Know,

$$P(A) + P(\bar{A}) = 1$$

$$\frac{n(A)}{n(s)} + \frac{3}{8} = 1$$

$$\frac{20}{n(s)} + \frac{3}{8} = 1$$

$$\frac{20}{n(s)} = 1 - \frac{3}{8}$$

$$\frac{20}{n(s)} = \frac{8-3}{8}$$

$$\frac{\cancel{20}^{4}}{n(s)} = \frac{\cancel{5}^{1}}{8}$$

$$32 = n(s)$$

$$\boxed{n(s) = 32}$$

∴ Number of defective bulbs

$$\Rightarrow n(\bar{A}) = n(S) - n(A)$$

$$n(\bar{A}) = 32 - 20$$

$$\boxed{n(\bar{A}) = 12}$$

---

11) The King and queen of diamonds, queen and jack of hearts, jack and King of spades are removed from a deck of 52 playing cards and then well shuffled. Now one card is drawn at random from the remaining cards. Determine the probability that the card is (i) a clavor (ii) a queen of red card (iii) a King of black card.

$$\left.\begin{array}{lll} \blacklozenge \;\; \text{Diamond} & \Rightarrow & K, Q = 2 \\ \heartsuit \;\; \text{Hearts} & \Rightarrow & Q, J = 2 \end{array}\right\} \text{Red colour}$$

$$\begin{array}{lll} \spadesuit \;\; \text{Spades} & \Rightarrow & J, K = \underline{2} \quad \text{Black Colour} \\ & & \quad\;\; \underline{6} \end{array}$$

$$\boxed{n(S) = 52 - 6 = 46}$$

(i) Let 'A' be the probability that the card is a clavor.

$$n(A) = 13$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{13}{46}$$

(ii) Let 'B' be the probability that the card is a queen of Red card.

$n(B) = 0$

$P(B) = \frac{n(B)}{n(s)} = \frac{0}{46} = 0$

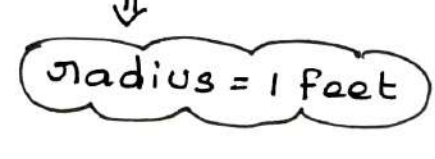
(iii) Let 'c' be the probability that the card is a King of black card.

$n(c) = 1$

$P(c) = \frac{n(c)}{n(s)} = \frac{1}{46}$

---

12) Some boys are playing a game, in which the stone thrown by them landing in a circular region (given in the figure) is considered as win and landing other than the circular region is considered as loss. What is the probability to win the game? $(\pi = 3.14)$



$\Downarrow$

radius = 1 feet

Area of a Circle $= \pi r^2$

$= 3.14 \times (1)^2$

$= 3.14$ Sq. Feet

Total Area $= l \times b$

$= 4 \times 3$

$= 12$ Sq. Feet

$\Rightarrow \boxed{n(S) = 12} \Leftarrow$

Let A be the probability to win the game

$n(A) = 3.14$

$$P(A) = \frac{n(A)}{n(S)} = \frac{3.14}{12}$$

$$= \frac{3.14}{12} \times \frac{100}{100}$$

$$= \frac{314}{1200}$$

$$\boxed{P(A) = \frac{157}{600}}$$

---

13) Two Customers Priya and Amuthan are visiting a particular shop in the same week (monday to saturday). Each is equally likely to visit the shop on any one day as on another day. What is the probability that both will visit the shop on (i) the same day (ii) different days (iii) consecutive days?

S = { (M, M) (M, T), (M, W) (M, Th) (M, F) (M, S)
(T, M) (T, T) (T, W) (T, Th) (T, F) (T, S)
(W, M) (W, T) (W, W) (W, Th) (W, F) (W, S)
(Th, M) (Th, T) (Th, W) (Th, Th) (Th, F) (Th, S)
(F, M) (F, T) (F, W) (F, Th) (F, F) (F, S)
(S, M) (S, T) (S, W) (S, Th) (S, F) (S, S) }

$n(S) = 36$

(i) Let A be the probability that both will visit the shop on the same day.

$A = \{(M,M)\ (T,T)\ (W,W)\ (Th,Th)\ (F,F)\ (S,S)\}$

$n(A) = 6$

$P(A) = \frac{n(A)}{n(S)} = \frac{6}{36}$

(ii) Let B be the probability that both will visit the shop on different days.

$n(B) = 30$

$P(B) = \frac{n(B)}{n(S)} = \frac{30}{36}$

(iii) Let 'c' be the probability that both will visit the shop on Consecutive days.

$C = \{(M,T)\ (T,W)\ (W,Th)\ (Th,F)\ (F,S)\}$

$n(c) = 5$

$P(c) = \frac{n(c)}{n(S)} = \frac{5}{36}$.

---

14) In a game, the entry fee is ₹150. The game consists of tossing a coin 3 times. Dhana bought a ticket for entry. If one or two heads show, she gets her entry fee back. If she throws 3 heads, she receives double entry fees. otherwise she will lose. Find the probability that she (i) gets double entry fee

(ii) just gets her entry fee

(iii) Loses the entry fee.

A Coin tossed 3 times:

$S = \{HHH, HHT, TTH, HTT, THH, HTH, THT, TTT\}$

$n(S) = 8$

One or two heads → entry fee back

3 heads → double the entry fees

No heads → Loses the entry fee

(i) Let A be the probability that she gets double entry fee.

$A = \{HHH\}$

$n(A) = 1$

$$P(A) = \frac{n(A)}{n(S)} = \frac{1}{8}$$

(ii) Let B be the probability that she just gets entry fee.

$B = \{HHT, TTH, HTT, THH, HTH, THT\}$

$n(B) = 6$

$$P(B) = \frac{n(B)}{n(S)} = \frac{6}{8}$$

(iii) Let C be the probability that she loses the entry fees.

$C = \{TTT\}$

$n(C) = 1$

$$P(C) = \frac{n(C)}{n(S)} = \frac{1}{8}$$

EXERCISE 8.4

1) If $P(A) = \frac{2}{3}$, $P(B) = \frac{2}{5}$ $P(A \cup B) = \frac{1}{3}$. then find $P(A \cap B)$

$$P(A \cap B) = P(A) + P(B) - P(A \cup B)$$

$$= \frac{2}{3} + \frac{2}{5} - \frac{1}{3}$$

$$= \frac{2 \times 5}{3 \times 5} + \frac{2 \times 3}{5 \times 3} - \frac{1 \times 5}{3 \times 5}$$

$$= \frac{10 + 6 - 5}{15}$$

$$= \frac{16 - 5}{15}$$

$$\boxed{P(A \cap B) = \frac{11}{15}}$$

3 | 3, 5, 3
5 | 1, 5, 1
1, 1, 1

L.C.M
$= 3 \times 5$
$= 15$

---

2) A and B are two events such that, $P(A) = 0.42$, $P(B) = 0.48$ and $P(A \cap B) = 0.16$ Find (i) P(not A) (ii) P(not B) (iii) P(A or B)

$P(A) = 0.42$, $P(B) = 0.48$ $P(A \cap B) = 0.16$

(i) P(not A)

$= P(\bar{A})$

$= 1 - P(A)$

$= 1 - 0.42$

$= 0.58$

(ii) P(not B)

$= P(\bar{B})$

$= 1 - P(B)$

$= 1 - 0.48$

$= 0.52$

(iii) $P(A \underset{\downarrow}{\text{or}} B)$

$= P(A \cup B)$

$= P(A) + P(B) - P(A \cap B)$

$= 0.42 + 0.48 - 0.16$

$= 0.90 - 0.16$

$= 0.74.$

---

3) If A and B are two mutually exclusive events of a random experiment and P(not A) = 0.45, P(A∪B) = 0.65, then find P(B).

A and B are mutually exclusive,

P(not A) = 0.45

$\Rightarrow \boxed{P(\bar{A}) = 0.45}$

$P(A \cup B) = 0.65$

$P(A) = 1 - P(\bar{A})$

$= 1 - 0.45$

$\boxed{P(A) = 0.55}$

$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.65 = 0.55 + P(B) - 0$

$0.65 - 0.55 = P(B)$

$0.10 = P(B)$

$P(B) = 0.10$

---

4) The probability that atleast one of A and B occur is 0.6. If A and B occur simultaneously with probability 0.2, then find $P(\bar{A}) + P(\bar{B})$.

P(atleast one of A and B) = 0.6

$$\Downarrow$$

$$\boxed{P(A \cup B) = 0.6}$$

P(A and B occurs simultaneously) = 0.2

$$\Downarrow$$

$$\boxed{P(A \cap B) = 0.2}$$

$$P(A \cup B) = \underset{\Downarrow}{P(A)} + \underset{\Downarrow}{P(B)} - P(A \cap B)$$

$$0.6 = 1 - P(\bar{A}) + 1 - P(\bar{B}) - 0.2$$

$$0.6 = 2 - 0.2 - P(\bar{A}) - P(\bar{B})$$

$$0.6 = 1.8 - P(\bar{A}) - P(\bar{B})$$

$$P(\bar{A}) + P(\bar{B}) = 1.8 - 0.6$$

$$\boxed{P(\bar{A}) + P(\bar{B}) = 1.2}$$

---

5) The probability of happening of an event A is 0.5 and that of B is 0.3. If A and B are mutually exclusive events, then find the probability that neither A nor B happens.

$$\boxed{P(A) = 0.5} \quad \boxed{P(B) = 0.3}$$

A and B are mutually exclusive

$P(A \cup B) = P(A) + P(B)$

$= 0.5 + 0.3$

$\boxed{P(A \cup B) = 0.8}$

To find:

P(neither A nor B happens)

$P(A \cup B)' = 1 - P(A \cup B)$

$= 1 - 0.8$

$P(A \cup B)' = 0.2$

---

6) Two dice are rolled once. Find the Probability of getting an even number on the first die or a total of face sum 8.

TWO DICE:

$S = \{(1,1)\ (1,2)\ (1,3)\ (1,4)\ (1,5)\ (1,6)$
$(2,1)\ (2,2)\ (2,3)\ (2,4)\ (2,5)\ (2,6)$
$(3,1)\ (3,2)\ (3,3)\ (3,4)\ (3,5)\ (3,6)$
$(4;1)\ (4,2)\ (4,3)\ (4,4)\ (4,5)\ (4,6)$
$(5,1)\ (5,2)\ (5,3)\ (5,4)\ (5,5)\ (5,6)$
$(6,1)\ (6,2)\ (6,3)\ (6,4)\ (6,5)\ (6,6)\}$

$n(S) = 36$

Let A be the probability of getting an even number on the first die.

$A = \{(2,1)\ (2,2)\ (2,3)\ (2,4)\ (2,5)\ (2,6)$

(4,1) (4,2) (4,3) (4,4) (4,5) (4,6)

(6,1) (6,2) (6,3) (6,4) (6,5) (6,6)}

$n(A) = 18$

$P(A) = \frac{n(A)}{n(S)} = \frac{18}{36}$

Let B be the probability of getting a total of face sum 8

$B = \{(2,6)\ (3,5)\ (4,4)\ (5,3)\ (6,2)\}$

$n(B) = 5$

$P(B) = \frac{n(B)}{n(S)} = \frac{5}{36}$

$A \cap B = \{(2,6)\ (4,4)\ (6,2)\}$

$n(A \cap B) = 3$

$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{3}{36}$

$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{18}{36} + \frac{5}{36} - \frac{3}{36}$

$= \frac{18+5-3}{36}$

$= \frac{23-3}{36}$

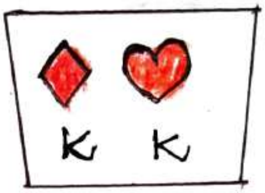
$\boxed{P(A \cup B) = \frac{20}{36}}$

7) From a well-shuffled pack of 52 cards, a card is drawn at random. Find the Probability of it being a red King or a black queen.

Cards:

$n(S) = 52$

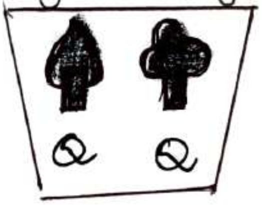
Let A be the probability of getting a Red king.



$n(A) = 2$

$$P(A) = \frac{n(A)}{n(S)} = \frac{2}{52}$$

Let B be the probability of getting a Black queen.



$n(B) = 2$

$$P(B) = \frac{n(B)}{n(S)} = \frac{2}{52}$$

$A \cap B = 0$

$$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{2}{52} + \frac{2}{52} - 0$$

$$= \frac{2+2}{52}$$

$$P(A \cup B) = \frac{4}{52}$$

---

8) A box contains cards numbered 3, 5, 7, 9, ... 35, 37. A card is drawn at random

from the box. Find the probability that the drawn card have either multiples of 7 or a prime number.

$S = \{3, 5, 7, 9, \ldots 35, 37\}$

$n(S) = 18$

Let A be the probability that the drawn card have multiple of 7.

$A = \{\overset{\checkmark}{7}, 21, 35\}$

$n(A) = 3$

$P(A) = \frac{n(A)}{n(S)} = \frac{3}{18}$

Let B be the probability that the drawn card have a prime number.

$B = \{3, 5, \overset{\checkmark}{7}, 11, 13, 17, 19, 23, 29, 31, 37\}$

$n(B) = 11$

$P(B) = \frac{n(B)}{n(S)} = \frac{11}{18}$

$A \cap B = \{7\}$

$n(A \cap B) = 1$

$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{1}{18}$

$$\Rightarrow P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{3}{18} + \frac{11}{18} - \frac{1}{18} = \frac{14-1}{18} = \boxed{\frac{13}{18}}$$

9) Three unbiased coins are tossed once. Find the probability of getting atmost 2 tails or atleast 2 heads.

Three Coins:

$S = \{HHH, TTT, HHT, TTH, THH, HTT, HTH, THT\}$

$n(S) = 8$

Let A be the probability of getting atmost 2 tails ↓

$A = \{TTH, HTT, THT, HHT, THH, HTH, HHH\}$

$n(A) = 7$

$$P(A) = \frac{n(A)}{n(S)} = \frac{7}{8}$$

Let B be the probability of getting atleast 2 heads ↑

$B = \{HHT, THH, HTH, HHH\}$

$n(B) = 4$

$$P(B) = \frac{n(B)}{n(S)} = \frac{4}{8}$$

$A \cap B = \{HHT, THH, HTH, HHH\}$

$n(A \cap B) = 4$

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{4}{8}$$

$$\Rightarrow P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{7}{8} + \cancel{\frac{4}{8}} - \cancel{\frac{4}{8}} = \boxed{\frac{7}{8}}$$

10) The probability that a person will get an electrification contract is $\frac{3}{5}$ and the Probability that he will not get plumbing contract is $\frac{5}{8}$. The probability of getting atleast one contract is $\frac{5}{7}$. What is the Probability that he will get both?

$$P(\text{getting electrification contract}) = \frac{3}{5}$$

$$\Rightarrow \boxed{P(A) = \frac{3}{5}}$$

$$P(\text{not getting plumbing Contract}) = \frac{5}{8}$$

$$\Rightarrow \boxed{P(\bar{B}) = \frac{5}{8}}$$

$$\therefore P(B) = 1 - P(\bar{B})$$

$$= 1 - \underset{\nwarrow}{}\frac{5}{8}$$

$$= \frac{8-5}{8}$$

$$\Rightarrow \boxed{P(B) = \frac{3}{8}}$$

$$P(\text{atleast one contract}) = \frac{5}{7}$$

$$\Rightarrow \boxed{P(A \cup B) = \frac{5}{7}}$$

$\therefore$ P (getting both)

$$\Rightarrow P(A \cap B) = P(A) + P(B) - P(A \cup B)$$

$$= \frac{3}{5} + \frac{3}{8} - \frac{5}{7}$$

$$= \frac{3\times 56}{5\times 56} + \frac{3\times 35}{8\times 35} - \frac{5\times 40}{7\times 40}$$

$$= \frac{168+105-200}{280}$$

$$= \frac{273-200}{280}$$

$$\therefore \boxed{P(A\cap B) = \frac{73}{280}}$$

$$\begin{array}{r|l} 2 & 5,8,7 \\ \hline 2 & 5,4,7 \\ \hline 2 & 5,2,7 \\ \hline 5 & 5,1,7 \\ \hline 7 & 1,1,7 \\ \hline & 1,1,1 \end{array}$$

L.C.M

$= 2\times 2\times 2\times 5\times 7$

$= 280$

---

11) In a town of 8000 people, 1300 are over 50 years and 3000 are females. It is known that 30% of the females are over 50 years. What is the probability that a chosen individual from the town is either a female or over 50 years?

$n(s) = 8000$ (People)

$\boxed{\text{Over 50 years} = 1300}$

$\boxed{\text{females} = 3000}$

30% females are over 50 years

$$\frac{30}{1\cancel{00}} \times 30\cancel{00} = 900$$

$\Rightarrow$ $\boxed{\text{Females over 50 years} = 900}$

Let A be the probability that a chosen individual from the town is a female.

$n(A) = 3000$

$$P(A) = \frac{n(A)}{n(S)} = \frac{3000}{8000}$$

Let B be the probability that a chosen Individual from the town is over 50 yrs.

$$n(B) = 1300$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{1300}{8000}$$

A Females	B 50 years
Below 50 years } = 2100 over 50 years } = 900 ✓ 3000	over 50 yrs male } = 400 over 50 yrs female } = 900 ✓ 1300

∴ $A \cap B$ = female over 50 years

$$n(A \cap B) = 900$$

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{900}{8000}$$

$$\Rightarrow \boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$$

$$= \frac{3000}{8000} + \frac{1300}{8000} - \frac{900}{8000}$$

$$= \frac{3000+1300-900}{8000}$$

$$= \frac{4300-900}{8000} = \frac{34\cancel{0}\cancel{0}}{80\cancel{0}\cancel{0}} = \frac{34}{80}$$

$$\Rightarrow \boxed{P(A \cup B) = \frac{34}{80}}$$

12) A coin is tossed thrice. Find the probability of getting exactly two heads or atleast one tail or two consecutive heads.

A coin tossed thrice:

$S = \{HHH, TTT, HHT, TTH, THH, HTT, HTH, THT\}$

$n(S) = 8$

Let A be the probability of getting exactly two heads.

$A = \{HHT, THH, HTH\}$

$n(A) = 3$

$P(A) = \frac{n(A)}{n(S)} = \frac{3}{8}$

Let B be the probability of getting atleast one tail. ↑

$B = \{TTT, HHT, TTH, THH, HTT, HTH, THT\}$

$n(B) = 7$

$P(B) = \frac{n(B)}{n(S)} = \frac{7}{8}$

Let C be the probability of getting two consecutive heads.

$C = \{HHH, HHT, THH\}$

$n(C) = 3$

$P(C) = \frac{n(C)}{n(S)} = \frac{3}{8}$

$A \cap B = \{HHT, THH, HTH\}$

$n(A \cap B) = 3$

$$\boxed{P(A \cap B)} = \frac{n(A \cap B)}{n(S)} = \boxed{\frac{3}{8}}$$

$B \cap C = \{HHT, THH\}$

$n(B \cap C) = 2$

$$\boxed{P(B \cap C)} = \frac{n(B \cap C)}{n(S)} = \boxed{\frac{2}{8}}$$

$A \cap C = \{HHT, THH\}$

$n(A \cap C) = 2$

$$\boxed{P(A \cap C)} = \frac{n(A \cap C)}{n(S)} = \boxed{\frac{2}{8}}$$

$A \cap B \cap C = \{HHT, THH\}$

$n(A \cap B \cap C) = 2$

$$\boxed{P(A \cap B \cap C)} = \frac{n(A \cap B \cap C)}{n(S)} = \boxed{\frac{2}{8}}$$

$$\Rightarrow \boxed{\begin{aligned} P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) \\ - P(A \cap C) + P(A \cap B \cap C) \end{aligned}}$$

$$= \frac{3}{8} + \frac{7}{8} + \cancel{\frac{3}{8}} - \cancel{\frac{3}{8}} - \frac{2}{8} - \cancel{\frac{2}{8}} + \cancel{\frac{2}{8}}$$

$$= \frac{3}{8} + \frac{7}{8} - \frac{2}{8}$$

$$= \frac{10-2}{8} = \frac{\cancel{8}^{\,1}}{\cancel{8}}$$

$$\Rightarrow \boxed{P(A \cup B \cup C) = 1}$$

13) If A, B, C are any three events such that probability of B is twice as that of probability of A and probability of C is thrice as that of probability of A and if $P(A \cap B) = \frac{1}{6}$, $P(B \cap C) = \frac{1}{4}$, $P(A \cap C) = \frac{1}{8}$, $P(A \cup B \cup C) = \frac{9}{10}$, $P(A \cap B \cap C) = \frac{1}{15}$, then find P(A), P(B) and P(C)?

Given

$$\boxed{\begin{array}{l} P(B) = 2\,P(A) \\ P(C) = 3\,P(A) \end{array}}$$

Find:

P(A), P(B), P(C)?

$P(A \cap B) = \frac{1}{6}$, $P(B \cap C) = \frac{1}{4}$

$P(A \cap C) = \frac{1}{8}$, $P(A \cup B \cup C) = \frac{9}{10}$, $P(A \cap B \cap C) = \frac{1}{15}$

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$$

$$\frac{9}{10} = P(A) + 2\,P(A) + 3\,P(A) - \frac{1}{6} - \frac{1}{4} - \frac{1}{8} + \frac{1}{15}$$

$$\frac{9}{10} = 6\,P(A) - \frac{1}{6} - \frac{1}{4} - \frac{1}{8} + \frac{1}{15}$$

$$6\,P(A) = \frac{9}{10} + \frac{1}{6} + \frac{1}{4} + \frac{1}{8} - \frac{1}{15}$$

$$6\,P(A) = \frac{9 \times 12}{10 \times 12} + \frac{1 \times 20}{6 \times 20} + \frac{1 \times 30}{4 \times 30} + \frac{1 \times 15}{8 \times 15} - \frac{1 \times 8}{15 \times 8}$$

$$6\,P(A) = \frac{\overbrace{108 + 20 + 30 + 15}-8}{120}$$

$$6\,P(A) = \frac{173 - 8}{120}$$

$$6\,P(A) = \frac{165}{120}$$

$$P(A) = \frac{\cancel{165}^{\cancel{33}\;11}}{\cancel{120}_{\cancel{24}\;8} \times 6}$$

$$P(A) = \frac{11}{8 \times 6}$$

$$\Rightarrow \boxed{P(A) = \frac{11}{48}}$$

$$P(B) = 2\,P(A)$$

$$= 2\left(\frac{11}{48}\right)$$

$$\Rightarrow \boxed{P(B) = \frac{22}{48}}$$

$$P(C) = 3\,P(A)$$

$$= 3\left(\frac{11}{48}\right)$$

$$\Rightarrow \boxed{P(C) = \frac{33}{48}}$$

$$\begin{array}{r|l} 2 & 10, 6, 4, 8, 15 \\ \hline 2 & 5, 3, 2, 4, 15 \\ \hline 2 & 5, 3, 1, 2, 15 \\ \hline 3 & 5, 3, 1, 1, 15 \\ \hline 5 & 5, 1, 1, 1, 5 \\ \hline & 1, 1, 1, 1, 1 \end{array}$$

L.C.M

$= 2 \times 2 \times 2 \times 3 \times 5$

$= 120$

---

14) In a class of 35, students are numbered from 1 to 35. The ratio of boys to girls is 4 : 3. The roll numbers of students begin with boys and end with girls. Find the probability that a student selected is either a boy with prime roll number or a girl with

composite roll number or an even roll number.

$$S = \{1, 2, 3, 4, \ldots\ldots 35\}$$

$$\boxed{n(S) = 35}$$

Ratio of boys to Girls = 4:3

$$\therefore \quad 4x + 3x = 35$$

$$7x = 35$$

$$x = \frac{\cancel{35}^{5}}{\cancel{7}}$$

$$\boxed{x = 5}$$

$\Rightarrow$ No. of boys $= 4x = 4(5) = 20$

No. of Girls $= 3x = 3(5) = 15$

$\therefore$ Boys Roll number $= \{1, 2, 3, \ldots\ldots 20\}$

Girls Roll number $= \{21, 22, \ldots\ldots 35\}$

Let A be the probability that a student selected is a boy with prime roll number.

$$A = \{2, 3, 5, 7, 11, 13, 17, 19\}$$

$$n(A) = 8$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{8}{35}$$

Let B be the probability that a student selected is a girl with composite roll number.

$$B = \{21, 22, 24, 25, 26, 27, 28, 30, 32, 33, 34, 35\}$$

$$n(B) = 12$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{12}{35}$$

Let c be the probability that a student selected is an even roll number.

$$C = \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34\}$$

$$n(C) = 17$$

$$P(C) = \frac{n(C)}{n(S)} = \frac{17}{35}$$

$$\boxed{A \cap B = 0}$$

$$B \cap C = \{22, 24, 26, 28, 30, 32, 34\}$$

$$n(B \cap C) = 7$$

$$\boxed{P(B \cap C)} = \frac{n(B \cap C)}{n(S)} = \boxed{\frac{7}{35}}$$

$$A \cap C = \{2\}$$

$$n(A \cap C) = 1$$

$$\boxed{P(A \cap C)} = \frac{n(A \cap C)}{n(S)} = \boxed{\frac{1}{35}}$$

$$\boxed{A \cap B \cap C = 0}$$

$$\Rightarrow P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$$

$$= \frac{8}{35} + \frac{12}{35} + \frac{17}{35} - 0 - \frac{7}{35} - \frac{1}{35} + 0$$

$$= \frac{8}{35} + \frac{12}{35} + \frac{17}{35} - \frac{7}{35} - \frac{1}{35}$$

$$= \frac{\overbrace{8+12+17}-\overbrace{7-1}}{35}$$

$$= \frac{37-8}{35}$$

$$\boxed{P(A \cup B \cup C) = \frac{29}{35}}$$